AN INVERSE MODEL WITH UNCERTAINTY QUANTIFICATION TO ESTIMATE THE ENERGY PERFORMANCE OF AN OFFICE BUILDING

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ABSTRACT

Buildings account for 40% of primary energy consumption in the United States. Reducing energy consumption becomes more and more important for economic and ecological reasons. In order to reduce energy consumption in buildings, energy specialists not only explore the technologies that are suitable for new construction, they also show great interests in retrofitting existing buildings. To estimate the energy savings after retrofitting an existing building with certain energy efficient measures, other research efforts have developed various types of baseline building models, ranging from simple statistical models to complex calibrated computer models.

In this paper, we present the development of an inverse model using a Gaussian Process Regression (GPR) method to access the baseline building energy consumption in the pre-retrofit phase, which is a less time-consuming and easy to accomplish process. This method is applied to an office building as a case study and the accuracy of the regression model is compared with a linear regression model and tested with measurement data.

INTRODUCTION

In the United States, the building sector accounted for about 41% of primary energy consumption in 2010 (DOE 2010). Enhancing building efficiency represents one of the easiest, most immediate and most cost effective ways to reduce the nation's energy consumption and carbon emissions. One method to help save energy in buildings is retrofitting the existing building. Energy efficiency retrofit decision are typically made based on predictions of how much energy and money a retrofit will save and the expected payback period of certain energy conservative measures (ECM). In performance contracting provided by the typical Energy Service Company (ESCO), the payment for a retrofit is based on predicted and actual savings associated with ECMs.

Figure 1 illustrates the general methodology defined in the ASHRAE guideline 14, Measurement of Energy and Demand Savings (ASHRAE 2002). The energy savings brought by ECMs is equal to the estimated baseline building energy use minus the measured post-retrofit building energy use.

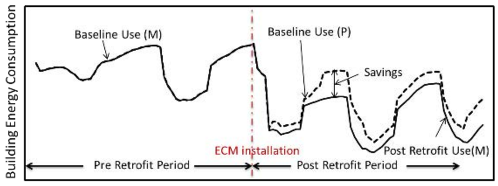

Figure 1 Energy saving calculation (M-Measurement, P-Model Prediction)

The general steps to develop an inverse building model and estimate savings from ECMs are:

Step 1: Measure energy use and influential variables during the pre-retrofit period.

Step 2: Develop a regression model of pre-retrofit energy use as a function of influential variables.

Step 3: Measure energy use and influential variables during the post-retrofit period.

Step 4: Use the values of the influential variables from the post-retrofit period in the pre-retrofit model to predict how much energy the building would have used if it had not been retrofitted.

Step 5 Subtract measured post-retrofit energy use from the predicted pre-retrofit energy use to estimate savings. (ASHRAE 2002)

Therefore, defining a baseline prior to a retrofit is essential. Generally speaking, the two most commonly used approaches to define the baseline, are a calibrated simulation (forward modeling) and a data-driven regression analysis (inverse modeling).

One traditional way to estimate the baseline energy consumption in a retrofit project is to build a predictive building energy model (typically using a whole building simulation program such as EnergyPlus (EnergyPlus 2013) and eQuest (eQuest, 2013)), then calibrate the model with utility bills and other measurement data in the pre-retrofit phase. However, this method has two major limitations: (1) it requires detailed building information which is not easy to get and (2) creating and calibrating the predictive building energy model is time-consuming and labor intensive. Another commonly used approach is inverse modeling which statistically derives a relationship between a set of inputs (in particular the ambient conditions) and outputs (e.g., energy consumption). In practice, inverse models trained by building energy consumption data have been deployed for measurement and verification, and ongoing commissioning of building performance (Claridge 2004).

