DESIGN OPTIMIZATION OF SQUARE SKYLIGHTS IN OFFICE BUILDINGS

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ABSTRACT

This research focuses on design optimization of horizontal roof apertures known as skylights in office buildings. It is one of the steps towards design, evaluation, and optimization of roof-daylighting systems in office buildings, which will correlate architectural design features and parameters with illumination quality and quantity and overall energy performance. This research builds on previous work published by the authors that addressed daylighting illumination performance and the lighting electricity reductions achieved through the use of skylights. This study extends the previous work to account for the energy and cost impacts of the thermal effects of the daylighting systems, suggests optimum aperture area and reports savings in energy consumption and operation costs in two distinct climates.

INTRODUCTION

Although skylights have been deployed in building for a long time, their energy performance is still under question. Improper design of skylights often results in: excessive light absorption in light-wells, large variations of the illumination across the task surface, and glare. In addition to the problems caused by these design errors, horizontal apertures are inherently subjected to extreme and sudden changes in the level of solar radiation and incident daylight, which can cause thermal overloading problems.

Previous work by the authors offered a range of promising design suggestions to reduce most of the design errors (Ghobad et al. 2012). It included information about light-well depth, ceiling shape, and aperture spacing. This paper will expand on that previous work by: (1) providing information about performance of horizontal apertures in two different locations with distinct climatic conditions (2) Providing a comprehensive assessment of the energy performance of properly designed horizontal apertures, including thermal effects (3) providing design optimization for square skylights.

The main questions to be addressed in this paper are: (1) How can each design be optimized to reach the best results in terms of daylighting and energy saving? (2) What are the potential savings in building operating energy and operating cost that can be achieved by implementing different designs for horizontal apertures?

BUILDING PARAMETERS

The baseline parameters for the building in this paper are:

- An office space with 30-ft x 30-ft (9.14m x 9.14m) dimension was modeled to represent a section cut from an infinite rooflit open office

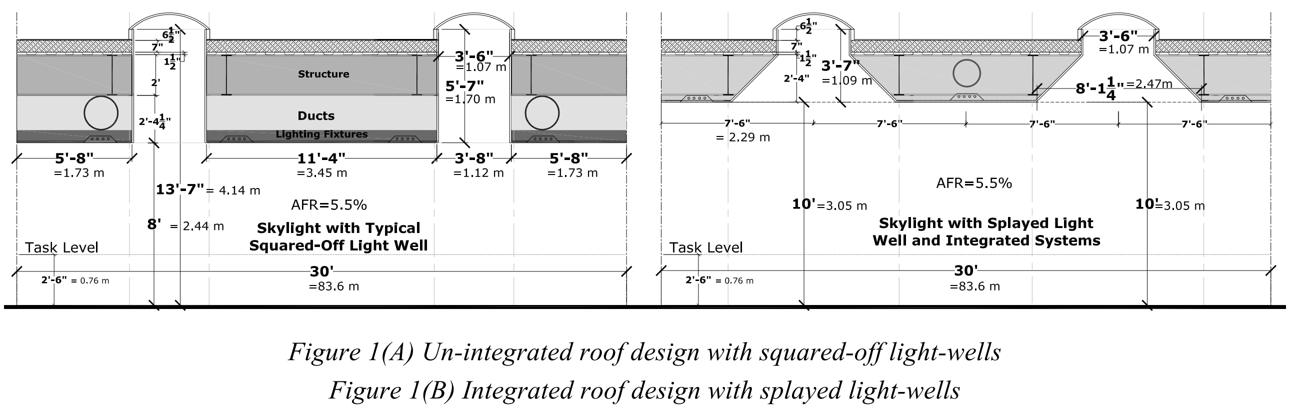

Figure 1(A) Un-integrated roof design with squared-off light-wells
Figure 1(B) Integrated roof design with splayed light-wells

area. To avoid complicating the outputs with wall or partition effects, this space has been surrounded on all sides by eight other identical spaces in the daylighting model. No partitions are assumed in this study because introducing partitions or walls will complicate the analysis and substantially alter the results. This roofing configuration can accommodate some private offices, but it generally lends itself better to open office arrangements, which is what was assumed in this study.

