ON THE INTEGRATION OF HYGROTHERMAL BRIDGES INTO WHOLE BUILDING HAM MODELING

Julien Berger[1,2], Simon Rouchier[3] Sihem Tasca-Guernouti[1], Monika Woloszyn[2] and Catherine Buhe[2]
[1]CETE de l'Ouest, Nantes, France
[2]LOCIE, CNRS UMR 5271, Université de Savoie, Chambéry, France
[3]CETHIL, CNRS UMR 5008, Université de Lyon, Lyon, France

ABSTRACT

This paper focuses on the issue of integrating two or three dimensional models of heat and moisture transfer in porous materials with whole building HAM modeling. To assess this aim, this paper proposes a reduced order model to simulate 2D Heat, air and moisture (HAM) behaviour in porous material. The reduction method used is the proper orthogonal decomposition. The method is applied on a block of concrete. Reduced order model and large original model are compared. The results illustrate the significant value of model reduction for modelling 2D heat and mass transfers.

NOMENCLATURE

c	specific heat (J/kg K)
g	flow of moisture (kg/m^{2}s)
k	thermal conductivity (W/m K)
K_l	liquid permeability (s)
L_v	enthalpy of evaporation/condensation (J/kg)
P_v	vapour pressure (Pa)
P_c	capillary pressure (Pa)
P_{sat}	saturation pressure (Pa)
R_v	gas constant for vapour (J/kg K)
RH	relative humidity (%)
S_h	heat source (W/m^3)
S_m	moisture source (W/m^3)
t	time
T	Temperature (K)
w	moisture content (kg/m^3)
δ_v	vapour permeability (kg/m s Pa)
ξ	moisture storage capacity
ρ_l	density of water (kg/m^3)
ρ_0	density of material (kg/m^3)

INTRODUCTION

Buildings may be affected by damages due to direct or indirect actions of moisture, such as mould growth, corrosion, reduction of thermal resistance of the insulation layers etc. The development of damage depends on material properties as well as on hygrothermal conditions in construction. The latter are governed by heat, air and moisture balances with the outdoor climate and with the indoor conditions, including impact of factors such as ventilation, internal sources, heating system etc.

To predict risks of moisture damages in building, many tools are available in literature to perform building simulation. Most of existing tools can be grouped into two categories (Woloszyn and Rode, 2008) :

- Two or three dimensional (2D-3D) tools for building envelope, using indoor and outdoor air as boundary conditions,
- Tools for whole buildings, including one-dimensional (1D) transfers through the envelope, and able to calculate the indoor conditions.

In buildings, thermal bridges represent sensitive areas for moisture damages. This is due to the lower surface temperature. Moisture risks are therefore more frequent in these sensitive points. In order to take into account such sensitive areas, the HAM (heat, air, moisture) model associated with the prediction of moisture damages should be two-dimensional. Moreover, it should be able to represent interactions between construction and indoor conditions, including the impact of ventilation and of heating system. Such multi-dimensional simulation of whole building HAM behaviour is formally feasible. However it is complex and very time consuming. Therefore another approach, based on model reduction techniques, is explored in this paper.

The main idea is to replace the full set of ordinary differential equations resulting from finite-volume or finite-element discretization of coupled 2D HAM problem by a reduced number of equations correctly representing main outputs. The main advantage is an important reduction of the computational time. Such techniques are successfully implemented in some building simulation tools to represent heat transfer through building envelope. For instance, (Gao et al., 2008)) proposed a way to integrate reduced order model (ROM) of 3-D transfers in thermal bridges in whole building HAM modelling. However the application of these techniques to non-linear modelling such as coupled HAM transfers is still an ongoing field of research. The issue of this paper is to propose a ROM simulating 2D HAM behaviour in layers. After a brief state-of-the-art of model reduction techniques for non linear model, a method of reduction will be proposed for 2D HAM model.

STATE OF THE ART

Methods for model reduction

For modelling purpose, heat and moisture transfers in layers are generally translated in finite volume or finite elements problems (in 1, 2 or 3 dimensions). A mesh of M nodes is considered and for transient problems, M values must be computed at each time step. And for non-linear problems, it becomes computationally more expensive when M increase. Reduction techniques aim to approach this kind of high-dimensional problems by a low-dimensional one.

