# COMMUNITY-SCALE RESIDENTIAL ENERGY DEMAND SIMULATION FOR SMART-GRID APPLICATIONS

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## ABSTRACT

This study reports the application of a stochastic simulation model that estimates community-scale residential electricity demand and photovoltaic (PV) generation to simulate the voltage of medium- and low-voltage distribution networks. This model enables the evaluation of the impact of PV diffusion and energy management technologies using, for example, battery and electric vehicles, on voltage management. This study shows a case study in which the developed model was applied to a community consisting of 732 households to estimate the network voltage and power generation loss that occurs when the voltage exceeds the legal limit. This study also discusses challenges in modeling the high-temporal-resolution energy demand of residential buildings.

## INTRODUCTION

Installed capacities of grid-connected PV are increasing dramatically. According to the Photovoltaic Power Systems Programme report of the International Energy Agency (IEA PVPS), the total capacity of grid-connected PV in all IEA PVPS countries was 62 GW in 2011. The European Photovoltaic Industry Association (EPIA) expects the total capacity to reach 350 GW by 2016. Similar to that of PV systems, it is expected that the capacity of grid-connected distributed generators (DGs) will increase in the coming decades. A large portion of DGs will likely be connected to low-voltage (LV) electric distribution lines.

However, till date, electric distribution systems have been designed without considerations of large-scale capacity requirements for future DG connections. The voltage level is determined by local electricity demand and generation in addition to the specifications of distribution lines, and this level increases as DG electricity generation increases. There is a legal voltage limit in many countries; thus, when generated electricity is much greater than electricity demand, the voltage exceeds the legal limit. Thus, large-scale diffusion of DGs could degrade the quality of existing electric distribution systems.

This issue could prove serious in residential areas in Japan. In residential areas, the upper and lower voltage limits are set at 101 ± 6 V by the Electricity Business Act. In residential areas, the electricity demand during daytime on weekdays is low, because majority of inhabitants are at work or school. Conventionally, a power conditioner attached to a PV system detects the voltage and disconnects from the grid when the voltage exceeds the legal limit.

There are a number of studies on the influence of PV diffusion on voltage distribution (Viawan et al., 2007; Renders et al., 2010; Svenda et al., 2012). However, these studies are somewhat limited in scope, because they usually assume a typical electricity demand. This approach does not consider specific conditions in residential areas and energy management conducted in residential buildings to change the electricity load of the grid by using, for example, a home-energy-management system or managing an electric vehicle battery charging system. To perform such analyses, a model that integrates the electric voltage and electricity demand of residential areas is required.

Based on this background, we propose a community-scale model that estimates residential energy demand, PV generation, the electric voltage of distribution lines, and curtailed PV generation of each household. We also discuss the required specifications of residential models that are relevant in the electrical engineering field. Especially, we focus on high-temporal-resolution electricity demand and diversity among households.

## SIMULATION MODEL

Figure 1 shows the flowchart representing the simulation model. This model has a database of building archetypes that combines electricity demand and the generation of a residential household. When a household is defined in the model, one of the archetypes is applied to the model as the input dataset. The archetype combines the specification of the house, residential appliances, DG equipment, and occupant behaviour. For example, for detached and apartment houses, 240 houses were defined with data consisting of house geometry, size, insulation level, and types of water heaters, as listed in Table 1. Based on the input database, electricity demand is calculated for each household at a 5-minute time resolution as specified by the residential energy demand model (Shimoda Y., 2007).

In addition to energy demand, electricity generation by DGs is calculated for each household. The input information of the DG model consists of DG specifications, meteorological data, and electricity demand if necessary.

*Table 1 Residential housing database*

| Data item | Data set |
|---|---|
| Detached house | $40m^2$, $50m^2$, $70m^2$, $90m^2$, $113m^2$, $146m^2$ |
| Apartment house | $20m^2$, $30m^2$, $52m^2$, $73m^2$, $91m^2$, $119m^2$ |
| Insulation level | No insulation, 1980 standard, 1992 standard, 1999 standard |
| Water heater | Electric, Gas, Oil, CO2HP<br>Latent heat recovery gas water heater |