- The height of roof, from finished floor to top of the curb, is 13'-7" (4.14 m) in all cases. Therefore, distance between the lower edge of the glazing to task-level remains constant in all cases. The cases presented in this paper are comparable in terms of building envelope area and overall building height, which is a reasonable basis for a comparison. A previous paper by the authors presented simulation results comparing configurations with a constant ceiling height (Ghobad et al., 2012). The authors believe that many building owners would rather have the larger airier volume than to lower the roof. Furtheromore, admitting the light higher allows fewer apertures to be used, which will reduce the complexity and cost of the construction.
- The model has four square apertures located at the center of each quarter of the space (figure 1 and 2) resulting in a uniformly spaced grid throughout the building.
- Other vertical and horizontal dimensions are shown in Figure 1 and table 1.

The parametric variations in the study are:

1. Building locations: Boston and Miami. These two locations were selected because they represent two substantially different climates in terms of daylight availability and thermal conditions in the United States.
2. Depth and shape of the light-well through which the daylighting is entering:

- The basecase, having a squared-off light-well that is a vertical shaft with a vertical dimension of 5'-7" (1.70 m) and a flat ceiling everywhere between the light-wells (Figure 1A). The deep light-well shaft is a manifestation of the allocation of deep layers to each of the the primary systems in typical roof construction (Ghobad et al. 2012). This deep roof leaves 8' (2.4 m) clearance for the ceiling.
- A system that has been refined for daylighting purposes (Figure 1B), in which:
    - o The ceiling has been splayed outward around the lower edges of the light-well. For nomenclature clarity, we will say that the light-well is the vertical shaft. The sloped surface will be referred to as the sloped portion of the ceiling, having a vertical dimension of 2'-4" (0.71 m) and being set at a slope of 45º.
    - o The structure and the ductwork have been integrated to reduce the vertical dimension of the light-well shaft to 3'-7" (1.09 m). In this configuration, the ceiling height has been increased to 10' (3.04 m) because of the reduced depth of the roof.

3. The glazing area, expressed as the Aperture to Floor area Ratio (AFR):

- 2%, 3.5%, and 5.5%

The flat roof is composed of the following components in all cases:

1. Glazing material composed of two Lexan plastic sheets which are translucent with 58.6% and 71.4% visible transmittance resulting in a final 42% visible transmittance and solar heat gain coefficient of 0.317. The U-value of the double layer Lexan sheets are 2.59 W/m^2K, which meets the American Society of Heating Refrigerating and Air-conditioning Engineers (ASHRAE) Standard 90.1-2010 requirements.

2. Curbs 6 1/2" (0.17 m) high and 2.5" (0.06 m) thick composed of 1.5" (0.04 m) wood and 1" (0.03 m) Styrofoam for insulation purposes. The total U-value of the curb is 0.98 W/m^2K; the total U-value of the assembly is calculated based on an area-weighted average of the components (table 2), which are: glazing, edge effects for the glazing, framing, and curbs. Table 2 shows the results.
3. A layer of rigid insulation 7-inch (0.18 m) thick with U-value of 0.187 W/m^2K to comply with ASHRAE Standard 90.1-2010.
4. A layer of 1.5-inch (0.04 m) corugated steel decking.
5. A structural spanning layer that extends over the entire footprint of the building, with 2-ft (0.61 m) depth to accommodate the deepest spanning member.

Table 1
Dimensions of models

FLOOR AREA ILLUMINATED	AFR	NUMBER OF APERTURES	CLEAR GLAZING EDGE
m^2	%		m
83.6	2	4	0.63
	3.5		0.86
	5.5		1.07

6. An electric-lighting and hung-ceiling layer that extends over the entire footprint of the building, with a depth of 4 inches (0.05 m).