Currently, the most popular approaches in inverse modeling of buildings are based on regression techniques. This is typically done in an ad-hoc manner relying on the assumption that the nonlinear energy behavior arising from complex multivariable relationships between ambient conditions, occupancy levels, and building operating conditions can be captured adequately. In this paper, we present the development of an inverse model using a GPR method to access the baseline building energy consumption in the pre-retrofit phase, which is a less time-consuming and easy to accomplish process. The GPR approach, which follows a Bayesian setting, is capable of modeling complex nonlinear behavior in multivariate problems. In addition, the ASHRAE defined linear change-point regression model (Kissock 2003) is also developed for the comparison with the proposed GPR model. The GPR model structure and model uncertainty quantification aspects are discussed as well. Finally, the proposed GPR approach is demonstrated with an office building case study using real-time monitored building data.

In this study, the purpose of developing regression based inverse models trained by building operation data is 1) to predict transient cooling and heating requirements for the building, and 2) for measurement and verification and ongoing commissioning of building performance. The proposed regression was derived from GPR method. In the post-retrofit phase, both forward modeling and inverse modeling can be used to estimate the baseline building energy performance, calculate the energy savings after retrofits, and predict energy consumption. However, the forward modeling requires more information related to the building compared with the inverse modeling; the information is usually not easy to get. Additional calibration work needs to be done to make sure the forward model is close enough to the actual building utility data. The inverse model only utilizes the parameters that can be measured in buildings and provides reliable results.

In this study, the first step for inverse modeling is data collection. The common driving variables considered in the inverse models include environmental variables such as ambient dry bulb temperature and wet bulb temperature, ambient air humidity ratio, solar radiation, and the building energy usage, such as building electricity consumption for lighting, plug load, air handling unit (AHU), condensing unit (CU), chiller, cooling tower, and domestic hot water (DHW) gas consumption. The next step is to develop the inverse models using a linear regression model and a GPR model. Then, the model validation and model comparison are performed. Finally, conclusions arising from this case study are presented.

METHODOLOGY

The goal is to develop a building inverse model using correlations and regressions with the objective to use the model for predicting baseline building energy consumption in the post-retrofit phase.

Several predominant inverse models have been used by building contractors, building consultants and building owners to calculate the savings from energy efficiency retrofits. These inverse models include constant-base degree-days models, variable-base degree-days models, bin method models, correlation and regression models and artificial intelligence models (Abushakra 1997). The different models are briefly summarized below.

Constant-base degree-days models

Degree days are calculated as the sum of the differences between daily average temperatures and the base temperature (ASHRAE 2009). Heating degree days and cooling degree days are used extensively in calculations related to building energy consumption. Heating degree day are a measure of how much (in degrees), and for how long (in days), the outside air temperature is below a certain level. They are commonly used in calculations of the heating energy consumption. On the other hand, cooling degree days are a measure of how much (in degrees), and for how long (in days), the outside air temperature is above a certain level. They are commonly used in calculations of cooling energy consumption.

The constant-base degree-days model is used to use the degree day as an independent variable while building total electricity consumption as dependent variable. Figure 2 shows an example of a constant-base degree-days model.

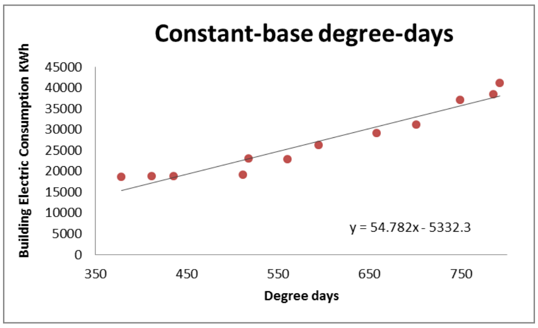

Figure 2 Constant-base degree-days model

Variable-base degree-days model

In many single-zone buildings, such as residential and small commercial buildings, space-heating energy use increases as outdoor air temperature decreases below a balance-point temperature. The heating balance-point temperature is defined as the temperature at which the heat gain from internal occupants and equipment balances heat loss through the building envelope. At outdoor air temperatures above the balance-point temperature, no thermal energy is needed for space heating; however thermal energy may also be required for hot water or cooking. Similarly, cooling energy use frequently increases as outdoor air temperature increases above some cooling balance-point temperature, below which no space cooling is necessary. However, the type of energy used for cooling, such as electricity, may also be used for non-cooling applications. Thus, monthly electricity consumption in many single-zone residences forms the pattern shown in Figure 3 (Kissock 2002).