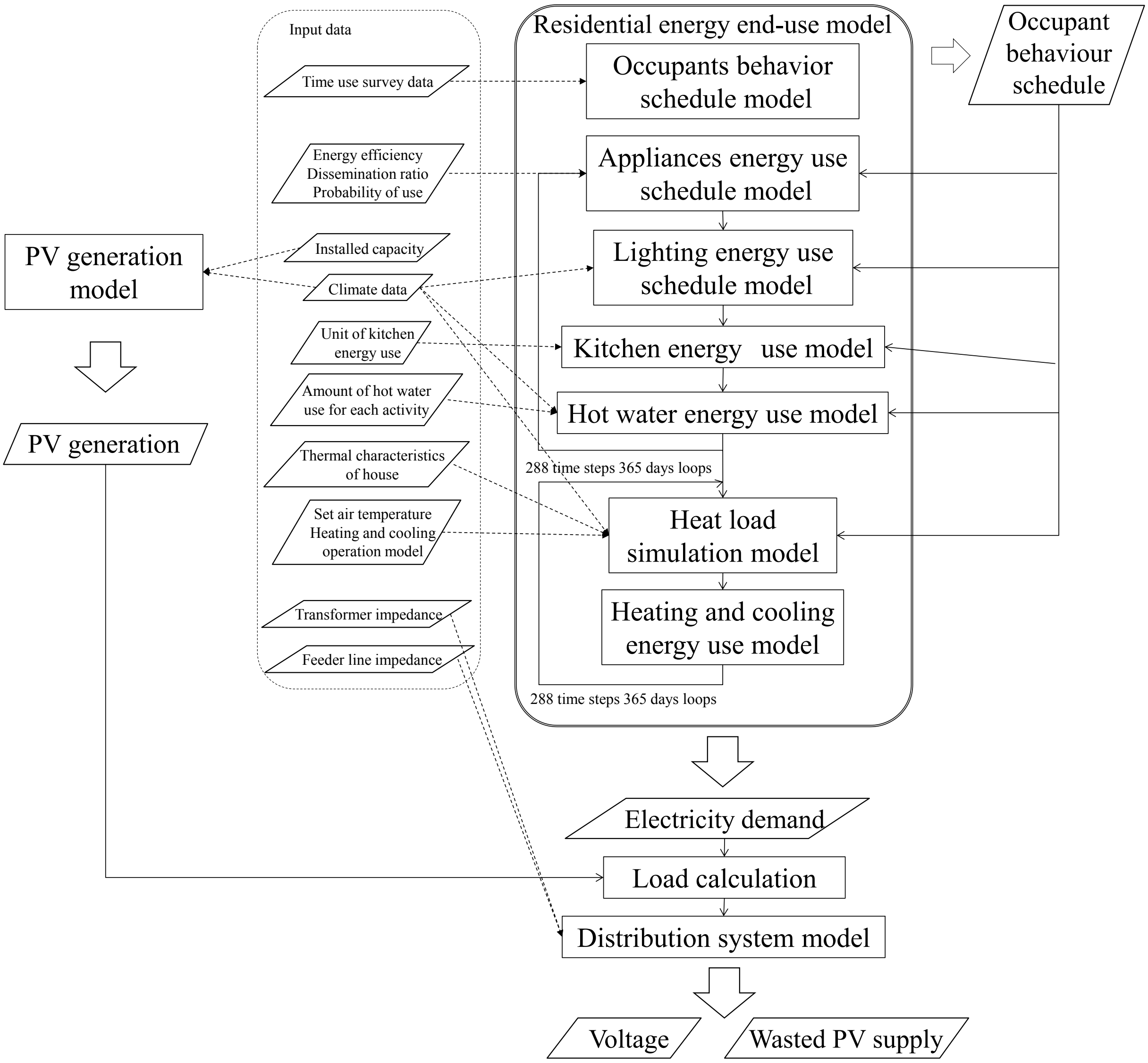


*Figure 1 Simulation Flowchart*

Finally, the electric distribution system model calculates the voltage at each node of the distribution network. Figure 2 shows the system configuration of the electric distribution network. In a typical residential distribution network, electricity is supplied through a voltage transformer, which converts high voltage (HV) to medium voltage (MV). We assume that this transformer is a distribution substation and that the model simulates electricity flowing through the substation. MV electricity is transferred to nodes indicated by the # symbol. This MV line is referred to as a "feeder line". Each node of the feeder line is equipped with a voltage transformer, called a pole transformer, which converts MV to low voltage (LV), and this transformer connects 10–20 houses. Thus, several hundred households can be connected.

Based on the total electricity demand and generation of each house and node, the voltage at each node on MV and LV lines is calculated by solving the power flow equations described later.

By integrating the three components, the residential energy demand model, the DG model, and the electric distribution system model, the proposed simulation model is capable of evaluating the impact of the diffusion of DGs on voltage at residential feeder grids and analyzing the contribution of energy management on the demand side.

