QUANTITATIVE COMPARISON OF MASSIVE WALLS THERMAL RESPONSE AMONG COMMERCIAL SOFTWARE

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ABSTRACT

Simulating heat conduction in massive walls with commercial software is reported to cause numerical instability or reduced accuracy. As contribution to the discussion, we have simulated one-dimensional heat conduction in massive walls and their dynamic thermal responses to a step, a sinusoid and time series in TRNSYS, EnergyPlus, Delphin and Matlab. As reference, we have used EN ISO 13786:2007 and a self-written Matlab response factor method implementation. We have compared transient and steady-state wall surface temperatures and heat fluxes for two different accuracy settings using suitable metrics. Errors up to 1 kWh/(m^2 month) have been observed.

INTRODUCTION

We have done this work for the European project 3encult on energy efficiency in historical buildings (3encult, 2010-2014). A major aim of the project is to develop a strategy for the conservation of listed buildings combined with energy efficiency measures. The conservation issue makes a correct assessment of energy performance even more important than for not listed buildings. Such an assessment includes an accurate calculation of the heat conduction through walls with dynamic simulation programs. In case of homogeneous wall compositions, small boundary effects and negligible thermal bridges, one-dimensional (1-d) heat conduction models are adequate. Different numerical methods and software implementing these methods have been developed and compared in the literature. Several articles on software validation take into account ASHRAE building stock and summer climate with the aim of finding an accurate heat transfer model for massive walls and defining some useful parameters to evaluate their behaviour (Asan, 2006). However, few publications deal with the typical properties of such walls. Cellura et al. (Cellura, Giarrè, Lo Brano, & Orioli, 2003) analysed the errors of different implementations of the Conduction Transfer Function (CTF) method for wall thicknesses up to 100 cm. Chen et al. (Chen, Zhou, & Spitler, 2006) compared the analytic frequencies of heat conduction through a wall with those obtained with numerical methods on ASHRAE building stock. Li et al. (Li, Chen, Spitler, & Fisher, 2009) compared the CTF coefficients calculated with three different popular methods. The authors proposed a strategy to assess the errors of the CTF coefficients based on wall properties. They reported improved performance of frequency-domain regression (FDR) compared to state-space (SS) and direct root-finding (DRF) methods. Acceptable errors were reported for SS and DRF methods for $1/(\mathrm{Fo} \cdot S_{ie})$ less than 600 in case of a single-layer and less than 1200 in case of a multi-layer slab. Fo denotes the Fourier number and S_{ie} the thermal structure factor as defined in the paper.

However, there still remains the need to assess the accuracy of commercial dynamic simulation software in the calculation of energy performance in historical buildings. In addition to the SS and DRF methods, we have considered other methods such as the response factor (RF), the finite difference (FD) and the finite control volume (FCV) method to take a broader view of possible issues.

METHODS

Throughout the paper, we consider a single exterior wall of a building with varying thickness L made of one or two homogeneous layers with fixed thermal properties. We focus on 1-d heat conduction, purposely neglecting radiation exchanges. Fourier's law and energy conservation yield the following equation for 1-d heat conduction in a homogeneous material (Cannon, 1984):

$$\frac{\partial T}{\partial t}(x,t) = \alpha \frac{\partial^2 T}{\partial x^2}(x,t) \quad (1)$$

The initial conditions are given by the steady-state for constant outdoor and indoor air temperature with (constant) heat flux:

$$q_0 = 1/\left(\frac{1}{h_{ex}} + \frac{1}{G} + \frac{1}{h_{in}}\right)(T_{a,ex} - T_{a,in}) \quad (2)$$

Boundary conditions are:

$$\begin{aligned} T_{s,ex}(t) &= f(t) \\ T_{s,in}(t) &= 20 \end{aligned} \quad (3)$$

$f(t)$ denotes the forcing function (FF).

In the following, we present the numerical methods considered in this paper to solve Equation 1.

Response factor method

The idea of the RF method (Stephenson & Mitalas, 1967) is to approximate outdoor and indoor temperature fluctuations by a series of triangular pulses, each with a base width of $2\cdot\Delta t$ and a height corresponding to the temperature; the less the time difference Δt between two consecutive pulses, the better the approximation. Δt is called time base. The response factors (RFs) X_j, Y_j and Z_j, j=0,1,2,…, represent the responses at time $j\cdot\Delta t$ of a monolayer wall to a single triangular temperature pulse at time zero: at the external (X_j) / internal (Y_j) surface to an outdoor pulse and at the internal surface to an indoor pulse (Z_j). In Equation 4, we give the RF X_0 as an example. :

$$X_0 = \frac{\lambda L}{\alpha\,\Delta t}\left(\frac{\alpha}{L^2}\Delta t + \frac{1}{3} - \frac{2}{\pi^2}\sum_{k=1}^{\infty}\frac{\phi_k}{k^2}\right)$$
$$\phi_k = \exp\left(-\frac{k^2\pi^2\alpha}{L^2}\Delta t\right) \quad (4)$$

