

## A SIMPLE GENERAL LUMINOUS EFFICACY MODEL OF GLOBAL IRRADIANCE

Sokol Dervishi<sup>1,2</sup>, Ardeshir Mahdavi<sup>1</sup>

<sup>1</sup>Department of Building Physics and Building Ecology, Vienna University of Technology, Austria

<sup>2</sup>Department of Architecture, Epoka University, Tirana-Albania

### ABSTRACT

This paper reports on the development of a luminous efficacy model of global horizontal irradiance. The model is intended to be both simple and applicable to multiple locations. Two sets of measured data were applied toward the development and evaluation of the model. The first set was used to derive the luminous efficacy model. The second set was then used to compare the model predictions with measurements. The proposed luminous efficacy model involves, as the main influencing variable, the Clearness Factor, which is an original derivative from the Clearness Index. Two further variables (Humidity Ratio and the solar altitude) are included in the model formulation. The paper includes the result of the statistical analysis of the relationship between the model predictions and the measured data from the second set of empirical data.

### INTRODUCTION

To perform detailed design analyses and evaluation pertaining to daylight conditions in architectural spaces, appropriate models of sky luminance distribution are needed. In the past, various sky luminance distribution models have been developed. However, such models typically require global and diffuse illuminance data for the relevant location. But measured data on global and – especially – diffuse external illuminance are generally not available for most locations. Hence, methods are needed that facilitate the derivation of illuminance and luminance data from the more widely available irradiance and radiance data via reliable luminance efficacy models. Luminous efficacy denotes the ratio of illuminance to irradiance.

Several authors have suggested models to derive luminous efficacy for different sky conditions. Littlefair (1988), Aydinli and Krockman (1983) presented polynomial relations of different degrees using solar altitude as the only independent input variable for beam luminous efficacy. Another model, which also uses solar altitude as independent variable, was proposed by Robledo and Soler (2001). Littlefair (1988) established diffuse luminous efficacy as an interpolation between overcast and clear sky using sky clearness as an indicator. Using Littlefair's model, Chung (1992) and Robledo and

Soler (2001) developed local luminous efficacy models (based on data from Hong Kong and Madrid, respectively) for overcast and intermediate skies. Perez et al. (1990) developed a luminous efficacy model of all sky types as a function of the solar zenith angle ( $Z$ ), atmospheric precipitable water content ( $W$ ) and the sky brightness index ( $\Delta$ ). The coefficients of these variables were specified as a function of sky clearness ranges. Munner and Kinghorn (1997) derived global luminous efficacy as a polynomial model for all sky types in which the clearness index ( $k_t$ ) is the only independent variable. Clearness index is defined as the ratio of global horizontal irradiance ( $I_g$ ) to extraterrestrial irradiance ( $I_e$ ) and the sinus of the solar altitude. Ruiz and Soler (2001) developed a different model for global luminous efficacy for all sky types using clearness index ( $k_t$ ), and the sun altitude ( $\alpha$ ) as independent variables.

Despite the above advances in model development, certain problems remain. An important open question is the applicability of these models for different locations. Short of detailed calibration, which requires both measured irradiance and illuminance data, the performance of a number of existing models was not found to be satisfactory (see Mahdavi and Dervishi 2011). In these studies, we have found that certain highly detailed models with multiple coefficients perform below expectation, if these coefficients are not calibrated based on local high-resolution long-term measurements.

In our view, these experiences highlight the desirability of a balanced approach pertaining to models' algorithmic simplicity and ease of use on the one side and its predictive capability on the other side. In this context, an unfortunate circumstance must be mentioned. As opposed to the general spirit of scientific inquiry, in which the continuous and rigorous testing and evaluation of existing models are energetically encouraged, in building performance research at times a hostile attitude is displayed, if existing models are critically examined. The typical reactions appear to be often either to question the fidelity of the underlying observations, or to suggest that the data of one specific location is not grounds for questioning the performance of general models (even though it appears that data from one location

are sometimes conveniently used to "validate" a model).

We can address the former comment by referring, amongst other things, to the systematic nature of the measurements, their long-term regime, the presence of sensor redundancy, and the conducted extensive data quality check (see, for instance, the details in the "model development" section below). As to the latter point, no unusual circumstance applies to the location of the measurements quoted. Rather, there appears to be a misunderstanding as to what constitutes scientifically a "validation". Likewise, the notion of an independent (preferably double blind) model validation seems to be still not well-understood, as occasionally the papers by the model developers are quoted as proof the models' validity.