Table 2

Effect of frames and curbs on overall U-values of the glazing

AFR	AREA OF GLAZING IN EACH SKYLIGHT	U AVERAGE OF GLAZING	GLAZING UA	AREA OF CURBS IN EACH SKYLIGH	U AVERAGE OF CURB	CURB UA	OVERALL UA FOR EACH SKYLIGHT	ASSEMBLY AVERAGE U-VALUE
%	m²	W/m²K	W/K	m²	W/m²K	W/K	W/K	W/m²K
2	0.40	3.41	1.35	0.48	0.98	0.47	1.83	4.60
3.5	0.74	3.20	2.37	0.63	0.98	0.62	2.99	4.04
5.5	1.14	3.09	3.54	0.77	0.98	0.75	4.29	3.75

SIMULATION

The analysis is performed in several stages. The roof daylighting models were drawn in Rhinoceros. DIVA-for-Rhino was used for daylighting and whole-building simulation. DIVA 2.0 is a plug-in to Rhino that exports scene geometries, material properties, and sensor grids into the format required to enable the use of Radiance, DAYSIM and EnergyPlus (Lagios et al. 2010). Radiance (Ward, 1994) and DAYSIM (Reinhart and Walkenhorst, 2001) are validated simulation tools for daylighting.

In simulation process, (1) illuminance distribution across the task surface was computed for a single moment in time using Radiance. (2) Annual interior illumination was assessed using DAYSIM. (3) Whole-building energy simulation was performed with EnergyPlus. (4) Operating energy was computed for the different categories of use in the building. (5) Total energy operating costs (in dollars) were calculated for each daylighting system to make comparisons and suggest design optimizations.

Annual interior illumination was assessed by DAYSIM. Electric lighitng schedules, generated in format of Excel CSV files from DAYSIM, were the most important inputs to EnergyPlus to assess electric light savings due to the use of natural light. For single-time simulations, Radiance parameters were: ambient bounces (ab) 8, ambient division (ad) 3600, ambient super-samples (as) 900, ambient resolution (ar) 600, ambient accuracy (aa) 0.05. These parameters were adjusted until smooth curves were achieved and illuminance values converged to consistent results. For annual simulations lower parameters were selected due to much longer time requirement: ab 7, ad 2500, as 625, ar 300, aa 0.05.

The illuminance target in models was 300 lux, which provides suitable lighting condition for computer-based office work (IES). Illuminance levels were collected in a 25x25 grid at task level 2.5' (0.76 m) above the finished floor in Radiance simulation program.

As formerly mentioned, the models represent a section cut from an open office space; the purpose was to allow various sitting arrangements for occupants by providing adequate illuminance at all points of the space. Therefore, a photosensor was located at the center of the model where the least illuminance occured (figure 2). Future research will address more electric lighting zones with additional sensors to control the electric lights more finely to the needs of the various parts of the space.

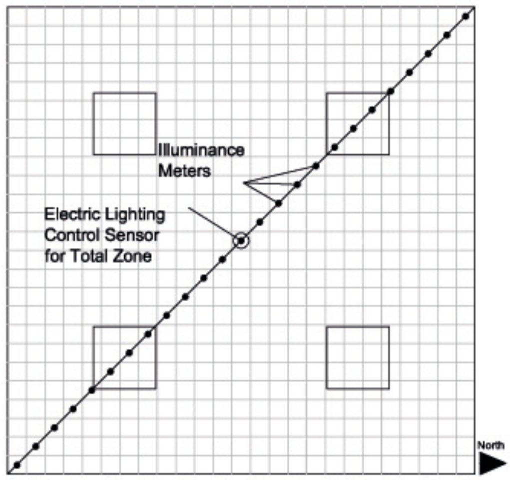

Figure 2 Floor plan with location of illuminance meters and photosensor

Electric lighting was a continuous dimming control system that controled 100% of lighting fixtures in the models. Figure 3 shows the electric power input to lighting fixtures as a function of daylight illuminance at the photosensor's location. Based on the electric lighting power input and electric lighting density, EnergyPlus calculated electric lighting consumption for the interior spaces.

In simulations, the standby power was assumed to be zero rather than the typical 20%-30% power drawn for dimming control for fluorescent luminaires (figure 3). This assumption was made regarding emerging improvements in the field of LED lighting, which draws much lower electric power when enough daylight is available. Studies already in the works by the authors will address fluorescent fixtures, with the appropriate standby power for that technology.