The other RFs are given by analogous formulas and are reported in the literature (Underwood & Yik, 2004). $X_0=\lambda/L+$... has the same unit as thermal transmittance. It follows that the RFs are numerically equal to heat fluxes produced by unit triangular pulses of 1 Kelvin.

$$X_0 = \frac{\lambda L}{\alpha\,\Delta t}\left(\frac{\alpha}{L^2}\Delta t + \frac{1}{3} - \frac{2}{\pi^2}\sum_{k=1}^{\infty}\frac{\phi_k}{k^2}\right)$$
$$\phi_k = \exp\left(-\frac{k^2\pi^2\alpha}{L^2}\Delta t\right) \quad (5)$$

The RFs of a two-layer wall can be computed from the RFs of the single layers (Underwood & Yik, 2004):

$$\begin{aligned} S &= X^{(2)} + Z^{(1)} \\ X &= -\left(Y^{(1)} * Y^{(1)}\right) *^{-1} S + X^{(1)} \\ Y &= \left(Y^{(1)} * Y^{(2)}\right) *^{-1} S \\ Z &= -\left(Y^{(2)} * Y^{(2)} - S * Z^{(2)}\right) *^{-1} S \end{aligned} \quad (6)$$

$X^{(j)}$, $Y^{(j)}$, $Z^{(j)}$ denote the RFs of the j-th layer and X, Y, Z the RFs of the wall. The layers are numbered in ascending order from exterior to interior. We have used the discrete convolution / deconvolution operators defined by:

$$(a * b)_j = \sum_{k=0}^{j} a_k b_{j-k}$$
$$b *^{-1} a = x \text{ such that } a * x = b \quad (7)$$

The RF method has been implemented in MATLAB. Series as the one in Equation 4 are truncated when the terms summed in reverse order (for higher precision) stop altering the result. The number N of computed RFs is determined such that the difference between the steady-state heat flux caused by a temperature unit step of either external or internal air temperature at time zero and the thermal conductance of the wall is less than 0.001 W/m^2:

$$\left(G - \sum_{k=0}^{N} RF_k\right)\cdot 1\,\mathrm{K} < 0.001\,\mathrm{W/m^2} \quad (8)$$

In Equation 7, *"RF"* has to be replaced in sequence by X_k, Y_k and Z_k. The thermal conductance of the wall has been calculated analytically by summing the layers' transmittances, $G^{(j)}=\lambda^{(j)}/L^{(j)}$.

Wall surface temperatures and heat fluxes at time $t_j=j\cdot\Delta t$ have been computed by solving the following two linear equations

$$\begin{bmatrix} q_{ex}(t_j) \\ q_{in}(t_j) \end{bmatrix} = \sum_{k=0}^{j}\begin{bmatrix} X_k & -Y_k \\ Y_k & -Z_k \end{bmatrix}\begin{bmatrix} T_{s,ex}(t_{j-k}) \\ T_{s,in}(t_{j-k}) \end{bmatrix} = \begin{bmatrix} h_{ex}\left(T_{a,ex}(t_j) - T_{s,ex}(t_j)\right) \\ h_{in}\left(T_{s,in}(t_j) - T_{a,in}(t_j)\right) \end{bmatrix}, j=1,2,\ldots \quad (9)$$

with respect to $T_{s,ex}(t_j)$ and $T_{s,in}(t_j)$ for j=1,2,… and then computing $q_{ex}(t_j)$ and $q_{in}(t_j)$.

EnergyPlus FD and Delphin FCV method

EnergyPlus (EnergyPlus v7.2.0, 2012) offers two FD schemes. We have used the semi-implicit Crank-Nicholson scheme based on an Adams-Moulton solution approach. Delphin (Delphin v5.6.8, 2012) uses a variable-order, variable-step multistep method of the CVODE integrator of the SUNDIALS package. The order varies between one and five according to integration error estimates. For the numerical solution of the balance equations, the FCV method is applied. For orthogonal, equidistant grids, the FCV method yields the same discretized equations as the FD method. Advantages of the FCV method are the applicability to unstructured grids and the mass-conserving formulation of fluxes over control volume boundaries. For a better comparison, we have set up the FD and FCV method with the same number of nodes.

EnergyPlus CTF and TRNSYS CTF method

EnergyPlus uses the state space (SS) method to calculate the CTF coefficients. The internal states, that is, the nodal temperatures, can be eliminated. The result is a matrix equation that directly relates the heat fluxes at the wall surfaces to the interior and exterior air temperatures.