We specifically mention here this tendency toward suppressing contributions with "non-conformist" model observations with some emphasis, as we have made similar experiences in other instances and believe it is a detriment to progress in our field. For example, in other studies, we addressed the performance of a large number of models to derive the diffuse component of solar irradiance based on global horizontal irradiance data (Dervishi and Mahdavi 2012, Vazifeh et al. 2013).

But the performance of none of the models could be considered acceptable, if we, for example, would follow the formulation in ASHRAE 2002. Thereby, we should consider models' performance acceptable, if the MBE is less than  $\pm 10\%$  and CV(RMSE) within  $\pm 30\%$  (for hourly measurements). Almost half of the results involved a relative error of at least  $\pm 20\%$ .

Nonetheless, as mentioned before, our main objective here is not to present a model that can be conclusively shown to surpass all existing models in predictive accuracy (this would be obviously an unjustified claim for a model developed based on data from one location). Rather, the aim is to explore a balanced approach pertaining to models' algorithmic simplicity and ease of use on the one side and its predictive capability on the other side. Our intention in model development was to generate a luminous efficacy model (of global solar radiation) based on a minimum set of input data. The model is intended to be simple and easy to implement. Even though the model development was based on data from one location (microclimatic station of the Department of Building Physics and Building Ecology of the Vienna University of Technology in Vienna, Austria), it involves variables that make it – in principle – applicable to multiple locations. Two sets of measured data were applied toward the development and evaluation of the model. The first set was used to derive the luminous efficacy model. The second set was then used to compare the model predictions with measurements. The proposed luminous efficacy model involves, as the main influencing variable, the Clearness Factor, which is

an original derivative from the Clearness Index. Two further variables are included in the model formulation. These are the Humidity Ratio and the solar altitude. Moreover, the model includes a location-dependent variable, which may be conveniently derived from the latitude information. The paper includes the result of the statistical analysis of the relationship between the model predictions and the measured data from the second set of empirical data. The results of this analysis display a good agreement between predictions and measurements.

## APPROACH

The empirical basis of the model comparison was long-term measurements at the microclimatic monitoring station of the Department of Building Physics and Building Ecology (Vienna, Austria). To derive the luminous efficacy model of global solar radiation, a first database containing measured irradiance and illuminance values over a one-year period was established (01.08.2010 to 30.07.2011, first 15 days of each month). To evaluate the predictive performance of the developed model, a second database of measured irradiance and illuminance values was used, covering the same period, but using the second 15 days of each month. Measurements of global irradiance and illuminance were performed every 15 minutes during the daylight hours, covering a variety of sky conditions, from sunny, to partly cloudy, to overcast. The specifications of the deployed irradiance and illuminance sensors are given in Tables 1 and 2. Parallel to radiometric and photometric measurements, a weather station at the same location (see Table 3 for related specifications) monitored other external environmental parameter such as air temperature and air relative humidity.

Collected data was made subject to a comprehensive quality check. Specifically, measurements at very low sun altitudes (less than 5 degrees) and those involving very low global horizontal irradiance values (below  $50 \text{ W.m}^{-2}$ ) were excluded, given the uncertainty in the sensor accuracy for this radiation intensity range. Subsequent to the data quality check, 6293 pairs of measured irradiance and illuminance values in the first database and 6141 pairs in the second database were included in the study.

Table 1  
The specification of the applied pyranometer

Spectral range	305-2800 nm
Sensitivity	10-35 $\mu\text{V}/(\text{W}\cdot\text{m}^{-2})$
Impedance (nominal)	79-200 $\Omega$
Response time (95%)	18 s
Non-linearity	$\pm 2.5\%$ ( $<1000 \text{ W}\cdot\text{m}^{-2}$ )
Temp. difference of sensitivity	6% ( $-10$ - $+40^\circ$ )
Directional error	$< \pm 25 \text{ W}\cdot\text{m}^{-2}$ ( $1000 \text{ W}\cdot\text{m}^{-2}$ )
Tilt error	$< \pm 2\%$
Zero-offset due to temp. changes	$< 4 \text{ W}\cdot\text{m}^{-2}$ at 5 K/h temp. change
Operating temp.	$-40$ - $+80^\circ$
ISO-9060 Class	