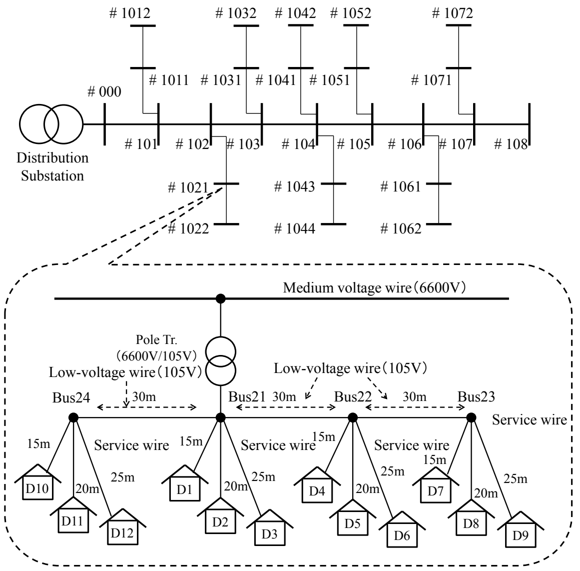


*Figure 2 Considered electric distribution network*

**Residential energy demand model**

This model first stochastically generates occupant behaviour data that describes the manner in which occupants in the household spend a day (time use) and the locations at which various behaviours occur. This behaviour information is then converted to the operation of home appliances and equipment such as water heaters. Then, the information is converted to energy consumption by considering the specifications of home appliances and equipment.

For example, if one or more family members are watching TV in the living room, TV and lighting in the room are included in the estimation, and their energy consumption is calculated by considering their specifications. Similarly, heating/cooling load is calculated by a dynamic thermal simulation for the room, and the load is converted to electricity consumption for any air conditioner operating in the room.

Occupant behaviour is generated on the basis of time-use data (TUD) developed by the Broadcasting Culture Research Institute of Japan (2006) based on their survey of the time use of 12,600 Japanese residents. TUD contains the mean and standard deviations of duration for 27 types of activities in a day (such as sleeping, working, and watching TV) and the probability distribution of each activity for 24 hours on weekdays, Saturdays, and holidays. By using the mean and standard deviations, duration for all behaviour is first determined (discrete behaviour). Then, discrete behaviour is placed on a timeline by employing a probability distribution over time. Routine activities (i.e., sleeping, working or studying at school, having a meal and taking a bath) are placed first on the timeline and then other activities are arranged to fill the gaps between routine behaviours. This modeling approach was originally proposed by Tanimoto et al. (2008). The applied model in this study is based on Tanimoto's model and questionnaire survey conducted by Shimoda (2004). Details of the model can be found elsewhere (Yamaguchi et al., 2012).

The abovementioned TUD is developed for each of eight demographic classifications: working male, working female, housewife, primary school student, junior high school student, high school student, elderly male, and elderly female. Thus, according to the demographic structure of a family, a corresponding TUD is given to each family member. This model has an archetype for each of 19 family compositions that are listed in Table 2.

*Table 2 Archetype of family composition*

| Number of persons per household | Family structure |
|---|---|
| 1 | Single male |
| | Single female |
| | Single-aged male |
| | Single-aged female |
| 2 | Working couple |
| | Couple |
| | Aged couple |
| | Working mother and child |
| | Mother and child |
| 3 | Working parents and child |
| | Parents and child |
| | Working mother and 2 children |
| | Mother and 2 children |
| 4 | Working parents and 2 children |
| | Parents and 2 children |
| 5 | Working parents and 3 children |
| | Parents and 3 children |
| 6 | Working parents, grandparents, and 2 children |
| | Parents, grand parents, and 2 children |

As for lighting, it is assumed that all occupied rooms (except when the occupant is asleep) and corridors are illuminated during the night. During the day, occupied rooms can be regarded as rooms that are always illuminated, never illuminated, or depend on daylight brightness. The probability of these situations is determined on the basis of a questionnaire survey conducted by the author. Daylight brightness is calculated from terrestrial meteorological observation data.

Energy demand for water heating is estimated by considering the amount of hot water, the hot water temperature, and city water temperature. The city water temperature is expressed as a function of outside air temperature. For heating and cooling models, dynamic heat load simulations are conducted by utilizing building data and meteorological data. A thermal circuit network method is adopted to calculate the heat load for heating and cooling. The internal heat gain is calculated by utilizing the energy consumption by home appliances and occupant behaviour.

**Calculation of PV electricity generation in the DG model**

The PV system model calculates the power output based on an arbitrarily oriented, sloped, and sized system for multicrystalline silicon solar cells.