The CTF method, implemented in the TRNSYS (Thermal Energy System Specialists (TESS)) building model (Type 56), is a further development of the RF method (Stephenson & Mitalas, 1971). The method is explained in (Delcroix, Kummert, Daoud, & Hiller, 2012) and (Giaconia & Orioli, 2000). The wall surface heat fluxes are calculated as shown in Equation 10, using the convolution operator defined in Equation 7 as shorthand notation. For the convenience of the reader, we have written out the computation of the internal heat flux. The CTF coefficients a=(a_0, a_1, …), b, c, d are computed with the DRF method (Mitalas & Arseneault, 1972).

$$d * q_{in} = b * T_{s,ex} - c * T_{s,in}$$
$$d * q_{ex} = a * T_{s,ex} - b * T_{s,in}$$
$$q_{in}(t_j) = \sum_{k=0}^{j} b_k T_{s,ex}(t_{k-j}) - \sum_{k=0}^{j} c_k T_{s,in}(t_{k-j}) - \sum_{k=1}^{j} d_k q_{in}(t_{k-j}) \quad (10)$$

Simulations

Tables 1 and 2 show the wall layers used and the simulated wall compositions.

Table 1
Wall layers

LAYER	λ [W/mK]	ρ [kg/m³]	c [J/kgK]
Brickwork	0.6	1560	850
Insulation	0.043	91	840

Table 2
Simulated wall compositions

WALL NO.	EXTERIOR LAYER	INTERIOR LAYER
1	40 cm brickwork	-
2	70 cm brickwork	-
3	15 cm insulation	70 cm brickwork
4	70 cm brickwork	15 cm insulation

As there is no radiation exchange, we have assumed constant convective surface heat transfer coefficients for each wall surface in accordance with EN ISO 13786:2007: h_{ex}=17.76 W/(m^2 K) for the exterior and h_{in}=3.07 W/(m^2 K) for the interior, respectively. We have set the wall emissivity to zero if the software allows it, otherwise to 1e-9. As geometric reference, we have used a Cartesian coordinate system with the *yz*-plane parallel to the wall surfaces and *x*=0 on the external and *x*=*L* on the internal wall surface. Two accuracy scenarios have been considered (Table 3). Simulations with EnergyPlus FD have been performed only for Scenario A as EnergyPlus allows only simulations with the FD method for a time step smaller or equal than 3 minutes. We have chosen three time series $f(t)$ for the external air temperature to assess different aspects of the wall's response (Table 4). Simulations have been run for all walls, forcing functions, software and accuracy scenarios, for a total of 120 runs. The first month has been simulated with constant $f(t)$ to reach the steady-state. For the massive walls, depending on the initial conditions set by the software, the steady-state could not always be reached in one month. In those cases, an additional month has been simulated before the excitation. The weather file has been retrieved from the EnergyPlus website (DOE).

Table 3
Parameters of the accuracy scenarios

PARAMETER	SCENARIO A	SCENARIO B
TRNSYS		
Time step	0.5 min	0.5 h
Active Layer (AL)	Yes	No
Time base	Wall 1: 6 min Wall 2: 15 min Wall 3: 45 min Wall 4: 45 min	Wall 1: 0.5 h Wall 2: 1.5 h Wall 3: 2.5 h Wall 4: 2.5 h
Delphin		
No. of nodes	Wall 1: 45 Wall 2: 78 Wall 3: 92 Wall 4: 92	Wall 1: 45 Wall 2: 78 Wall 3: 92 Wall 4: 92
Tolerances	Trel=Tabs=1e-8	Trel=1e-5 Tabs=1e-8
Max time step	0.5 min	0.5 h
Max order	5	5
Output time step	1 min	0.5 h
EnergyPlus FD		
No. of nodes	As in Delphin	NA
Time step	1 min	NA
Inverse Fourier coefficient	3	NA
Relaxation factor	1	NA
Intra-layer temperature convergence criterion	ΔT<0.02 K	NA
EnergyPlus CTF		
Time step	1 min	0.5 h

Table 4
External air temperature time series

FF NO.	FF NAME	31 DAYS	28 DAYS
1	Step	0 [°C]	10 [°C]
2	Sinusoid	0 [°C]	Amplitude: 5 [°C] Period: 1 day
3	Weather file temperature time series	2 [°C]	Dry-bulb air temperatures in Bologna in February

As reference, we have used our RF method implementation in MATLAB with a time step and a time base of 30 seconds (RF0.5) or EN ISO 13786:2007.

RESULTS

Accuracy of our RF method implementation

First, we have checked the convergence of our code by performing simulations on Wall 2 for different time steps tending to zero. For FF 1, the maximum difference in external heat flux between a simulation with Δt=1 min (RF1) and one with Δt=30 sec (RF0.5) has been 2.7 W/m^2 one minute after the jump of the FF (where the analytical external flux is infinite). The

difference at the end of month 2 has been less than machine accuracy (less than 1e-15). For FF 2, the maximum difference between RF1 and RF0.5 has been 3.8e-4 W/m^2 (relative error 9.0e-6) in external heat flux and 2.2e-4 K (relative error 4.7e-5) in external surface temperature. As the internal wall surface heat fluxes and temperatures are smoother, the relative errors have been less.
All simulations performed and especially those shown in this paper have been useful to check our RF method implementation for systematic error.