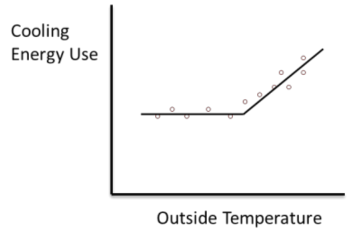

Figure 3 Fitted model of building cooling energy use against balance-point temperature

Bin method models

The bin method model is based on Box–whisker–mean plots theory. It is convenient to use Box–whisker–mean plots to display the main features of a set of data and these plots facilitate the comparison of multiple data sets (Wu 2009). The calculation of retrofit savings pertains to the energy conservation retrofits and other energy consuming systems that are primarily influenced by schedule-dependent loads. In a typical before-after measurement analysis a baseline energy method is determined and then used to predict energy use in the post-retrofit period. In the bin method, the bin model predicts average hourly pre-retrofit electricity use during any hour of day in the post-retrofit period, and compares the value with measured hourly post-retrofit energy use for every hour of day to get the total energy saving of a certain energy conservation retrofit (Thamilseran 1995) as shown in Figure 4.

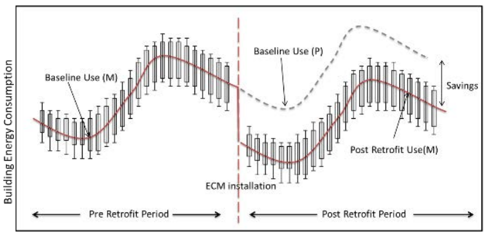

Figure 4 Box−whisker− mean plots

Correlation and regression models

Regression methods are most often used because they are simple to develop, easy to use, and retain the ability to calculate the associated uncertainty. The simplest is the two-parameter linear model, which is used to regress the hourly, daily or monthly weather-dependent energy use against temperature. The physical basis for this correlation is the fact that most of the time, the building envelope and ventilation heating loads are a strong function of the average outdoor dry bulb temperature. Studies have also been performed with change-point regressions of energy consumption against ambient temperature in residential (Fels 1986) and commercial (Claridge et al. 1992) buildings. These studies suggested that buildings that have thermostatically controlled HVAC systems usually exhibit behavior that can be explained with a change-point regression model driven by ambient temperatures. There are four different regression models as explained below:

Simple least squares linear regression

A general least-squares regression method would generate the model with a set of parameters which minimizes the sum of the squared error between predicted value and actual observations. Figure 5 shows an example of a linear regression model.

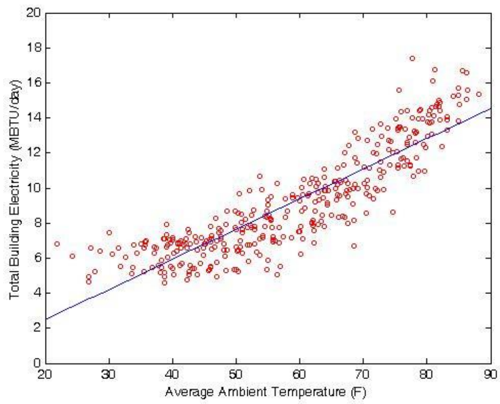

Figure 5 Linear regression model

Change-point model

Space-heating energy use increases as outdoor air temperature decreases below the change point temperature. When the temperature is above the change-point temperature, the boiler gas consumption will remain constant. Thus, monthly gas consumption forms a pattern as shown in Figure 6 below（Kissock 2003）.