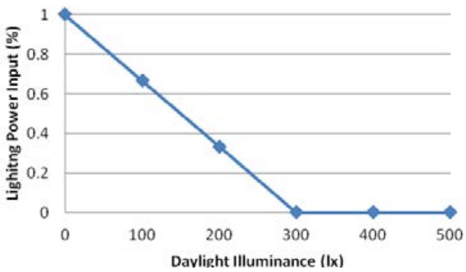

Figure 3 Power input curve of the continuous dimming lighitng control used for DAYSIM schedules

The thermal models were generated along with the daylight models in Rhino, with the same dimensions but less architectural details and in a separate layer. DIVA generates the idf file, which contains geometric information of the models. The idf file is modified in EnegyPlus 7.0.0.036 and further parameters such as construction materials, internal loads, operation schedules, and HVAC system were inserted.

All the 30' (9.14 m) x 30' (9.14 m) modules were simulated as single thermal zones with four adiabatic walls and an adiabatic ground, representing an interior space of a large well-insulated rooflit office building. The installed lighting power density for the building was modelled as 9.68 Watt/m^2 based on ASHRAE 90.1-2010 for commercial buildings. Office equipment for each module was composed of four computers, a printer, a scanner and a copier, resulting in 886 watts heat generation. Four people occupied each thermal zone during weekdays from 9 am until 5 pm and required total of 0.0378 (cubic meter per second) ventilation (ASHRAE 90.1-2010). No air infiltration existed in the models because of having four adiabatic walls.

The HVAC system was composed of the following components: outdoor air mixing box, AC unit (cooling coil), gas furnace, humidifier, fan, air splitter, air terminal with reheat, and mixing box. The cooling system employed a direct-expansion DX cooling coil with single speed. The AC system used electricity with COP (Coefficient of Performance) of 3. The heating system was a natural gas furnace with COP of 0.8.

Table 3 shows heating and cooling setpoint temperatures. The thermostat performed based on operative temperature because it is a more accurate indicator for thermal comfort rather than mean air temperature (ASHRAE 55-2010). For the purposes of this study, operative temperature was defined as:

$$T_{opt} = 0.55_{\ MRT} + 0.45_{\ Mean\ Air\ Temp} \quad (1)$$

which corresponds to a still air situation. For situations with somewhat more air movement, a weighting of 50-50 between MRT and mean air temperature is commonly used.

Table 3

Heating and cooling setpoints

HEATING SETPOINT		**COOLING SETPOINT**	
0:00- 4:00	17 °C	0:00- 5:00	32 °C
4:00- 19:00	22 °C	5:00- 20:00	24.5 °C
20:00-24:00	17 °C	20:00- 24:00	32 °C

RESULTS AND DISCUSSION

Daylighting

Figures 4 shows Illuminance levels in modules with 5.5% AFR square apertures received at 25 sensors located on a diagonal axis at task surface (see the plan view in figure 2). The illuminance distribution across the task surface was computed for 12 pm September 21 with sunny sky condition. In figure 4, the horizontal axis represents number of sensors and vertical axis depicts illuminance levels in lux.

Single-time illuminance simulations reveal that average illuminance levels are higher in horizontal apertures located in Miami than the ones located in Boston. Average illuminance in the diagonal axis of task surface in a space roofed with square apertures with 5.5% AFR in Boston is about 80% of the average illuminance of the same room in Miami. This fact is due to higher solar altitude angles at equinox noon in Miami (90°) compared to Boston (71°). Miami is located on 25° North latitude and Boston is located on 42° North latitude.

Not surprisingly, the base-case configuration with the deep, squared-off light-well is the poorest performer both in terms of the low amount of light reaching the task surface and the extreme variations in illuminance levels. The low quantity of light is attributable to the high numbers of bounces and the high absorption of light on the surfaces of the light-well. The high variations in the illuminance on the task surface are attributable to the light-well selecting against low-angle light and easily passing the light rays moving nearly vertically down through the light-well (Ghobad et al. 2012). This tends to create high illuminance directly below the skylights and relative darkness between the skylights. Average illuminance in square apertures with sloped and integrated light-wells is 1.8 times the average illuminance in un-integrated and squared-off light-wells.