Comparison of step FF simulations

We have calculated delay and settling times of $T_{s,ex}$ and $T_{s,in}$ of both scenarios and all walls, forcing functions and software, and have compared them with those of the reference. Delay time is the time required for the response to reach the average between initial and final value the very first time. We have defined settling time as the time required to remain within a range of 2% of the difference between initial and final value. Delay and settling times of q_{ex} are not well-defined as the flux is infinite at the jump of the forcing function. q_{in} is proportional to $T_{s,in}$ (as $T_{a,in}$ is constant) and thus has the same delay and settling times as $T_{s,in}$. Table 5 shows the reference values of RF0.5.

Table 5
Delay and settling time reference values

DELAY TIME			SETTLING TIME	
Wall no.	$T_{s,ex}$	$T_{s,in}$	$T_{s,ex}$	$T_{s,in}$
1	22m	1d0h53m	1d15h58m	3d21h12m
2	24m	2d15h40m	2d22h59m	9d11h56m
3	2m	5d22h39m	57m	22d17h23m
4	26m	4d13h3m	6d10h22m	18d12h43m

Table 6 shows the delay time differences in minutes with respect to the reference. Positive values indicate higher delay times as those reported in Table 5, negative values lower delay times.

Table 6
Delay time differences for $T_{s,ex}$ for both scenarios

SOFTWARE	WALL NO.	SCEN A [min]	SCEN B [min]
TRNSYS	1	+3	+8
TRNSYS	2	+7	-24
TRNSYS	3	0	-2
TRNSYS	4	+20	-26
Delphin	1	+6	+38
Delphin	2	+5	+36
Delphin	3	+1	+28
Delphin	4	+5	+34
E+ FD	1	0	+8
E+ FD	2	-1	+6
E+ FD	3	0	+28
E+ FD	4	-1	+4
E+ CTF	1	1	+8
E+ CTF	2	-22	-24
E+ CTF	3	0	-2
E+ CTF	4	-24	-26

Delay times for $T_{s,in}$ vary by less than 2% in all simulated cases. Settling times for $T_{s,ex}$ vary by less than 2% in most cases. Table 7 reports those cases where settling times have varied by more than 2%. As in Table 6, positive values indicate higher settling times as those reported in Table 5, negative values lower settling times.

Table 7
Settling times for $T_{s,ex}$ in case of more than 2% difference

SOFT-WARE	WALL NO.	SCENARIO	[min]
TRNSYS	3	A	-13
E+ CTF	3	A	+44
TRNSYS	3	B	+123
Delphin	3	B	+33
E+ CTF	3	B	+63

Settling times for $T_{s,in}$ vary by less than 2% in all cases except one. For E+ CTF, Wall 1 and Scenario B, the difference has been 4%.

Comparison of sinusoid FF simulations

For all simulated runs, we have calculated the periodic thermal transmittance Y_{12} and the decrement factor f in two ways: numerically and according to EN ISO 13786:2007. The reference values are shown in Table 8 together with the thermal transmittance of the wall.

Table 8
Periodic thermal transmittance and decrement factor calculated according to EN ISO 13786:2007

WALL NO.	U [W/m^{2}K]	$\lvert Y_{12}\rvert$ [W/m^{2}K]	$arg(Y_{12})$	f [-]
1	0.95	0.097	-13h44m	0.10
2	0.65	0.0066	-1d0h0m	0.010
3	0.20	0.00029	-1d3h59m	0.0015
4	0.20	0.00066	-1d2h57m	0.0033

In accordance to EN ISO 13786:2007, the negative time shift indicates that the internal wall surface heat flux lags behind the external air temperature. Of course, the best damping with the highest time shift is achieved for the externally insulated wall. As $\lvert Y_{12}\rvert$ is very small, errors in the numerical computation of $\lvert Y_{12}\rvert$ vary considerably according to whether the steady-state before and after the excitation is reached or not. Therefore, we have simulated two additional

months both before and after the excitation, and errors have been below 5% in all cases.
The errors in the phase of Y_{12} are less than 2% for Scenario A. For Scenario B, the errors range from 1 to 30 minutes.

Comparison of real temperature FF simulations

We have used the Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and time integral over February as metrics. The RMSE gives a relatively high weight to large errors and is always larger than or equal to the MAE. We have calculated the difference between the RMSE and the MAE to obtain the variance in the individual errors in the time series. Integrating the absolute differences in the heat fluxes over time gives the absolute error in energy transmitted through the wall surface over the simulated time period. Results for the external wall surface heat flux q_{ex} are reported in Table 9. q_{ex} has been chosen for the purpose of demonstration as the errors are more evident than for q_{in}.