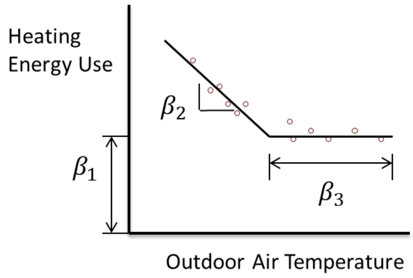

Figure 6 Chang Point Heating Model

Equation 1 shows the relationship between building heating energy consumption and outdoor air temperature.

$$Y_H = \beta_1 + \beta_2 (X - \beta_3)^- \quad (1)$$

β_1 the constant term, which describe the independent fuel consumption

β_2 the slope term, which indicate the heating energy change per degree temperature

β_3 the change point

$()^-$ indicate that the values of the parenthetic term shall be set to zero when they are positive

Space cooling energy use increases as outdoor air temperature increases above some cooling change point temperatures, below which no space cooling is necessary. However, the type of energy used for cooling, such as electricity, may also be used for non-cooling applications. Thus, monthly electricity bills show an energy use pattern similar to that in Figure 7 below（Kissock 2003）.

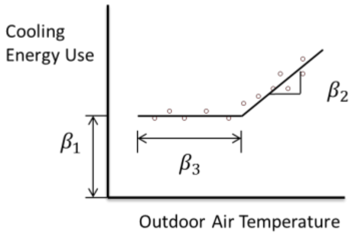

Figure 7 Chang Point Cooling Model

Equation 2 below shows the relationship between building heating energy consumption and outdoor air temperature.

$$Y_C = \beta_1 + \beta_2(X - \beta_3)^+ (2)$$

β_1 the constant term, which describe the independent electricity consumption

β_2 the slope term, which indicate the cooling energy change per degree temperature

β_3 the change point

$()^+$ indicate that the values of the parenthetic term shall be set to zero when they are negative

Change-Point Multivariable Regression Model (CP-MVR)

The change-point (CP) model, as described above, can provide good fits between building energy use and ambient temperatures. However, other variables may also influence building energy use. A simple multivariable regression model could capture the effect of multiple independent variables; however, it could not model energy use that varies at ambient temperature change points (Kissock 2003). The CP-MVR model retains the ability to model energy use with temperature change points, while including the effects of additional independent variables as shown in Equations 3 and 4.

$$Y_C = \beta_1 + \beta_2(X - \beta_3)^+ + \beta_4 X_2 + \beta_5 X_3 + \beta_6 X_4 (3)$$

$$Y_H = \beta_1 + \beta_2(X - \beta_3)^- + \beta_4 X_2 + \beta_5 X_3 + \beta_6 X_4 (4)$$

Gaussian Process Regression

Gaussian process regression is an approach that has recently received attention in many application domains. A key advantage of this approach is that it constructs the model by specifying the structure of the covariance matrix of the explanatory variables rather that the algebraic structure of the input-output relationship itself, as is done in traditional parametric regression approaches. Gaussian processes regression is a highly flexible method that can quantify different sources of uncertainty in a natural way (Heo 2012).

A Gaussian process is a collection of random variables, any finite number of which have consistent joint Gaussian distributions. A Gaussian process is fully specified by its mean function μ(x) as shown in Equation 5 and covariance function $k(x, x')$ as shown in Equation 6, which indicates how correlated the function value y is at x and x'.