PV generation ($E_P$ [kW]) is estimated by Equation (1).

$$E_p = \left[P_{AS} \cdot H_A \cdot K \cdot \{1 + \alpha(T_C - 25)\}\right] / G_S , \quad (1)$$

where $P_{AS}$ [kW] is the capacity of the PV system, $H_A$ [kW/m²] is the amount of inclined solar radiation that is calculated by global solar radiation with the model proposed by Perez et al. (1993), K [–] is the performance ratio (= 0.78), α [–] is the temperature correction factor (= –0.005), $T_C$ [°C] is the temperature of the PV surface, and $G_S$ [kW/m²] is the solar radiation intensity under standard test-cell conditions (= 1 kW/ m²).

The temperature of the PV surface is defined by Equation (2) (Yukawa M., 1996):

$$T_C = T_A + \left[ \frac{A}{B \times U^{0.8} + 1} + 2 \right] \times H_A - 2, \qquad (2)$$

where $T_A$ [°C] is the outdoor air temperature, A and B [–] are factors for calculating temperature (A = 50, B = 0.38), and U [m/s] is the wind velocity.

The PV capacity was determined for each compass direction according to the size of a house, as shown in Figure 3. It is assumed that each detached house has a PW capacity of 4 kW.

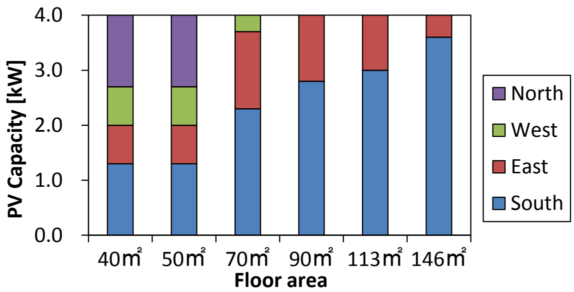


*Figure 3 PV capacity*

**Electric distribution system model**

MV and LV distribution network voltages are calculated by the method developed by Maran et al. (1989). The nominal voltage on MV and LV distribution networks are 6,600 V and 100 V, respectively. In this model, the following equations are iteratively solved to determine the line voltage V [V] until the active power P [W] and reactive power Q [W] at the edge of the feeder and LV lines converge to zero:

$$P_{k+1} = P_k - r_k \frac{P_k^2 + Q_k^2}{V_k^2} - PL_{k+1}, \qquad (3)$$

$$Q_{k+1} = Q_k - x_k \frac{P_k^2 + Q_k^2}{V_k^2} - QL_{k+1}, \qquad (4)$$

$$V_{k+1}^2 = V_k^2 - 2(r_k P_k + x_k Q_k) - (r_k^2 + x_k^2) \frac{P_k^2 + Q_k^2}{V_k^2} \qquad (5)$$

where k is a node, PL [W] and QL [W] are the active and reactive power loads, r [Ω] is the line resistance, and x [Ω] is the line reactance.

For iteration, the Newton–Raphson method is applied for both MV and LV distribution lines.

As mentioned earlier, when the electric voltage exceeds the legal limit defined in the Electricity Business Act (107 V), a power conditioner attached to a PV system prevents power generation. We define the wasted PV generation $P_W$ [kW] in Equation (6):

$$P_W = P_{PV}(V_{HOUSE} > V_{UL}), \qquad (6)$$

where $P_{PV}$ [kW] is the electricity generated by PV, $V_{HOUSE}$ is the calculated voltage at each house, and $V_{UL}$ is the upper voltage limit regulated in the Electricity Business Act (107 V).

## CASE SETTING

In this case study, 732 households were assumed to be connected to a feeder line. A building archetype and family composition were selected for each household. The location of each house on the distribution network was randomly selected.

We assumed that the simulated area is a new residential area in which each house has the latest appliances (TV, refrigerator, air conditioner, personal computer (PC), video tape recorder (VTR)), which were released from 2010 to 2011 in Japan. These appliances have been included in Japan's top runner programme for energy-efficient devices. Moreover, all detached houses were assumed to be equipped with a $CO_2$ heat pump water heater, while all apartment houses were assumed to be equipped with a gas stove burner with a latent heat recovery gas water heater.

As shown in Figure 2, we assumed that the MV distribution network consists of a feeder line and eight subfeeder lines with 25 nodes each. We assumed that an apartment building is connected to nodes #101, #102, #103, and #104. At other nodes, radial LV grids connect 12 detached houses via a pole transformer, as shown in Figure 2. Three detached houses are fed at each node via service wires assumed to be 15, 20, and 25 m, respectively. The tap rate of the pole transformers was assumed to be 6,600 V/105 V. The voltage at the distribution substation was assumed to be constant at 6,600 V. The impedances related to MV and LV distribution networks are tabulated in Table 3 and Table 4, respectively. The load factor is assumed to be 0.9.