Techniques of reduction consist in approximating the solution of a problem in sub-spaces of lower dimensions. Generally the approximated solution of the problem $\tilde{u}_m(x,t)$ is sought as a discrete one :

$$\tilde{u}_m(x,t) = \sum_{k=1}^{n} a_k(t)\phi_k(x) \quad (1)$$

where n, $n << m$, will be the order of reduction of our problem.

The first important step in reducing techniques is to build the basis $\{\phi(x)\}_{1\leq k\leq n}$. This step characterizes the methods of reduction. Two families of methods exist. One called *a posteriori*, needs an already-computed or experimental solution to build the reduced order model. Two beneficial approaches belong to this first technique. First, the reduced order model is created with simulations of the large original model on a short time interval. Then the reduced order model is used for simulation on larger time interval. The second approach is to create the reduced order model with the large original model on a large time interval. Then the reduced order model is used for problems on similar time interval but with different boundary conditions or material properties.

The second family of reduction techniques is *a priori* methods. These techniques do not need preliminary information on the studied problem. The basis of projection is not known ab initio and is built by an iterative process.

Numbers of methods exist for reducing linear problems. These methods are well established and were successfully used in various fields. See (Palomo Del Barrio, 2011) and (KIM, 2011) for review of this techniques. On the contrary, for non linear problems, few methods exists and it is still an open topic. In the following subsections, only reduction methods for non linear problems are presented.

Proper orthogonal decomposition (POD)

The Proper Orthogonal Decomposition (POD) is the most widely used technique. This statistic method enables to move from a high amount of random data to a deterministic reduced-order representation. The aim of this method is to capture the maximum of energy of a field of interest $u(x,t)$ in an optimal number of modes. Depending on the field of application, this method has different name : Karhumel-Love decomposition, Principal component analysis (PCA), POD etc. Numbers of works have been published using this method. In building physics, this technique has been used in computational fluid dynamics. For example, we can cite works of (Dumon et al., 2011) on resolution of Navier-Stokes equation, (Liberge and Hamdouni, 2010), on fluid-structure interactions and (Verdon, 2007), on particle dispersion in ventilated cavity. Thanks to POD, the subspace obtained for determining the reduced order model gives the optimum representation of the large original model (Liberge and Hamdouni, 2010). It means that it provides the most efficient way of capturing the dominant components of an infinite-dimensional process with only a really small finite number of modes. For this reason POD is widely used. As for all *a posteriori* methods, disadvantage of POD is that construction of reduced basis requires the full solution of the large original model. Therefore, POD needs many numerical computations.

A priori proper orthogonal decomposition

Recently, alternative *a priori* methods were developed based on POD in (Dauvergne and Palomo del Barrio, 2010). The basis of this method follows the POD. Matrix of energy **Q** of the system is defined and Lyapunov equation is solved to determine it. It is actually an *a priori* method because matrix of energy is calculated without using the results of large original model. Then the singular basis is obtained by proper orthogonal decomposition of **Q**. The reduced order model is obtained, as in the standard POD, by projecting the solution on the reduced singular basis. This method has been applied on phase-change problems (see works of Dauvergne).

Modal basis reduction (MBR)

Other methods are used in building physics such as modal basis reduction (MBR) (or modal identification). It was used to solve non linear problems of thermal conduction. The problem is written in a matrix form. The issue is to operate a change of basis. Identification techniques are used to search matrix of dimensions $n << m$ in the new basis. For identification, simulation of large original model of experimental tests are required. So this method is an *a posteriori* one. Identification are done by least squares, Marshall, amalgam or Litz method. Works of Girault ((Girault and Petit, 2005) and (Girault et al., 2004)) can be referred for example and details of this method.

Branch eigenmodes reduction (BER)

This method is an *a priori* one. This method is riveting when the field of interest (temperature) is not continuous (non linear boundary conditions). The solution of the problem $u_m(x,t)$ is projected on the modal basis. It is considered that only few modes have an influence in the reconstruction of $u_m(x,t)$. The feature of this basis is that boundary conditions between sub-domains need to assume Steklov condition or to be homogeneous. Main advantage of this method is that it can be applied with success to problems with non linearity on the boundary conditions. But the development of this method is numerically difficult (slow rate of convergence). For example of application on non-linear conditions, see (Laffay et al., 2009) and (Neveu and Khoury, 2000).