$$\mu(x) = E[f(x)] (5)$$

$$k(x, x') = E[(f(x) - \mu(x))(f(x') - m(x')] (6)$$

Given a data set, if we expect the underlying function to be linear, and can make some assumptions about the input data, we might use a least-squares method to fit a straight line (Change-point model). Rather than claiming the relationship between independent variable and dependent variable relates to some specific models (e.g. Change-point model), GP constructs the model by specifying the structure of the covariance matrix of the explanatory variables, which makes it highly flexible. In addition, GP regression is a Bayesian approach which assumes a GP prior on functions, which enables the modeler to quantify different sources of uncertainty in an easy and natural way. Typically, two inputs vectors x(i), x(j) are assumed to be correlated through a covariance function. There are many choices of covariance functions which may be reasonable. The only constraint on the covariance function is that it should generate a non-negative definite covariance matrix for any set of points. Formally, we are required to specify functions which will generate a non-negative definite covariance matrix for any set of points $(x^1, \dots, x^k)$. From a modeling point of view, we want to choose a covariance function such that points with nearby inputs will give the similar predictions. The stationary squared exponential (SE) covariance function is the most widely-used kernel within the kernel machines field, and its infinitely differentiable property enables the GP with this covariance function has mean square derivatives of all orders, and is thus very smooth (Rasmussen 2006). We chose the SE covariance function

(Equation 7) for one-dimensional GP regression application in this paper.

$$k(x, x') = \sigma_f^2 exp\left[\frac{-(x-x')^2}{2l^2}\right] \quad (7)$$

In Equation 7, the maximum allowable covariance is defined as σ_f^2. If x$\approx x'$, then $k(x, x')$ approaches this maximum, which means $f(x)$ is nearly perfectly correlated with $f(x')$. If x is distant from x', then $k(x, x') \approx 0$, so during an interpolation at new x values, distant observations will have negligible effects. How much effect this separation has will depend on the length parameter l. According to (Adler 1981, Theorem 4.1.1), the mean number of level-zero upcrossings for SE process in one dimension is $(2\pi l)^{-1}$. We could easily see that parameter l defines the characteristic length-scale (Rasmussen 2006).

In reality, the data are often noisy as well, from measurement errors and so on. Each observation y can be thought of as related to an underlying function $f(x)$ through a Gaussian noise model (Equation 8).

$$y = f(x) + N(0, \sigma_n^2) \quad (8)$$

$$k(x, x') = \sigma_f^2 exp\left[\frac{-(x-x')^2}{2l^2}\right] + \sigma_n^2 \delta(x, x') \quad (9)$$

$\delta(x, x')$ in Equation 9 is the Kronecker delta function.

$$K = \begin{bmatrix} k(x_1,x_1) & k(x_1,x_2) \dots k(x_1,x_n) \\ k(x_2,x_1) & k(x_2,x_2) \dots k(x_2,x_n) \\ & \cdot \\ & \cdot \\ & \cdot \\ k(x_n,x_1) & k(x_n,x_2) \dots k(x_n,x_n) \end{bmatrix} \quad (10)$$

$$K_* = [k(x_*,x_1) \quad k(x_*,x_2) \dots k(x_*,x_n)] \quad (11)$$

$$K_{**} = k(x_*,x_*) \quad (12)$$

Since the key assumption in GP modeling is that the data can be represented as a sample from a multivariate Gaussian distribution, we have

$$\begin{bmatrix} y \\ y_* \end{bmatrix} \sim N\left(0, \begin{bmatrix} K & K_*^T \\ K_* & K_{**} \end{bmatrix}\right) \quad (13)$$

T indicates matrix transposition. The probability $p(y_*|y)$ follows a Gaussian distribution.

$$y_*|y \sim N(K_*K^{-1}y, K_{**} - K_*K^{-1}K_*^T) \quad (14)$$

The best estimate for y_* is the mean of this distribution:

$$\bar{y}_* = K_*K^{-1}y \quad (15)$$

The uncertainty in the estimate is then captured in its variance (Rasmussen 2006):

$$var(y_*) = K_{**} - K_*K^{-1}K_*^T \quad (16)$$

Artificial Neural Network (ANN) Model

The concept of ANN model is inspired by biological neural networks. The ANN model consists of an input layer, an output layer and hidden layer. The input layer receives the information from outside environment, and the output layer shows the response. The hidden layer will first need to be trained by training data set to adapt its structure that represents the relationship between the inputs and the response during a learning phase. Neural networks have been used to model complex relationships between the inputs and the outputs or to find patterns in data. Yalcintas 2005 and Karatasou 2006 conducted studies for applications of using ANN models to predict building energy use. Figure 8 shows the general structure of ANN model to estimate building energy use.