*Table 3 Details of the MV distribution network*

| Node | R[Ω] | X[Ω] | Node | R[Ω] | X[Ω] |
|---|---|---|---|---|---|
| 101 | 0.110 | 0.142 | 1043 | 0.052 | 0.061 |
| 1011 | 0.052 | 0.061 | 1044 | 0.079 | 0.103 |
| 1012 | 0.246 | 0.122 | 105 | 0.110 | 0.142 |
| 102 | 0.110 | 0.142 | 1051 | 0.052 | 0.061 |
| 1021 | 0.052 | 0.061 | 1052 | 0.079 | 0.103 |
| 1022 | 0.169 | 0.112 | 106 | 0.110 | 0.142 |
| 103 | 0.110 | 0.142 | 1061 | 0.052 | 0.061 |
| 1031 | 0.052 | 0.061 | 1062 | 0.079 | 0.103 |
| 1032 | 0.079 | 0.103 | 107 | 0.110 | 0.142 |
| 104 | 0.110 | 0.142 | 1071 | 0.052 | 0.061 |
| 1041 | 0.079 | 0.084 | 1072 | 0.391 | 0.122 |
| 1042 | 0.449 | 0.139 | 108 | 0.122 | 0.070 |

*Table 4 Details of the LV distribution network*

| Equipment | Resistance | Reactance |
|---|---|---|
| Low voltage wire[Ω/km] | 0.237 | 0.401 |
| Service wire[Ω/km] | 0.313 | 0.343 |
| Pole transformer[Ω]* | 0.0313 | 0.209 |

*Primary + Secondary

## SIMULATION RESULTS

### Simulation result for a single household

First, we show the simulation result for a four member family that consist of a working male (labeled as Father), a housewife (Mother), and two children (Child 1 and 2) on three representative weekdays—January 20, April 15, and August 18—corresponding to three seasons (winter, spring, and summer, respectively).

Figure 4 shows the simulation results for occupant behavior on the abovementioned days. As shown in this figure, occupant behaviour is stochastically determined on the basis of the attributes of each occupant. The transition of behaviour is crucial to the model dynamic behaviour of energy demand, because the behaviour is combined with the operation of home appliances and equipment. As shown in the figure, behaviour transition is randomly generated.

Figure 5 shows the simulated time-series electricity demand and PV generation on the three days for the family corresponding to Figure 4. We assumed that the family live in a well-insulated detached house with a total floor area of 90 m . The electricity consumption of home appliances, lighting, water heating, and space heating and cooling was estimated. In addition to occupant behaviour, the simulation results reflect the variation of meteorological conditions. On January 20, a large amount of electricity was consumed for water heating using a heat pump water heater compared with the other days. On August 18, a large amount of electricity was consumed for space cooling, while heating energy consumption was modest on January 20, because the detached house was assumed to be well-insulated.

For this house, we assumed that 4 kW of PV equipment is installed. The red line in the figure shows the PV-generated electricity. As the generated electricity exceeds the electricity demand of the house, the difference between generation and demand is assumed to be to the LV line to which the house is connected.

### Simulation result for community-scale energy demand and PV generation

Community-scale electricity demand and PV generation were estimated by adding the electricity demand and PV generation of all houses connected to feeder lines. Figure 6 shows the total electricity demand and PV generation in the community. Here we assumed that 50% of the detached houses have a PV capacity of 4 kW. The total PV capacity of the community is 0.5 MW under this assumption. Compared with Figure 5, the estimated demand curve is smoother, because the electricity demand was averaged among houses. In addition, the amount of excess PV generation over the electricity demand became considerably small relative to the electricity demand, because the excess PV generation was distributed to the entire community.

Figure 7 shows the cumulative frequency of the total electric load in the community. The black line shows the load calculated assuming that no PV system was installed, while the red line shows the load calculated assuming that 50% of detached houses are equipped with 4-kW PV systems. As shown in the figure, the total electricity load shifted to the left because of the electricity generated by PV. PV generation exceeded electricity demand 8% of the year.