METHODOLOGY

Heat and moisture transfer

Let us consider a material. The conservation equation of energy in material can be expressed as :

$$\rho_0 * c(w) * \frac{\partial T}{\partial t} = \nabla(k * \nabla(T) - L_v \nabla(\mathbf{g}_v)) + S_h \quad (2)$$

with $c(w) = c_0 + \frac{rho_l}{\rho_0} * c_w$, $\mathbf{g}_v$ the mass flux of vapour and S_h heat source. The conservation equation of moisture transfer is :

$$\frac{\partial w}{\partial t} = -\nabla(\mathbf{g}_v + \mathbf{g}_l) + S_m \quad (3)$$

S_m is a source term, $\mathbf{g}_v$ is the mass flux of vapour and $\mathbf{g}_l$ is the mass flux of liquid water. Liquid transport is given by Darcy's law :

$$\mathbf{g}_l = -K_l * \nabla P_c \quad (4)$$

with K_l liquid permeability and P_c the capillary pressure. Mass flux of vapour is given by :

$$\mathbf{g}_v = -\delta_v * \nabla P_v \quad (5)$$

with δ_p the vapour permeability of the material. It can also be expressed as:

$$\nabla P_v = \frac{\partial P_v}{\partial P_c} \nabla P_c + \frac{\partial P_v}{\partial T} \nabla T \quad (6)$$

Kelvin's law describes the balance between liquid and vapour phase :

$$P_v = P_{sat} * exp(\frac{P_c}{\rho_l * R_v * T}) \quad (7)$$

$$\frac{\partial P_v}{\partial P_c} = \frac{P_v}{R_v * T * \rho_l} \quad (8)$$

The thermal part of (6) can be written as :

$$\frac{\partial P_v}{\partial T} = \left[\frac{\partial P_s}{\partial T} + P_s \left[\frac{1}{\rho_l R_v T} \frac{\partial P_c}{\partial T} - \frac{P_c}{\rho_l R_v T^2}\right]\right] exp(\frac{P_c}{\rho_l * R_v * T}) \quad (9)$$

With the Clausius-Clapeyron's law :

$$\frac{\partial P_s}{\partial T} = \frac{L_v}{R_v T^2} P_s \quad (10)$$

The dependency of capillary pressure (surface tension) to temperature is assumed negligible, i.e. $\frac{\partial P_c}{\partial T} = 0$ and :

$$\frac{\partial P_v}{\partial T} = \frac{P_v}{R_v * T^2 * \rho_l}(\rho_l * L_v - P_c) \quad (11)$$

Therefore mass conservation in material (3) can be written as :

$$\frac{\partial w}{\partial t} = -\nabla\left[(\delta_v \frac{\partial P_v}{\partial P_c} + K_l)\nabla P_c + \delta_v \frac{\partial P_v}{\partial T} \nabla T\right] + S_m \quad (12)$$

Assuming $\xi = \frac{\partial w}{\partial P_v}$ the sorption curve, we have :

$$\xi \frac{\partial P_c}{\partial t} = -\nabla\left[(\delta_v \frac{\partial P_v}{\partial P_c} + K_l)\nabla P_c + \delta_v \frac{\partial P_v}{\partial T} \nabla T\right] + S_m \quad (13)$$

Finally the heat and moisture transport in Ω can be expressed as the following equation :

$$\begin{bmatrix} \frac{\partial P_c}{\partial t} \\ \frac{\partial T}{\partial t} \end{bmatrix} = \nabla(\begin{bmatrix} -\delta_v \frac{\partial P_v}{\partial P_c} - K_l & -\delta_v \frac{\partial P_v}{\partial T} \\ L_v * \delta_p & -k \end{bmatrix} * \nabla \begin{bmatrix} P_c \\ T \end{bmatrix}) + \begin{bmatrix} S_m \\ S_h \end{bmatrix} \quad (14)$$

The problem is a system of two non-linear partial coupled differential equations. To build the large original model, the problem is generally transformed in an explicit scheme with a spatial and temporal discretisation. Let note $\mathbf{u}$ the matrix containing the values of temperature and capillary pressure at the different nodes of the scheme.

$$\mathbf{u} = \begin{bmatrix} T \\ P_c \end{bmatrix} \quad (15)$$

The discrete solution $\mathbf{u}^{m+1}$ at time t^{m+1} is computed from the linear algebraic system :

$$\mathbf{u}^{m+1} = \mathbf{H}^m \cdot \mathbf{u}^m + S \quad (16)$$

with S the source term matrix and H^m matrix of spatial and temporal discretisation and material properties. This matrix changes at each time step. Details of the explicit scheme are given in (Rouchier et al., 2012)

Proper orthogonal decomposition

As shown in the state-of-the-art, POD is an optimal reduction method. It can also be used for non-linear problems. Therefore POD is chosen here to treat non-linear heat and moisture transfers in materials. The present subsection presents details about this reduction method.