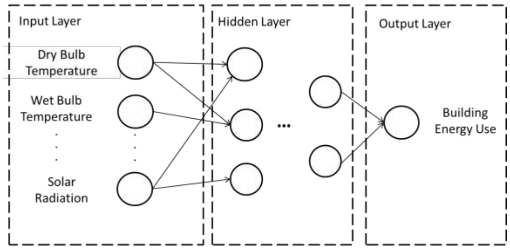

Figure 8 ANN model to estimate building energy use

CASE STUDY RESULT AND DISCUSSIONS

Building 101, as shown in Figure 9 below, located at 4747 South Broad Street in Philadelphia, PA, is managed by Cushman and Wakefield. The building is over 100 years old and was originally a Marine barracks at the Philadelphia Navy Yard. All of the building's mechanical systems were updated in 1998. The building is currently used as an office building. It is the current headquarter of the U.S. Department of Energy's Energy Efficient Building Hub. The building covers 61,700 square feet over four floors (basement through three floors). The building owned by the Philadelphia Industrial Development Corporation, has become one of the nation's most highly instrumented commercial buildings (http://www.eebhub.org/projects-list/navy-yard-building-101).

Figure 9 Building 101 in Philadelphia, PA

We have the daily monitored building energy consumption data between November, 2011 and January 2013. A few example data points are shown in Table 1. In this case study, we use the data from

November 2011 to October 2012 as training data and the data from November 2012 to January 2013 as test data. Figures 10, 11 and 12 below show the outdoor air temperature, building electricity consumption, boiler gas consumption over the monitoring period. Figure 13 illustrates the decomposition of different loads.

Table 1 Example measurement data points from Building 101

Date	DHW gas use (MBTU/day)	AHU electricity (MBTU/day)	Lighting (MBTU/day)	CU elctricity (MBTU/day)	Total building electricity (MBTU/day)	DHW gas (MBTU/day)	Boiler gas (MBTU/day)
1-Nov-11	0.10	1.78	4.62	0.05	6.45	0.10	7.02
2-Nov-11	0.10	1.78	4.65	0.05	6.48	0.10	5.55
3-Nov-11	0.12	1.83	4.52	1.12	7.48	0.12	5.63
4-Nov-11	0.10	1.98	4.36	0.05	6.39	0.10	6.89
5-Nov-11	0.09	1.94	3.43	0.05	5.43	0.09	8.83
6-Nov-11	0.08	1.95	3.31	0.05	5.32	0.08	8.44
7-Nov-11	0.09	1.98	4.41	0.05	6.45	0.09	6.87
8-Nov-11	0.11	1.91	4.67	1.75	8.33	0.11	4.93
9-Nov-11	0.10	1.88	4.67	1.92	8.48	0.10	3.62
10-Nov-11	0.10	1.87	4.69	1.28	7.83	0.10	5.87
11-Nov-11	0.11	2.10	4.31	0.05	6.47	0.11	11.22