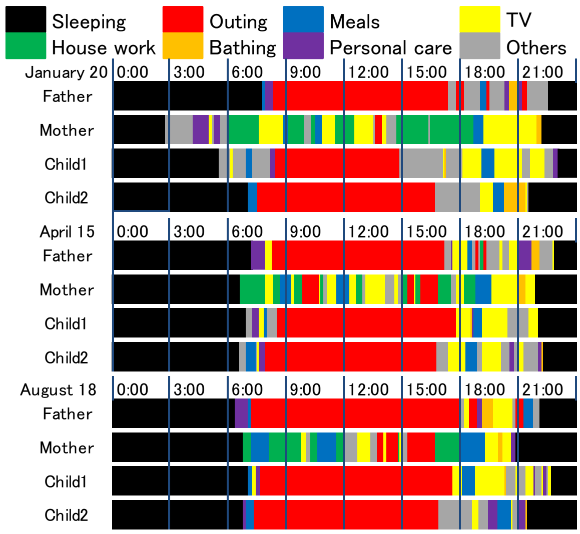


*Figure 4 Simulated occupant behaviour on January 20 (top), April 15 (middle), and August 18 (bottom).*

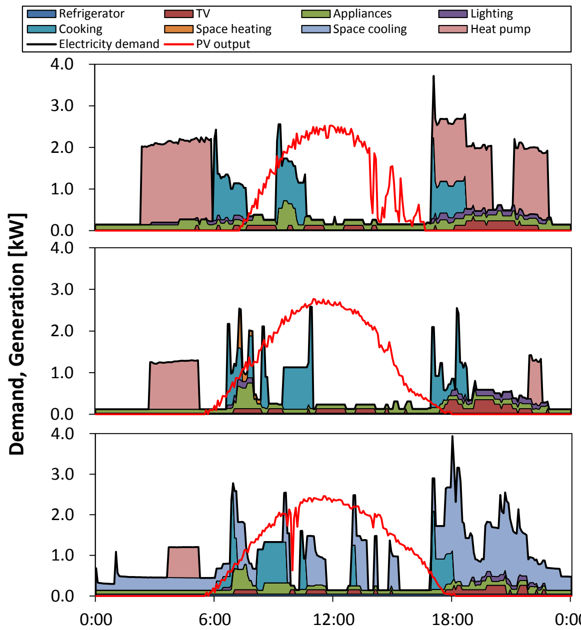


*Figure 5 Simulated electricity demand and PV power generation*

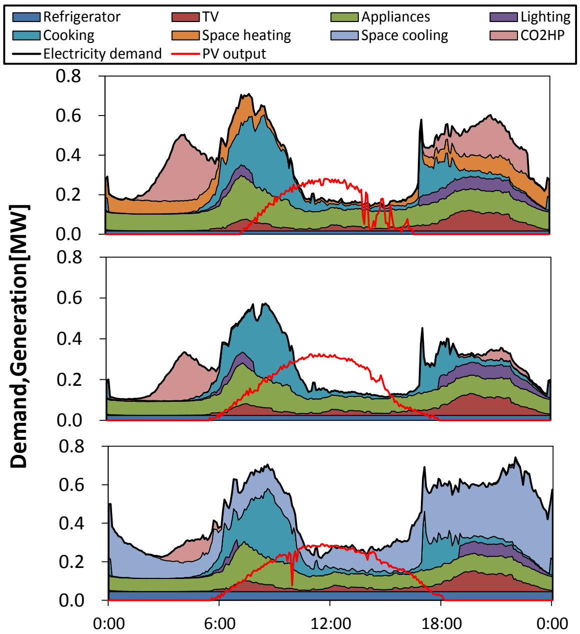


*Figure 6 Simulated time-series community-scale electricity demand and PV power generation*

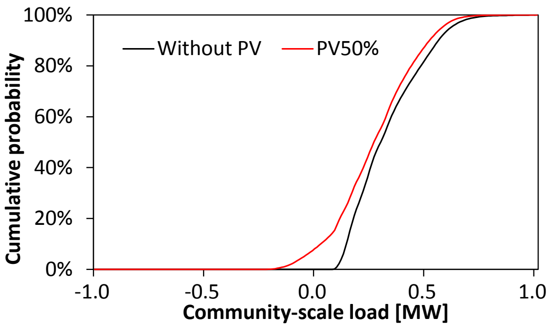


*Figure 7 Load time duration curve*

## Impact of PV diffusion on an LV network

Electricity demand and PV generation of each house were inputted to the electric distribution system model to calculate the voltage level at each house connected to LV and at each node of MV to which an LV is connected. Here we assumed that 50% of the detached houses are equipped with 4-kW PV systems and that PV systems are equipped at houses D1, D2, D4, D5, D8, D10, and D12 of node #1072 (see Figure 2).

Figure 8 shows the electricity demand and PV generation of the house located at the end of the feeder line, D8 of node #1072, on April 15. The figure also shows the estimated voltage level. The voltage is high during daytime, because the PV output significantly exceeded the electricity demand of the house. As shown in the figure, the voltage levels exceeded 107 V from 10 am to 3 pm. The voltage level at each house is obviously sensitive to its load.