Consider a material $\Omega = \Omega_0 \cup \partial\Omega$ as a 1-dimensional bounded region. Ω_0 represents the set of inner points and $\partial\Omega$ the boundary points. The field is divided into a mesh having M nodes. $u(x,t) = \{T(x,t), P_c(x,t)\}$ is the field of interest. It can be temperature or capillary pressure. $u(x,t)$ is known for each node x_i of the mesh for discrete time $t_m = m * \Delta t$ with $i \in [1, M]$ and $m \in [0, P]$. For simplifying annotation, we assume $u(x_i, t_m) \equiv u_i^m$. The main objective of POD is to search a deterministic function $\Phi(x)$, which gives the optimum representation of u_i^m, $\forall m$, (Liberge and Hamdouni, 2010)). For this, the following scalar quantity needs to be maximized :

$$\alpha = \frac{\sum_{m=1}^{P}[\sum_{i=1}^{M} \phi(x_i) u_i^m]^2}{\sum_{i=1}^{M}(\phi(x_i))^2} \quad (17)$$

which amounts to solve the following eigenvalue problem:

$$\mathbf{c}\Phi = \lambda\Phi \quad (18)$$

Vector **c** is the correlation matrix between two points, defined as :

$$c_{ij} = \sum_{m=1}^{P} u^m(x_i)u^m(x_j) \tag{19}$$

the *snapshots* matrix is defined as :

$$\mathbf{Q} = \begin{bmatrix} \mathbf{u}^1 & \mathbf{u}^2 & \dots & \mathbf{u}^P \end{bmatrix} = \begin{bmatrix} u_1^1 & u_1^2 & \dots & u_1^P \\ u_2^1 & u_2^2 & \dots & u_2^P \\ \vdots & \vdots & \ddots & \vdots \\ u_M^1 & u_M^2 & \dots & u_M^P \end{bmatrix} \tag{20}$$

Therefore the POD modes are solution of the eigenvalue problem

$$\mathbf{Q} \cdot \mathbf{Q}^T \cdot \Phi = \Lambda \cdot \Phi \tag{21}$$

With Φ the matrix containing the eigenvectors. Λ is the diagonal matrix composed of the eigenvalues of Q.

Building the POD reduced order model

To build the reduced order model of heat and moisture transfers, the eigenvalue problem (18) must be solved. The eigenvectors are calculated with the corresponding eigenvalues. $\tilde{\Phi}$ is the spatial POD basis, size $M \times N$, composed of the N eigenvectors corresponding to $\{\lambda_i\}_{1 \leq i \leq N}$ eigenvalues. Doing this, the POD decomposition is the best energy decomposition that can be obtained. These eigenvalues are chosen as the highest eigenvalues $\lambda_1 \geq \ldots \geq \lambda_i \geq \ldots \geq \lambda_N \geq 0$. The function $f(N)$ is the energy captured with the N modes chosen :

$$f(N) = \frac{\sum_{i=1}^{N} \lambda_i}{\sum_{j=1}^{M} \lambda_j} \tag{22}$$

N is sought as $f(N) > 0.999$. Therefore the spatial POD basis is

$$\tilde{\Phi} = \begin{bmatrix} \Phi_1 & \Phi_2 & \dots & \Phi_N \end{bmatrix} \tag{23}$$

or equivalent

$$\tilde{\Phi} = \begin{bmatrix} \Phi_1(x_1) & \Phi_2(x_1) & \dots & \Phi_N(x_1) \\ \Phi_1(x_2) & \Phi_2(x_2) & \dots & \Phi_N(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_1(x_M) & \Phi_2(x_M) & \dots & \Phi_N(x_M) \end{bmatrix} \tag{24}$$

N is the order of reduction of the problem. These N eigenvectors are chosen to approximate the solution u_i^m, $\forall m$ (equation (1)). Let us call $\tilde{u}_i^m$ the field of interest obtained by the reduced order model :

$$\tilde{U} = \tilde{\Phi} \cdot \zeta \tag{25}$$

with ζ new matrix of size $(M \times N)$. Then $\tilde{u}_i^m$ is solution of the reduced order model:

$$\tilde{\Phi} \cdot \zeta^{m+1} = \mathbf{H}^m \cdot \tilde{\Phi} \cdot \zeta^m + S \tag{26}$$

The size of solution of the algebraic system (26) is $(N \times 1)$ whereas the one of (16) is $(M \times 1)$. The size of the new reduced order model is smaller than the large original model. Table 1 gives the different size of each matrix.