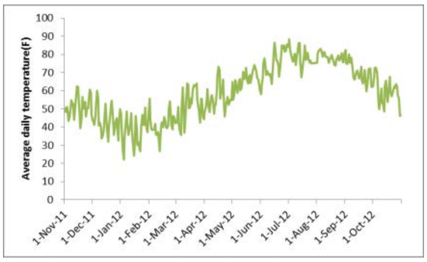

Figure 10 Annual outdoor air temperature

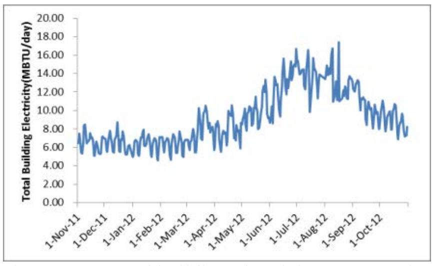

Figure 11 Annual building electricity consumption

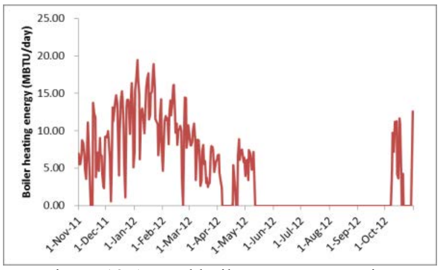

Figure 12 Annual boiler gas consumption

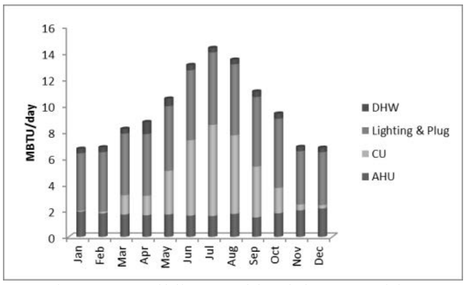

Figure 13 Building total load decomposition

Figures 14 and 15 show the linear regression models for the heating and cooling season that we developed for this building in this study.

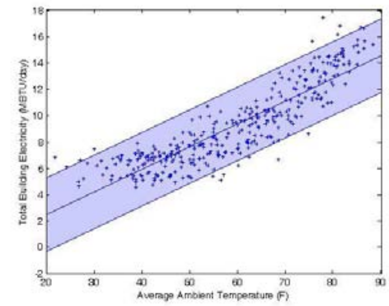

Figure 14 Linear regression cooling model
$\mathrm{Y} = 0.17\mathrm{x} - 0.96$

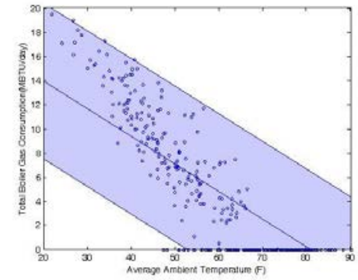

Figure 15 Linear regression heating model
$Y = -0.23x + 18.5$

In Figures 14 and 15, the blue line in the middle is the model prediction value. The blue marks are the training data points. The blue bound is the 95% confidence interval of the model.

The GP regression model is shown in Figures 16 and 17 below:

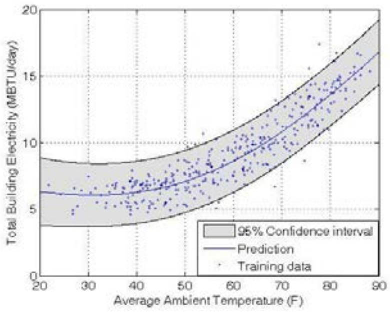

Figure 16 GP regression cooling model

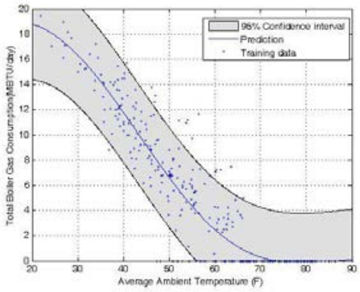

Figure 17 GP regression heating model

In Figures 16 and 17, the blue solid line is the mean function. The blue marks are the training data points. The grey area is the 95% confidential interval, which equals to $[\mu \pm 20\%\sigma]$.

From the Figures 16 and 17, we could see the predictive qualities of the GP model are strongly influenced by the range covered by the training and testing data set. As suggested by the structure of the covariance function, as the distance between training and testing sets increases, so does their correlation; and the GP model is required to extrapolate outside the training domain. Poor input variable scaling can lead to biasing preference to certain variables. The range of 95% confidential interval is relatively narrow near the testing data. The uncertainty of the prediction increases when the distance between training and testing data sets increases.

Model validation

The model was further validated with measurement data as shown in Figures 18 and 19.