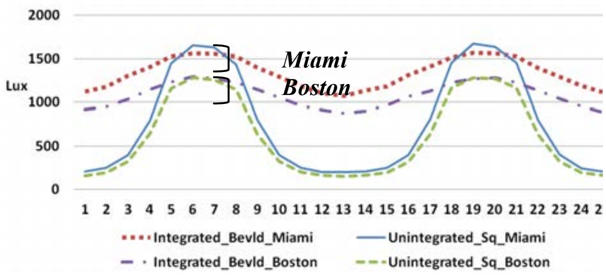

Figure 4 Illuminance distribution [lux] in 5.5% AFR

It is understood that with the use of less vertical dimension and sloped light-wells, less light fluctuations occur in the space. The problems caused by high variations in the daylight illuminance level have been previously discussed (Ghobad et al. 2012). Two important problems are increased level of electric lighting use to provide sufficient light for the spots where lowest daylight illuminance is available and excessive complication to provide uniform electric lighting in such spaces.

Figure 5 compares electric lighting energy use in skylights with un-integrated systems and squared-off light-wells, skylights with integrated systems and beveled light-wells, and the base case, which has no roof apertures.

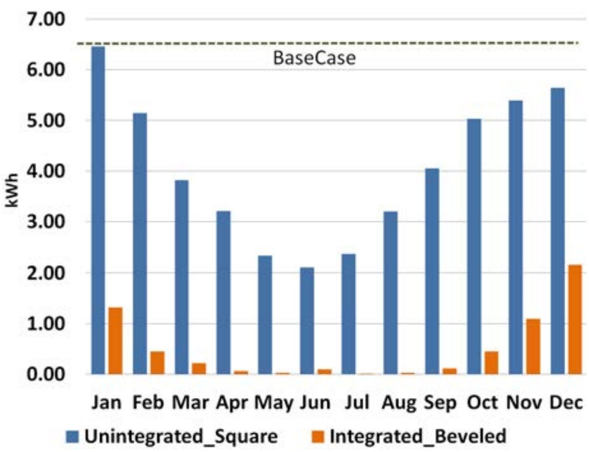

Figure 5 Daily average lighting electric use [kWh] in each month in square skylights with 5.5% AFR in Boston

Because of the apparent daylighting benefits of the skylights with integrated systems and beveled light-wells, this system was chosen as the focus for whole-building energy assessment in this paper.

A major purpose of this study is to find the optimum area of skylight apertures, where the advantages of reduced electric lighting due to daylighting overcome the disadvantages of increased conductive heat loss and increased solar heat gain through the glazing material. The following **A**perture-to-**F**loor-Area-**R**atios (AFRs) were investigated: 0.02 (2%), 0.035 (3.5%), and 0.055 (5.5%).

Figure 6 shows daylight illuminance distribution at 12 pm September 21 in Boston and Miami for various AFRs. Twenty-five sensors are located on the diagonal axis at task level in each module. Skylights were designed with integrated systems and splayed light-wells. Results show that the average daylight illuminance increases as the AFR increases (table 4). The increase in average illuminance is not directly proportional to the AFR, because of light-well effects.

Table 4

Average daylight illuminance in various AFRs

	AVERAGE DAYLIGHT ILLUMINANCE (LUX)		
AFR	2	3.5	5.5
Boston	290	593	1084
Miami	361	730	1337

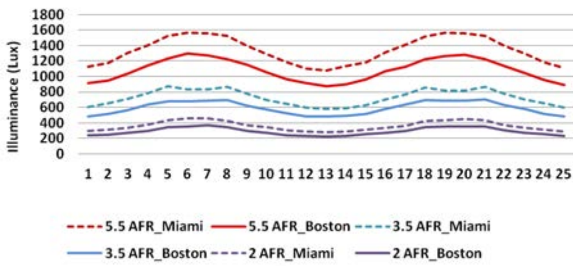

Figure 6 illuminance distribution [lux] on diagonal axis at task level for various AFRs in Boston and Miami

Electric lighting

Figure 7 shows daily average electric lighting use (by month), for square skylights in Boston and Miami. The base case is shown to compare spaces with horizontal apertures to the same space with no apertures on the roof.

For small AFRs (0 to 2%), the electric consumption goes down rapidly with each additional increment of aperture area. This rapid decrease primarily reflects the influence of beam sunlight, which is intense enough to displace substantial amounts of the electric light, even when the collecting aperture is quite small. At larger AFRs (above 2%), the lighting electricity consumption goes down less rapidly, indicating primarily the effect of diffuse skylight during those hours when beam sunlight is not available or is only weakly incident on the collection glazing.