Figure 9 shows the voltage profile of each house from D1 to D12 connected to node #1072 when the highest voltage level was observed during the representative days: January 20, April 15, and August 18. The voltage levels observed on April 15 were the highest, while those on August 18 were the lowest, which is attributed to the difference between PV generation and electricity demand. The voltage levels at all houses at node #1072 exceed the upper limit of 107 V on January 20 and April 15.

Figure 10 shows the cumulative frequency of the voltage level observed at node #1072. When 50% of detached houses are equipped with 4-kW PV systems, the voltage levels exceeded the upper limit by an average of approximately 316 hours per household.

The impact of PV diffusion on the end-user voltage level varies widely among households depending on the location of the house. The voltage level at house D1 was the smallest in Figure 9, while house D8 was affected most severely. The total time during which D1 exceeded the limit was 228 hours as compared with 547 hours for D8.

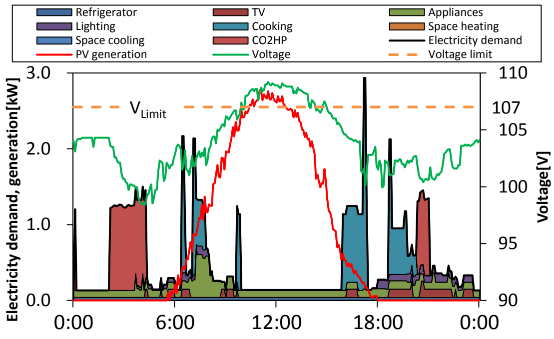


*Figure 8 Electricity demand, PV generation, and voltage profile on 15 April.*

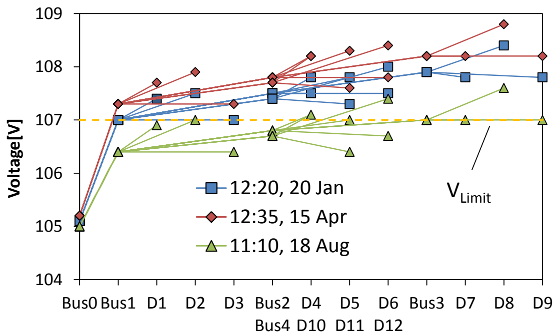


*Figure 9 Voltage profile of the LV distribution network (#1072)*

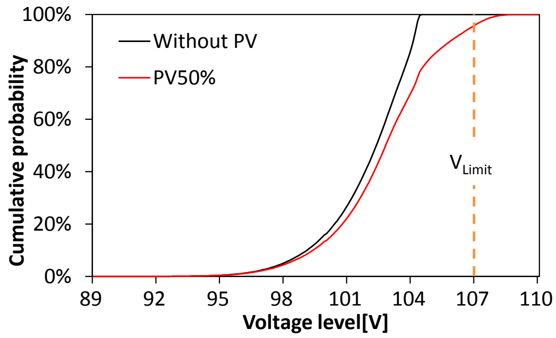


*Figure 10 Voltage time duration curve at node (#1072)*

## Impact of PV diffusion on the whole network

Figure 11 shows the total electricity demand and PV generation in the community. The total electricity demand was estimated to be 1281 MW/year, of which 87% of the demand was supplied from electric utility, while the remaining 13% was provided by PV. The annual PV generation was estimated to be 499 MW/year, which is equivalent to 39% of the total annual electricity demand. Only 13% of PV generation was consumed in the community. Another 50% was reversed to the electric grid. More importantly, 16% was curtailed by power conditioners when the voltage output was higher than the upper limit of 107 V.

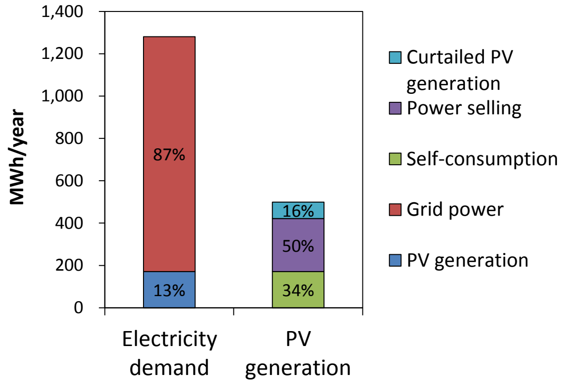


*Figure 11 Breakdown of electricity demand and PV generation of all detached houses*

We examined the influence of PV diffusion on the load of the electric grid and voltage level. Figure 12 shows the cumulative frequency of the electric load of the community calculated while assuming that 0%–100% of detached houses are equipped with 4-kW PV systems, which is equivalent to 0 MW to 1 MW of the generation capacity. Figure 13 shows the electricity generated by PV. The result was separated into the amount utilized in the community, that injected to the electric grid, and that curtailed by power conditioners. This result showed that when PV diffuses to more than 50% of detached houses, a large amount of PV generation is curtailed due to excessive voltage generation.