Table 2  
The specification of the applied illuminance meter

Measuring range	0 - 130 klx
Spectral sensitivity	360 - 760 nm
Dome	PMMA
Cosine error	$< 3\%$
Linearity	$< 1\%$
Absolute error	$< 10\%$
Operating temp.	$-20$ - $+60^\circ$

Table 3  
Overview of the monitoring station specifications

Outdoor temperature	Absolute Error: $< 0.3 \text{ K}$ ; Temperature range: $-30$ to $+70$ $^\circ\text{C}$ ; Response time $< 20 \text{ s}$ ( $\geq 1.5 \text{ m}\cdot\text{s}^{-1}$ )
Outdoor relative humidity	Absolute Error: $< \pm 2\%$ ; Humidity range: 0 to 100 %; Response time $< 10 \text{ s}$ ( $\geq 1.5 \text{ m}\cdot\text{s}^{-1}$ )
Wind speed	Absolute Error: $< 1\%$ ; Wind speed range 0 - 75 $\text{m}\cdot\text{s}^{-1}$

The collected data was statistically analyzed to explore the relationship between luminous efficacy of the global horizontal solar radiation. Thereby, several variables were taken into consideration, including the Clearness Factor (CF), solar altitude ( $\alpha$ ), and humidity ratio (HR). The statistical treatment resulted in a simple algorithm for the calculation of luminous efficacy as a function of the above variables. Calculations based on this algorithm were then compared with data from the aforementioned second set of empirical measurements. Toward this end, three common statistical indicators were used for the comparison: the relative mean bias deviation MBD (Equation 1), the relative error RE (Equation 2), and the root mean square deviation RMSD (Equation 3).

$$MBD = \frac{\sum_{i=1}^n \left( \frac{M_i - C_i}{M_i} \right)}{n} \cdot 100 \quad [\%] \quad (1)$$

$$RE = \left( \frac{M_i - C_i}{M_i} \right) \cdot 100 \quad [\%] \quad (2)$$

$$RMSD = \sqrt{\frac{\sum_{i=1}^n \left( \frac{M_i - C_i}{M_i} \right)^2}{n}} \cdot 100 \quad [\%] \quad (3)$$

In these equations,  $M_i$  is the measured diffuse luminous efficacy,  $C_i$  is the computed diffuse luminous efficacy, and  $n$  the total number of pairs of global irradiance and illuminance values.

### MODEL DEVELOPMENT

As mentioned before, the luminous efficacy of the global horizontal solar radiation  $\eta$  (Equation 4) is defined as the ratio of the horizontal global illuminance  $E_v$  and global horizontal irradiance  $E_e$ .

$$\eta = \frac{E_v}{E_e} \quad [\text{lm}\cdot\text{w}^{-1}] \quad (4)$$

The most simple way to define luminous efficacy would be to derive a mean value from the measurements. To illustrate this, Figure 1 shows the relationship between the measured illuminance and irradiance values (first data set). The correlation is strong (0.99) and yields a simple formula for the estimation of illuminance based on irradiance values between the 50 to 1000  $\text{W}\cdot\text{m}^{-2}$  range (Equation 5).

$$E_v = 120 \cdot E_e + 450 \quad [\text{lx}] \quad (5)$$

However, Figure 1 does not reveal the rather large uncertainty involved in the estimation of luminous efficacy for the lower irradiance range (see Figure 2). Such a highly simplified approximation amounts to a constant luminous efficacy and is limited in at least two ways. First, potential influence of candidate variables (e.g., sky conditions) is entirely neglected. Second, an application of the relationship to locations, other than the one for which the data was available, is not possible.

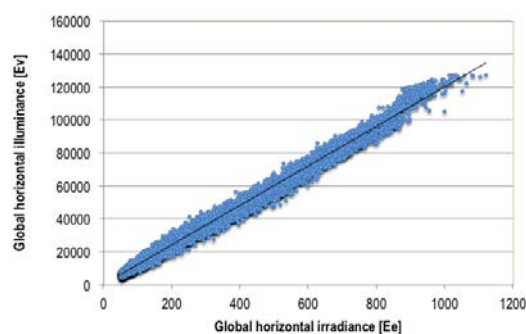


Figure 1 Measured global horizontal irradiance versus global horizontal illuminance