The reductions in lighting electricity were greater in Miami than Boston, because the lower latitude of Miami results in more availability of sunlight, particularly during the winter months when short days and cloudy conditions seriously limit the effectiveness of daylighting in Boston. The differences in the lighting electricity consumption curves at small apertures are primarily a result of differences in availability of beam sunlight. At small apertures (2% AFR), the lighting electricity consumption for Miami is substantially lower than for Boston. For larger apertures, diffuse skylight becomes more significant, and the major differences in the lighting electricity consumption curves result from differences in the number of hours of daylight. Boston has significantly higher lighting electricity consumption compared to Miami, because the higher latitude of Boston limits the number of hours of daylight during winter.

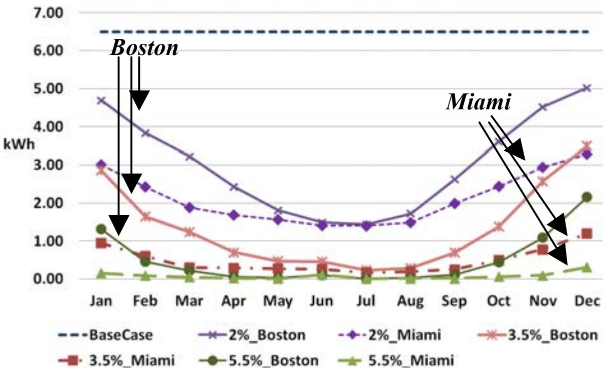

Figure 7 Daily average lighting electric consumption (by month) [kWh] in Boston and Miami

Heating and cooling energy consumption

In figure 8, daily average heating coil gas consumption (by month) [kWh] is plotted for various roof aperture areas, for both Boston and Miami. For small aperture areas, heating fuel consumption increases with increasing aperture area, resulting primarily from increased conductive losses associated with adding glazing to the roof and the replacement of electric light with sunlight, which has lower heat content compared to lighting fixtures. As the area of apertures increases, heating fuel consumption increases with a lower rate. The reason is that solar gains compensate the combined effect of reduced heat from the electric lights and increased conductive losses associated with increased glazing area.

In Boston, heating fuel consumption (figure 8) is more sensitive than cooling coil consumption (figure 9) to variations in the aperture area. This is because of high requirement for heating in Boston with significantly more heating degree days than Miami. In Miami, there is heating coil energy use in both heating and cooling seasons. The reason for having gas use for the furnace even in cooling season is due to the heating required for the dehumidification process in the HVAC loop.

In square skylights in Miami, the annual cooling energy follows a general trend. The annual cooling energy reduces first when square skylights are created in the roof because of the decrease in internal loads generated by fewer electric lights. At square apertures larger than 3.5% AFR, the annual cooling load increases, because of the increase in solar energy transmitted to the space.

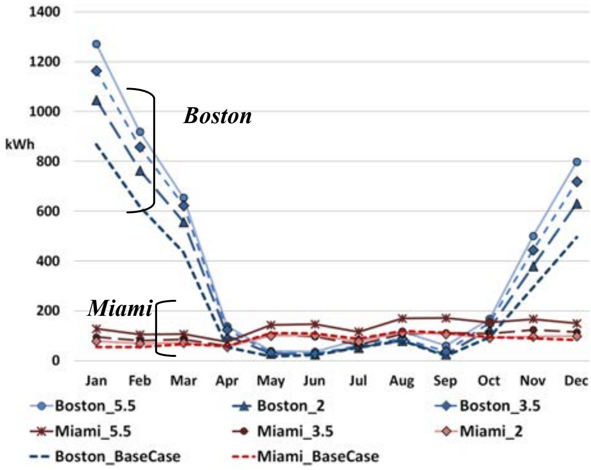

Figure 8 Monthly heating coil gas consumption [kWh]