Table 9
Comparison of RMSEs of q_{ex}. Column 4 shows the difference in value between Scenario A and B. Column 5 shows the ratio between Scenario B and A

		SCEN A	**B − A**	**B/A**
SOFT-WARE	**WALL NO.**	q_{ex} **[W/m²]**	q_{ex} **[W/m²]**	**[%]**
TRNSYS	1	0.1580	-0.0435	72%
TRNSYS	2	0.3260	0.9705	398%
TRNSYS	3	0.1295	0.1584	222%
TRNSYS	4	0.7497	1.2926	272%
Delphin	1	0.1580	-0.1151	83%
Delphin	2	0.3260	-0.0604	90%
Delphin	3	0.1295	0.0660	145%
Delphin	4	0.7497	-0.0574	91%
E+ FD	1	0.0871	NA	NA
E+ FD	2	0.0871	NA	NA
E+ FD	3	0.1025	NA	NA
E+ FD	4	0.0871	NA	NA
E+ CTF	1	0.5994	1.2982	317%
E+ CTF	2	1.5367	0.1667	111%
E+ CTF	3	0.0902	0.0397	144%
E+ CTF	4	1.5935	0.2371	115%

In terms of RMSE, Scenario A yields smaller errors than Scenario 2 in 28 (58%) of the total 48 cases (Walls 1-4, the three software TRNSYS, Delphin and E+ CTF, and the four time series for $T_{s,ex}$, $T_{s,in}$, q_{ex} and q_{in}). On average, the RMSEs of Scenario B are more than double (210% as big as) the RMSEs of Scenario A. In some cases, the RMSEs of Scenario B are more than 4 times (up to 412% as big as) the RMSEs of Scenario A.

In terms of MAE, results are similar. Scenario A yields smaller errors than Scenario B in 30 (63%) of the cases. On average, the MAEs of Scenario B are 205% as big as the MAEs of Scenario A. In some cases, the MAEs of Scenario B are up to 418% as big as the MAEs of Scenario A.

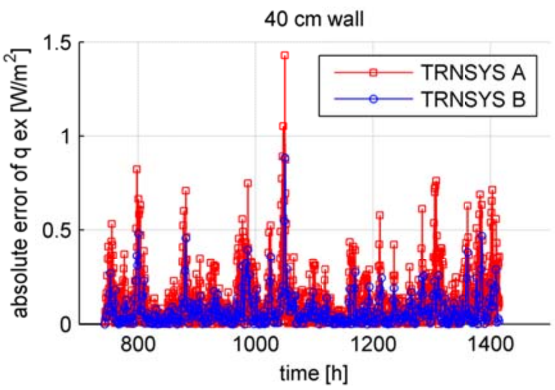

Figure 1 Absolute errors of q_{ex} for Wall 1 simulated in TRNSYS

We report some interesting cases. The Wall 1 temperatures and heat fluxes simulated in TRNSYS are more accurate in Scenario B than in Scenario A (Figure 1). The absolute errors in Scenario B are 12% to 73% as big as the errors in Scenario A. With regard to massive walls, we have observed the opposite: for Walls 2-4, Scenario A has been more accurate than Scenario B (detail shown in Figure 3).

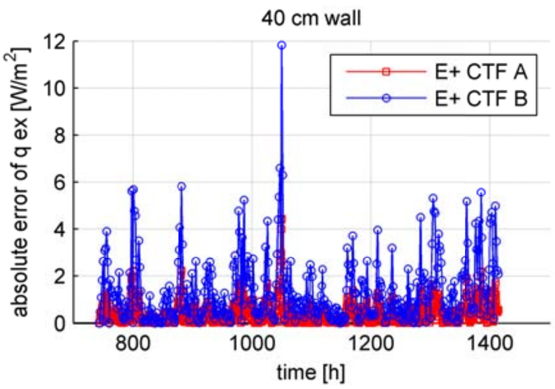

Figure 2: Absolute errors of q_{ex} for Wall 1 simulated in EnergyPlus with the CTF method

Scenario A is also better than Scenario B for the calculation of $T_{s,ex}$ and q_{ex} for all walls simulated with the EnergyPlus CTF method. We have observed the biggest improvement for Wall 1 (Figure 2). $T_{s,in}$ and q_{in} have been more accurate in Scenario A for Walls 1-3 and slightly less accurate for Wall 4 than in Scenario B. In the case of Delphin, results do not indicate a clear preference for either Scenario, but this is due to offsets in the solutions (see the Discussion).

The differences in energy due to errors in the computation of q_{ex} over one month are reported in Tables 10 to 12. Table 10 shows a comparison between the accuracy scenarios. Comparisons between software are reported in Tables 11 and 12 taking RF0.5 as reference. We have chosen to present the results for the external heat flux as the errors are more evident.