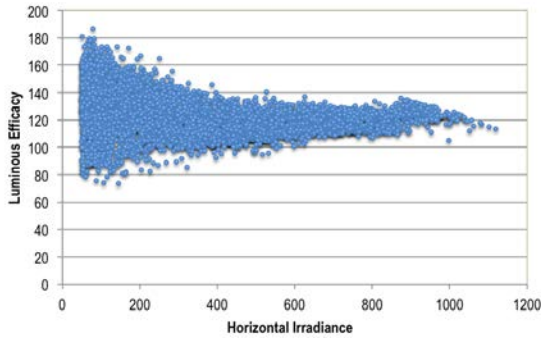


Figure 2 Global horizontal luminous efficacy as a function of global horizontal irradiance for the first data set

Thus, we further explored the data toward identification of potential influencing parameter. First, a relationship between luminous efficacy and a derivative function of sky clearness index, namely Clearness Factor (CF) could be discerned from the first data set (see Figure 3). CF is defined as follows (Equation 6):

$$CF = \sqrt{\log(k_g \cdot 10)} \quad (6)$$

Here,  $k_g$  denotes the clearness index as per the following equation (Reindl et al. 1990)

$$k_g = \frac{E_e}{E_0 \cdot \sin(\alpha)} \quad (7)$$

Moreover, correlational analysis of the first data set revealed certain – rather weak – dependencies of luminous efficacy on both solar altitude  $\alpha$  (Figure 4) and Humidity Ratio of the air (Figure 5).

Given these observations, we derived a general luminous efficacy model with additive terms taking Clearness Factor (CF), solar altitude ( $\alpha$ ), Humidity Ratio (HR) (both actual value and long-term mean), as well as location's latitude into considerations (Equation 8).

$$\eta = a + b + c \quad (8)$$

whereby,

$$a = 160 + (E_e / 10) - (60 + E_e / 10) \cdot CF \quad (9)$$

$$b = (\alpha - f_1) / f_1 + (35 - E_e \cdot 0.025) \quad (10)$$

$$c = 1000 \cdot (HR - f_2) \cdot (4 - 0.004 \cdot E_e) \quad (11)$$

In the above relationships,  $f_1$  denotes a function of the location's latitude ( $l$ ). It may be approximated as follows:

$$f_1 = (-0.009 l^2 + 0.294 l + 68) / 2 \quad (12)$$

Given Vienna's latitude (latitude 48, 14 N, 16, 20 E), the value of  $f_1$  for Vienna amounts to 33. Furthermore,  $f_2$  is a simple function (Equation 13) of the location's long-term mean Humidity Ratio ( $HR_m$  in kg water vapor per kg dry air). For Vienna,  $HR_m$  is roughly 0.007, resulting in a value of 7 for  $f_2$ .

$$f_2 = 1000 \cdot HR_m \quad (13)$$

Hence, the luminous efficacy model of Vienna may be formulated as follows (Equation 14):

$$\begin{aligned} \eta = & 160 + (E_e / 10) - (60 + E_e / 10) \cdot CF + \\ & + (\alpha - 33) / 33 + (35 - E_e \cdot 0.025) + \\ & + 1000 \cdot (HR - 7) \cdot (4 - 0.004 \cdot E_e) \end{aligned} \quad (14)$$

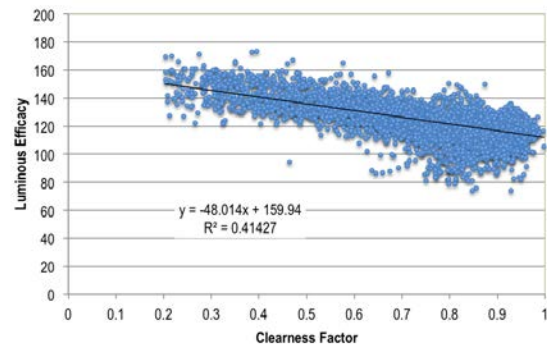


Figure 3 Luminous efficacy as a function of Clearness Factor for the first data set

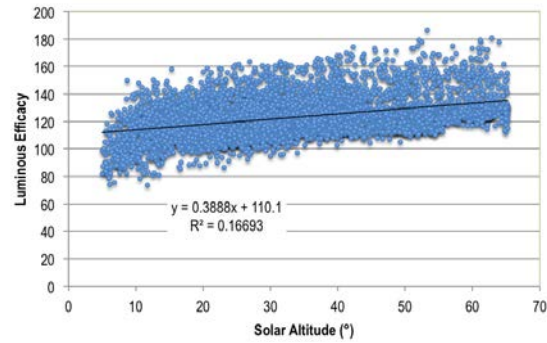


Figure 3 Global horizontal luminous efficacy as a function of solar altitude for the first data set