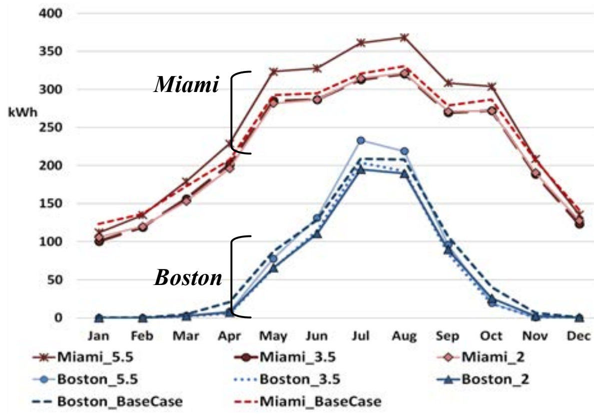

Figure 9 Monthly cooling coil electricity [kWh]

Energy use intensity

In figures 10 and 11, energy use per unit of floor area per year, EUI [kWh/m^2/yr], is categorized by type of energy consumption: equipment, fan, lighting, cooling, heating and humidifier in order to understand contribution of each category separately. The most potential saving by the use of horizontal apertures occured for electric lighting enery consumption in both Boston and Miami. Horizontal apertures create higher potentials for whole-building enrgy saving in Miami than Boston do due to better daylighting performance and lower heat loss through glazing in Miami.

Results show a general trend in all cases. At small AFRs, the energy consumption falls with increasing glazing area, because of the decrease in both lighting electricity and cooling electricity consumption. At larger AFRs, EUI rises slightly as increasing heating gas and cooling electricity negates the benefits in decreasing lighting electricity consumption.

For square apertures, the most energy efficiency occurs at 2% in Boston and 3.5% in Miami. The effectiveness of small area of glazing is a result of the extreme intensity of sunlight compared to the illumination level required in an office building.

In Boston, even the most energy efficient square aperture with 2% AFR requires significant heating energy to compensate the heat loss through the apertures. In skylights with 2% AFR, the reduction in electric lighting is 11.63 [kWh/m^2/yr] from the base case, which is higher than the increase in heating energy consumption, 9.17 [kWh/m^2/yr]. At larger skylights than 2% AFR, increase in furnace gas consumption negates the benefits of lighting electricity reduction.

In Miami, the benefits of electric use reduction is more significant than the changes in heating and cooling energy use up to an optimum AFR (3.5%). As the aperture area increases more than the optimum AFR, increase in cooling energy consumption overcomes the benefits of daylighting.

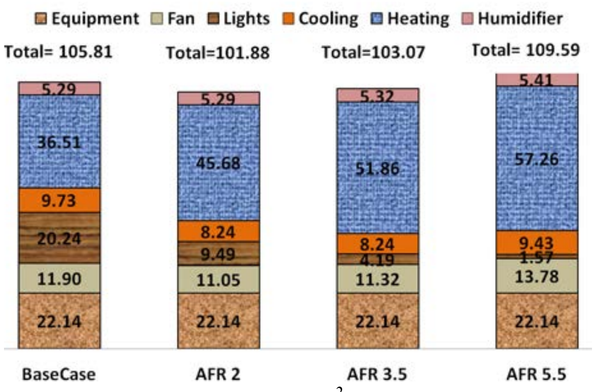

Figure 10 EUI [kWh/m ^{2} /yr] in Boston

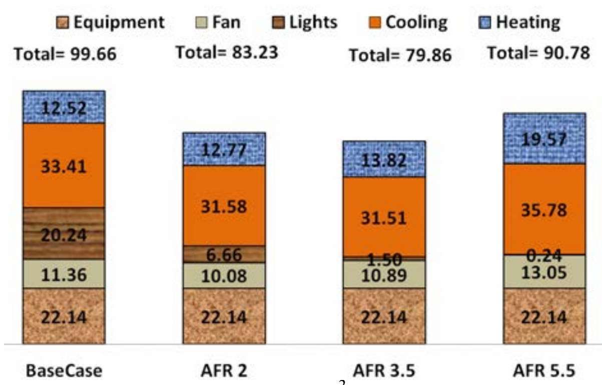

Figure 11 EUI [kWh/m ^{2} /yr] in Miami

BUILDING OPERATION COSTS

Figure 12 shows the annual operating cost for energy as a function of AFR in modules with 83.6 m^2 floor area with square skylights in Boston and Miami. The operation costs were calculated based on the local cost for electricity and gas, which were both higher in Boston than Miami. Contribution of electric and gas consumption to the total cost is reflected in figure 12. The cost per unit of energy at both sites was higher for electricity than it was for gas. As a result, the variations in electricity as a function of AFR were more significant from an energy economics point of view. Figure 12 also depicts the total savings associated with the area of apertures.