Table 10
Comparison of scenarios with respect to the error in energy due to errors in q_{ex}

ERRORS IN ENERGY [kWh/(m^2 month)]				
SOFT-WARE	WALL NO.	SCEN A	B – A	B/A [%]
TRNSYS	1	0.0662	-0.0175	73%
TRNSYS	2	0.1369	0.4337	417%
TRNSYS	3	0.0544	0.0689	227%
TRNSYS	4	0.3243	0.5727	277%
Delphin	1	0.3582	-0.0530	85%
Delphin	2	0.3133	-0.0268	91%
Delphin	3	0.0888	0.0236	127%
Delphin	4	0.3354	-0.0239	93%
E+ FD	1	0.0404	-	-
E+ FD	2	0.0404	-	-
E+ FD	3	0.0465	-	-
E+ FD	4	0.0404	-	-
E+ CTF	1	0.2731	0.6134	325%
E+ CTF	2	0.7089	0.0841	112%
E+ CTF	3	0.0359	0.0201	156%
E+ CTF	4	0.7335	0.1133	115%

In the following text, all errors are reported in kWh/(m^2 month). The errors vary between 0.036 and 0.90 for the external heat flux and between 0.00011 and 0.73 for the internal heat flux. The average for the external heat flux is 0.22 for Scenario A and 0.44 for Scenario B. The average for the internal heat flux is 0.15 for Scenario A and 0.036 for Scenario B.

Table 11
Scenario A: comparison of errors in energy among software due to errors in q_{ex}

ERRORS IN ENERGY [kWh/(m^2 month)] AMONG SOFTWARE – SCENARIO A				
Wall no.	TRNSYS	Delphin	E+ FD	E+ CTF
1	0.0662	0.3582	0.0404	0.2731
2	0.1369	0.3133	0.0404	0.7089
3	0.0544	0.0888	0.0465	0.0359
4	0.3243	0.3354	0.0404	0.7335

We have obtained the error of 0.90 for the TRNSYS simulation of Wall 4 and Scenario B (see Figure 3). We have attributed the error to the jaggedness of the curves caused by the large time base. Although the solution in Scenario A appears more jagged, the error is less because of the smaller time base.

Table 12
Scenario B: comparison of errors in energy among software due to errors in q_{ex}

ERRORS IN ENERGY [kWh/(m^2 month)] AMONG SOFTWARE – SCENARIO B			
Wall no.	TRNSYS	Delphin	E+FD
1	0.0486	0.3052	0.8864
2	0.5706	0.2865	0.7930
3	0.1233	0.1125	0.0560
4	0.8971	0.3115	0.8468

The errors in energy due to errors in q_{ex} in Scenario A simulated with E+ FD are between 0.040 and 0.047 for all walls. The errors due to q_{in} range from 0.00011 to 0.00065.

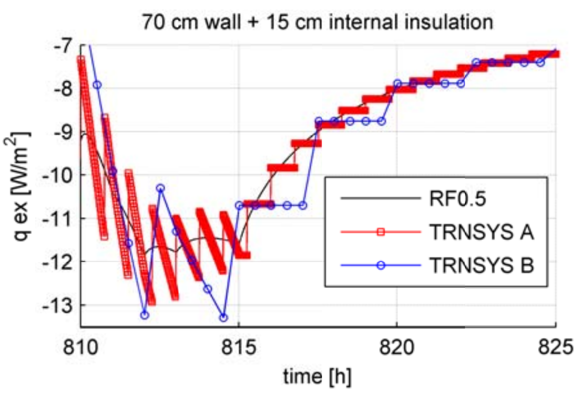

Figure 3 q_{ex} for Wall 4 simulated with TRNSYS

The simulation of Wall 4 with E+ CTF in Scenario B has an error in energy of 0.847 similar to that of TRNSYS, but for a different reason. The solution is not jagged, but slightly displaced, and peaks are underrated (Figure 4). The error in energy due to q_{ex} is less in Scenario A for all walls.

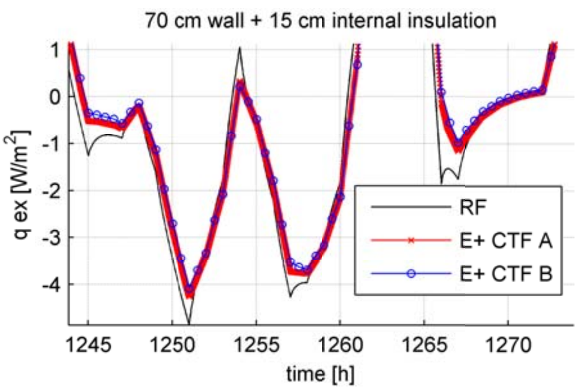

Figure 4 q_{ex} for Wall 4 simulated with E+CTF

DISCUSSION AND CONCLUSIONS

Numerical calculation of periodic thermal transmittance

The assessment of the periodic thermal transmittance in case of a wall with high internal mass and external insulation is very sensitive to the steady-state. Different strategies are used among software to compute the wall surface temperatures and heat fluxes before the excitation. Our implementation computes the steady-state analytically from the known air temperatures and convection coefficients. By default, Delphin and TRNSYS start from a different steady-state than that indicated in Table 4. Therefore, the computation of the periodic thermal transmittance will not be precise if the steady-state indicated in Table 4 is not reached before and after the excitation. For Wall 3 and RF0.5, the computed $|Y_{12}|$ has an error of 28% after 28 days of periodic external air temperatures because the steady-state has not been reached after one month. Simulating for another month, the error drops below 1%.