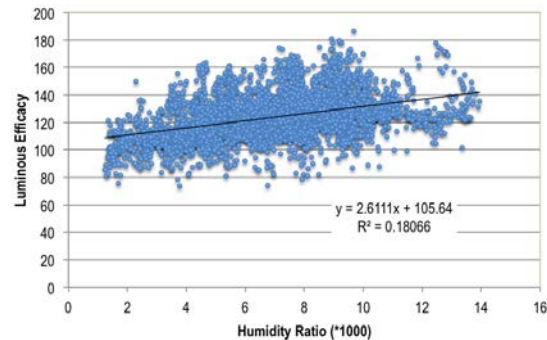


Figure 4 Global horizontal luminous efficacy as a function of Humidity Ratio for the first data set

## MODEL EVALUATION

To evaluate the performance of the proposed luminous efficacy model (referred to as Mahdavi and Dervishi Model), the second data set of measurements was used. Figure 5 shows the percentage of the results (pairs of measured and computed luminous efficacy values) with associated maximum relative errors. To put the performance of the proposed model in context, we have also included predictions based on the assumption of a constant luminous efficacy value (as per Equation 5), referred to here as the CLE Model. Table 4 shows the same information numerically for discrete values of relative error ( $\pm 5\%$ ,  $\pm 10\%$ ,  $\pm 15\%$ ,  $\pm 20\%$ ). Table 5 illustrates the performance of both Mahdavi and Dervishi Model and the CLE model for luminous efficacy in terms of MBD and RMSD.

Despite its simplicity (limited number of independent variable, whose values can be conveniently obtained), the Mahdavi and Dervishi model performs relatively well. The relevant statistical benchmark considered shows all a better performance of the proposed model as compared to the constant luminous efficacy assumption. An important and potentially significant feature of the proposed model is its adaptability for application in different geographical locations. We cannot yet verify this feature empirically. However, we hope through future collaborative research involving other institutions, such an examination of the broad applicability potential of the proposed model could be realized.

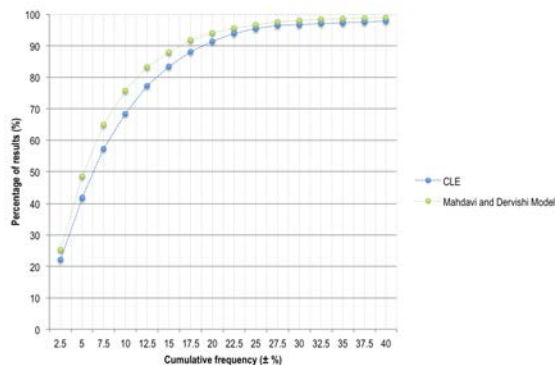


Figure 4 Percentage of the results (pairs of measured and derived global luminous efficacy values) with respective maximum relative error (RE) for CLE and Mahdavi and Dervishi model respectively

Table 4  
Percentage of results with corresponding maximum relative error (RE) for CLE and Mahdavi and Dervishi model

MODEL	$\pm 5\%$	$\pm 10\%$	$\pm 15\%$	$\pm 20\%$
CLE	41.7	68.4	83.5	91.5
Mahdavi and Dervishi	48.5	75.8	88.0	94.2

Table 5

Comparison of luminous efficacy models based on MBD and RMSD

MODEL	MBD (%)	RMSD (%)
CLE	-1.3	12.2
Mahdavi and Dervishi	0.3	10.4

## CONCLUSION

We presented a luminous efficacy model of global solar radiation to derive horizontal illuminance values from more widely available measured global horizontal irradiance data. Model development and evaluation was conducted using measurement data from Vienna, Austria. Two sets of measured data were applied. The first set was used to derive the luminous model. The second set was then used to evaluate the performance of the model. The results suggest a good match between predictions and measurements. Clearness Factor was identified as the main model variable. Two further variables (Humidity Ratio and solar altitude) were included in the model to fine-tune its performance. Ongoing and future research shall address the possibility to explore the model's adequacy for predicting illuminance level in a wide range of locations. In addition, further parameter such as the effect of aerosols and the sky ratio (diffuse irradiance over global irradiance) will be further evaluated.

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