In Boston, for commercial buildings, the price of electricity was \$0.0548 per kWh in Oct-May and \$0.0828 per kWh in June-Sep plus a monthly fee (NSTAR). Gas price in Boston was \$0.0196 per kWh. In Miami, electricity costs \$0.0469 per kWh and gas price for commercial buildings, which use 0-2000 annual therms, was \$0.0116 per kWh plus a monthly fee (FLP). In calculation of building operation costs, all the monthly charges were excluded because they included maintenance fees and generated disproportional relations between energy and costs.

The most significant observation in figure 12 is the striking similarity of the two cost curves. Benefits of skylights are higher in Miami, because of generally warm and sunny character of Miami and also lower operation costs in this city compared to Boston. For cold climates such as Boston, more stringent U-values are required to increase performance of skylights. However, using a triple glazed skylight with highly insulated frame will increase the cost of skylights.

In both locations, costs decrease rapidly with increasing aperture area, up to optimum aperture areas, which are 3.5% AFR in square apertures. Reductions in lighting electricity consumption and cooling electricity consumption contribute to these utility cost decreases (see figures 8 and 9). Beyond an optimum aperture area, increases in heating and cooling energy exceed the decreases in lighting electricity, and the costs increase with increasing aperture area.

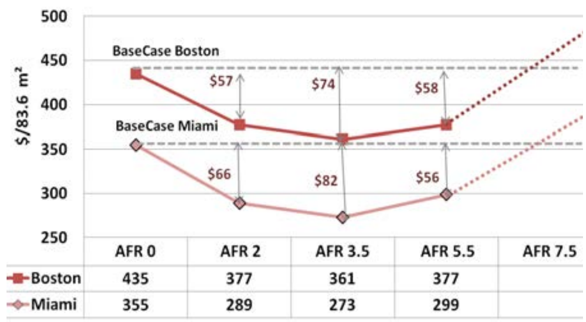

Figure 12 Building operation costs [\$ per module of 83.6 m^2] in Boston and Miami

The most potential cost benefits are achieved at 3.0 to 3.5% AFR at both climates. Skylights with 3.5% AFR can save \$0.08 per ft^2 (\$0.89 per m^2) of floor area per year in Boston and \$0.09 per ft^2 (\$0.98 per m^2) in Miami. Results show that an economically optimum skylight saves 72%-88% of annual lighting electricity consumption in Boston and Miami respectively. Using an oversized aperture does not contribute to any savings in either climate.

CONCLUSIONS

This study examined application of square skylights in flat roofs in open office spaces. Results showed that skylights produce high illuminance on a horizontal working plane even with a small aperture size. Having a small aperture size is a thermal benefit since less heat transfer occurs through the opaque part rather than the glazing part of the roof. As a result, skylights designed with 3-3.5% AFR contribute to the highest savings in building operation costs as illustrated in the results (see figure 12).

The shapes of the energy costs in figure 12 were influenced by assumptions in the study:

Electric charges did not include peak-power demand charges. Including peak-power demand charges will highlight benefits of daylighting because of reducing electric use at noon, which the highest demand for cooling.

The office building was modeled with 13’ 7” (9.14 m) roof height in this study. For fixed number and area of skylights, increasing the ceiling height would result in more even illumination at task surface; thus, less dark spots are created and fewer light fixtures are required in the space. As a conclusion, cost benefits of roof-daylighting systems will be higher in commercial buildings with higher ceilings.

In this study, diffusing glazing material with 42% visible transmittance (Vt) was used for skylights. Higher Vt would increase SHGC of the glazing material higher than energy code requirements such as ASHRAE 90.1-2010. Advent of glazing materials that can transmit higher levels of visible light without transmitting solar heat would increase efficiency of roof-daylighting systems.

The electric lighting control was a dimming control that performed ideally to generate electric light in proportion to reductions in available daylight. Electric lighting controls with ON/OFF switch will use higher electric lighting and create disturbing effect while sudden changes occur in sky illumination such as cloudy days. However, such electric lighting systems are simpler and less expensive, and will be investigated as this study continues to evolve.

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