Accuracy of computed CTF coefficients

It is common practice to use Equation 11 to verify that the CTF coefficients determined by EnergyPlus and TRNSYS yield the correct steady-state heat transfer:

$$\frac{\sum_{k=0}^{N_x} X_E^k}{1-\sum_{k=1}^{N_\varphi} \varphi_E^k} = \frac{\sum_{k=0}^{N_y} Y_E^k}{1-\sum_{k=1}^{N_\varphi} \varphi_E^k} = \frac{\sum_{k=0}^{N_z} Z_E^k}{1-\sum_{k=1}^{N_\varphi} \varphi_E^k} \approx G$$
$$\frac{\sum_{k=0}^{N_a} a_k}{\sum_{k=0}^{N_d} d_k} = \frac{\sum_{k=0}^{N_b} b_k}{\sum_{k=0}^{N_d} d_k} = \frac{\sum_{k=0}^{N_c} c_k}{\sum_{k=0}^{N_d} d_k} \approx G \qquad (11)$$

For the walls considered in this paper, this is not always the case. The smaller the time base and the longer the response of the wall, the more coefficients have to be calculated to capture the entire response of the wall. A smaller time base means that less time passes between two temperature pulses. Therefore, more coefficients are needed to record the response of the wall for the same amount of time. In EnergyPlus and TRNSYS, the number of calculated coefficients varies only to a certain extent; therefore, the terms in Equation 11 become generally less precise for small time bases. On the other hand, a large time base means that temperatures and fluxes are recorded less frequently, causing again inaccuracies. It is well known that the calculation of only a small number of CTF coefficients causes the simulation to become unstable or even diverge for too small time bases, especially in case of massive walls. Moreover, commercial programs are optimized for the common case in terms of speed and memory; therefore, round-off and truncation errors as well as numerically ill-conditioned algorithms like the computation of the coefficients of a polynomial from its roots are involved. Indeed, our implementation is stable for very small time bases such as 30 seconds, because much attention has been paid to the calculation of the series in Equation 4, only basic algebra has been used, and almost 100,000 RFs have been stored in case of the 70 cm brick wall with 15 cm insulation. TRNSYS does not simulate in that case reporting a stability error. EnergyPlus uses staggered CTF coefficient histories combined with interpolation to keep the accuracy for a decrease of the time step up to 1 min. The cross coefficients have an error of about 3% with respect to the analytical conductance of all walls, but the inner and outer coefficients are 2400% wrong in the worst case for Wall 4 and a time step of 1 min.

In the case of TRNSYS, all errors are below 0.001 W/m^2 as too small a time base cannot be used a priori. In our implementation, the errors are below 0.001 W/m^2 by design.

Comparison of real temperature time series simulations

In most cases, if accuracy is of concern, we recommend Scenario A. Of course, a time step of 30 seconds will be exaggerated for most applications. Time steps of 3 to 15 minutes should be accurate enough if systems with fast responses are controlled. Otherwise, time steps of half an hour or an hour will usually suffice. We have seen that in special cases Scenario B is even better. Scenario B is better for Wall 1 simulated in TRNSYS because the time base is equal to the time step and the actual flux is not changing rapidly within one time step. An issue that arises in TRNSYS, especially for small time bases, can be seen in Figure 3. The small time base produces a jagged curve that is accurately tracked due to the small time step. Although local values are not so reliable, the moving average follows the reference solution quite well. Note the arcs of the reference solution. These are caused by the linear interpolation of the hourly temperatures taken from the weather file. In case of massive walls, TRNSYS behaves better due to the smaller time base (Figure 3). The smaller time base has been possible thanks to the insertion of an active layer (AL). In this case, the time step could be larger as there is no need to track the jagged curve with such precision. Using the CTF method of EnergyPlus, this problem does not arise as the CTF coefficients are computed using staggered temperature and heat flux time histories and interpolation.

In Delphin, Scenario A is generally better than Scenario B, especially near non-differentiable points produced by the linear interpolation of the weather file temperatures (see Figure 5). For very smooth solutions there is no real need to use a very small time step. Delphin is clearly offset with respect to the reference solution because of a slightly different initial steady-state. Indeed, the RMSEs have been only 12% larger than the MAEs in that particular case. The initial steady-state found depends on the number and position of the intra-wall nodes. Shifting the solution upward so that the curves overlap, Delphin tracks the reference solution with very slight differences. We have found similar results for the E+ FD simulations.