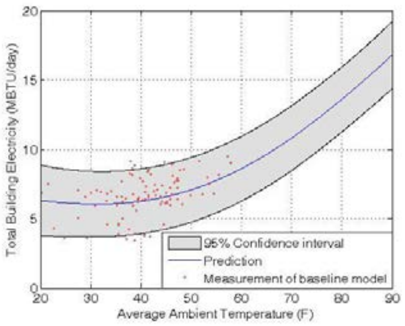

Figure 18 Validation of GPR cooling model

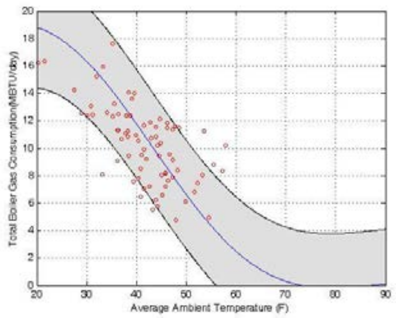

Figure 19 Validation of GPR heating model

The red marks in Figures 18 and 19 are the testing data point. We can see that most of the measurement data is falling in the range of the 95% confidence interval.

Model comparison

Figures 20 and 21 show the comparisons of the simple linear regression models and the GPR models. The red line is the simple linear regression function and the red area is the 95% confidence interval of the model. The blue line is the GPR model mean function and the grey area is the 95% confidence interval. The black data points are the measurement data of the baseline building including two heating dominated months (November and December 2012) and two cooling dominated months (June and July 2012). In the cooling model, we can see that with the same set of training data, and same level of accuracy, the GPR model provides a narrower range of uncertainty in the upper right part, and the mean function of the GPR model can better explain the cooling energy behavior of the building, since when the temperature is below a certain temperature, the total electricity is a constant value while the linear regression couldn't reflect this pattern. In the heating model, we can see a huge difference between the uncertainty ranges given by two different models with the same set of training data. The GPR model can provide more reliable and reasonable results compared to the simple linear regression model. We can also see that there are some other variables other than the ambient air temperature that can affect the building total energy consumption since at the same temperature; the total building electricity and gas consumption are widely distributed in Figures 15 and 16. Further study should develop a high dimension GPR model to capture the effect introduced by the other independent variables.

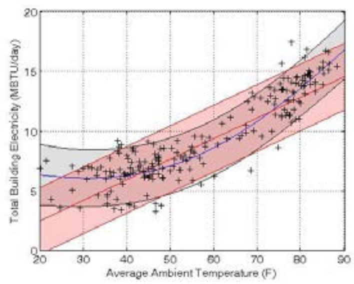

Figure 20 Cooling model comparison

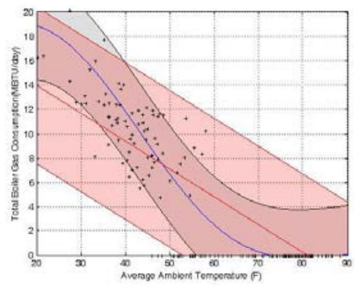

Figure 21 Heating model comparison

CONCULSIONS AND FUTURE WORK

In this study, we use both the linear regression method and GPR method to develop an inverse model for a case study building. From the comparisons, it is concluded that the inverse modeling is a realistic and efficient way to estimate the baseline building performance in the post-retrofit phase. The Gaussian Process approach leads to a highly flexible model, which can easily capture the complex building behavior. It can provide more realistic results compared with linear regression models. The predictive quality of the GP model is strongly influenced by the range covered by the training and testing data set. Future work is to apply certain energy conservative measures in retrofitting the existing building, use the GPR model for predicting the baseline building energy consumption, and compare it with measured building energy consumption in the post-retrofit phase to calculate energy savings as a result of retrofits.

ACKNOWLEDGEMENT

This work is funded by Energy Efficient Buildings Hub, sponsored by the Department of Energy under Award Number DE-EE0004261.

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