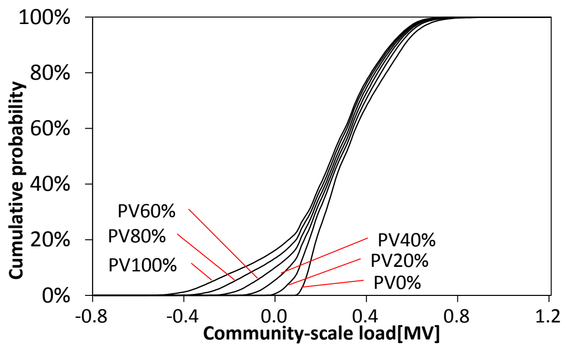


*Figure 12 Cumulative probability of community-scale loads at different PV penetration rates*

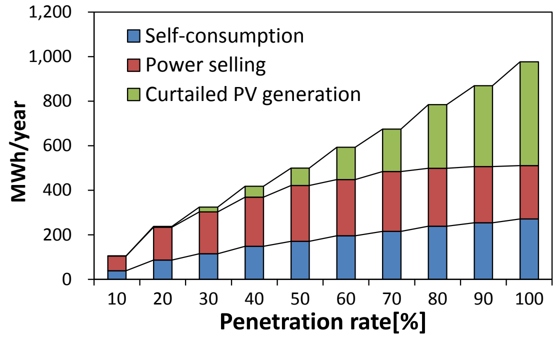


*Figure 13 Breakdown of PV generation of all detached houses at different PV penetration rates*

## DISCUSSION

The case study presented above demonstrates the capability of the model to simulate the voltage of MV and LV distribution networks by assuming various households in terms of family composition, building geometry and insulation, the configuration and the efficiency of home appliances and equipment, as shown in the simulation results for a single household (Figure 5). Since the community energy demand is modeled as the sum of the demand of each household located in the community, the results reflect the actual household structure to determine energy demand. The model is also capable of simulating the influence of technology dissemination, social change such as population aging, and the implementation of energy demand management by changing occupant behavior, appliance/equipment operation, and the flow of electricity using a PV system to charge a battery used by a plug-in hybrid vehicle (PHV).

Although the model demonstrates useful capabilities, it suffers from a disadvantage. Figure 14 shows the frequency distribution of the annual electricity demand calculated for the household archetypes used in the model. The figure also shows the observed annual electricity consumption for the same 381 households located in a specific region. The standard deviation calculated for the observed consumption is 2.44 MWh/year, which is significantly greater than that calculated from simulation (1.17 MWh/year).

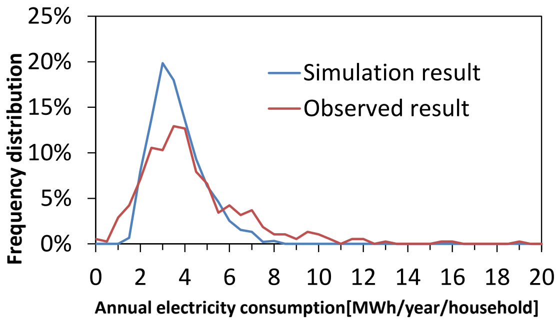


*Figure 14 Frequency distribution of annual electricity demand*

This discrepancy can be attributed to the model using various common conditions among households. One of the most important elements is the TUD used to simulate occupant behaviour. Since a common TUD is given to simulated persons with the same demographic classification, the behavioural differences among real-world households are ignored. Similarly, to convert behaviour (time use) to the operation of home appliance/equipment, a common conversion model is universally applied to all households. Thus, it is necessary to establish a methodology to generate input information including TUD and behaviour regarding appliance/equipment operation that is customized for each household.

## CONCLUSION

This study presents the application of a residential energy demand model that simulates energy demand and PV generation at a 5-minute time resolution into an analysis of the performance of an electric distribution system. The community-scale case study results show that large-scale diffusion of PV systems in a residential community significantly influences the voltage level of low- and medium-voltage lines. The simulation model is useful, because it enables the understanding of the influence of electricity demand and generation on voltage generation and consumption under seasonal variation and by the location of installed PV systems. The model is also capable of evaluating the effect of various energy demands that can be used to implement an efficient energy management system in a residential community. The model is disadvantageous in predicting diverse energy demand among real-world households, which is attributed to a number of conditions commonly applied to simulated households. To overcome this weakness, it is necessary to develop an algorithm to generate input information into the simulation model that reflects real-world conditions.

## ACKNOWLEDGEMENT

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