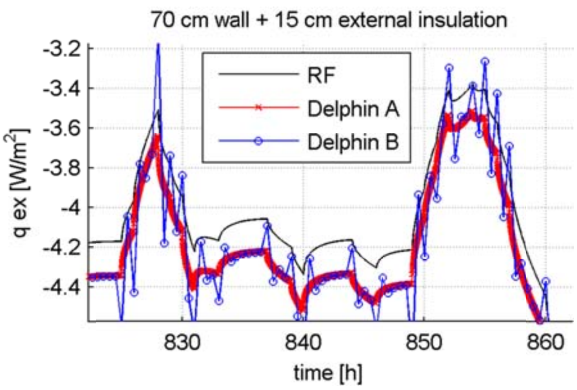

Figure 5 q_{ex} for Wall 4 simulated in Delphin

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NOMENCLATURE

α = wall thermal diffusivity [m²/s]
a_j = j-th CTF coefficient (TRNSYS) [W/(m² K)]
b_j = j-th CTF coefficient (TRNSYS) [W/(m² K)]
c = wall layer specific heat capacity [J/(kg K)]
c_j = j-th CTF coefficient (TRNSYS) [W/(m² K)]
d_j = j-th CTF coefficient (TRNSYS) [-]
$f(t)$ = external air temperature forcing function [K]
G = wall conductance [W/(m² K)]
h_{ex} = exterior convection coefficient [W/(m² K)]
h_{in} = interior convection coefficient [W/(m² K)]
λ = wall layer thermal conductivity [W/(m K)]
L = wall thickness [m]
q = heat flux [W/m²]
q_0 = initial steady-state heat flux [W/m²]
q_{ex} = heat flux at external wall surface [W/m²]
q_{in} = heat flux at internal wall surface [W/m²]
ρ = wall layer density [kg/ m³]
t = time [s]
Δt = time base, time step [s]
T = temperature [K]
$T_{a,ex}$ = external air temperature [K]
$T_{a,in}$ = internal air temperature [K]
$T_{a,in}$ = internal air temperature [K]
$T_{s,ex}$ = external wall surface temperature [K]
$T_{s,in}$ = internal wall surface temperature [K]
φ_E^j = j-th flux coefficient (EnergyPlus) [-]
x = space coordinate perpendicular to the wall surface [m]
X_j = j-th outside response factor [W/(m² K)]
X_E^j = j-th outside CTF coefficient [W/(m² K)]
Y_j = j-th cross response factor [W/(m² K)]
Y_E^j = j-th cross CTF coefficient [W/(m² K)]
Z_j = j-th inside response factor [W/(m² K)]
Z_E^j = j-th outside CTF coefficient [W/(m² K)]

REFERENCES

3encult. (2010-2014). Retrieved 2 15, 2013, from 3encult - Efficient Energy for EU Cultural Heritage: http://www.3encult.eu/

Delphin v5.6.8. (2012). Retrieved from Delphin - Simulation program for the calculation of coupled heat, moisture, air, pollutant, and salt transport: http://www.bauklimatik-dresden.de/delphin/index.php?aLa=en

EnergyPlus v7.2.0. (2012). Retrieved from EnergyPlus Energy Simulation Software: http://apps1.eere.energy.gov/buildings/energyplus/

Asan, H. (2006). Numerical computation of time lags and decrement factors for different building materials. *Building and Environment 41*, 615–620.

Cannon, J. R. (1984). *The one-dimensional heat equation*. Reading, MA/Cambridge, UK: Addison-Wesley/Cambridge University Press.

Cellura, M., Giarrè, L., Lo Brano, V., & Orioli, A. (2003). Thermal dynamic models using Z-transform coefficients: an algorithm to improve the reliability of simulations. *Eighth International IBPSA Conference.* Eindhoven, NL.

Chen, Y., Zhou, J., & Spitler, J. D. (2006). Verification for transient heat conduction calculation of multilayer building constructions. *Energy and Buildings 38*, 340–348.

Delcroix, B., Kummert, M., Daoud, A., & Hiller, M. (2012). Conduction transfer functions in TRNSYS Multizone Building model: current implementation, limitations and possible improvements. *SimBuild 2012 - IBPSA-USA.* Madison, Wisconsin.

DOE. (n.d.). *energyplus/weatherdata*. Retrieved from EnergyPlus Energy Simulation Software: http://apps1.eere.energy.gov/buildings/energyplus/weatherdata_about.cfm

Giaconia, C., & Orioli, A. (2000). On the reliability of ASHRAE conduction transfer function coefficients of walls. *Applied Thermal Engineering*, 21-47.

Li, X. Q., Chen, Y., Spitler, J., & Fisher, D. (2009). Applicability of calculation methods for conduction transfer function of building constructions. *International Journal of Thermal Sciences 48*, 1441–1451.

Mitalas, G., & Arseneault, J. (1972). *Fortran IV program to calculate z-transfer functions for the calculation of transient heat transfer through walls and roofs.* Ottawa, CA: Division of Building Research, National Research Council Canada.

Stephenson, D. G., & Mitalas, G. P. (1967). Room thermal response factors. *ASHRAE Transactions 73(2)*, III.2.1-III.2.10.

Stephenson, D., & Mitalas, G. (1971). Calculation of heat conduction transfer functions for multi-layer slabs. *ASHRAE Transactions 77(2)*, pp. 117-126.

Thermal Energy System Specialists (TESS). (n.d.). *Trnsys*. Retrieved from http://www.trnsys.com/

Underwood, C. P., & Yik, F. W. (2004). *Modelling methods for energy in buildings.* Malden, MA: Blackwell Publishing.