*Table 1: Size of different matrices of POD reduction, with $n << m$. **u** solution of the large original model and ζ solution of the reduced order model*

matrix	size	matrix	size
u	$(M \times 1)$	**Q**	$(M \times P)$
Φ	$(M \times M)$	Λ	$(M \times M)$
$\tilde{\Phi}$	$(M \times N)$	ζ	$(N \times 1)$

Difference between large original model and reduced order model is estimated by square mean quadratic error (SMQE) :

$$SMQE = \sqrt{\frac{1}{M * P} * \sum_{i=1}^{M} \sum_{k=1}^{P} e^2(i,k)} \tag{27}$$

with $e(i,k) = U - \tilde{U}$ the approximation error.

Strategy for building the POD reduced order model

The issue is to build a reduced order model representing system (14). The difficulties come from the coupled equations and the non-linear coefficient. To build this reduced model, the following strategy is proposed (see figure 1) :

- **step one**: The large original model under boundary conditions is solved for a time interval. With the solution u of the large original model, the *snapshots* matrix is created.
- **step two**: Then the POD basis is built choosing the N dominant modes (N eigenvectors associated to the N eigenvalues)
- **step three**: The new explicit scheme is constructed for the reduced order model with new solution ζ
- **step four**: Solutions of large original model and reduced order model are compared.

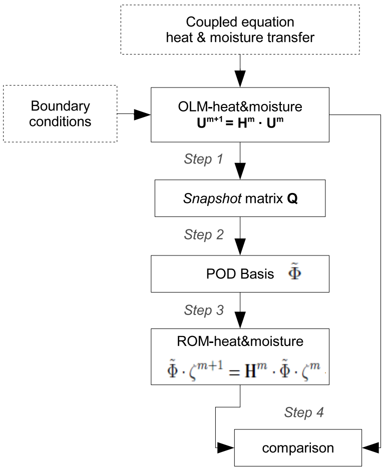

Figure 1: Strategy of construction of the reduced order model of heat and mass transfer. ROM means reduced order model and OLM means large original model.

MODEL REDUCTION IN PRACTICE

The POD method is illustrated on the following case (see figure 2). A block of concrete of 10×10 cm is considered with the following boundary conditions (see figure 3):

- side (1) : no transfer
- side (2) : dirichlet condition, temperature and relative humidity imposed : $T_2 = sinuswave$ and $RH_2 = sinuswave$.
- side (3) : dirichlet condition, temperature and relative humidity imposed : $T_3 = 19°\text{C}$ and $RH_3 = 0.8$
- side (4) : dirichlet condition, temperature and relative humidity imposed : $T_4 = 10°\text{C}$ and $RH_4 = 0.6$

Material properties used come from (Rouchier et al., 2012). Equations describing retention curve and moisture transport coefficient were determined by fitting measurements. Thermal properties were taken from literature and database ((Hagentoft et al., 2004)).

Initial conditions are $T(x, y, 0) = 293.15$ K and $P_c(x, y, 0) = -5 \cdot 10^{-8}$ Pa. The material is discretized with a mesh of 50×50 nodes. The time step is constant $\Delta t = 600$ with $t \in [0, 720]$, corresponding to 5 days. Here, no moisture or thermal sources are considered.

The large original model is computed using the software *MATLAB*, according to (Rouchier et al., 2012). With the results of this model, the *snapshots* matrix Q is computed following equation (20) and *snapshots* matrix of temperature Q_T and capillary pressure Q_{Pc}.

$$Q = \begin{bmatrix} Q_{Pc} \\ Q_T \end{bmatrix} \qquad (28)$$

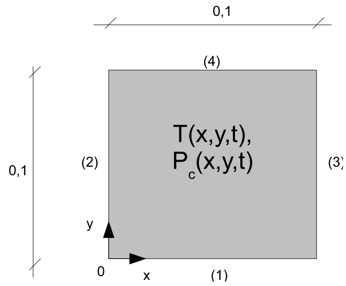

Figure 2: Case for the construction of the reduced order model

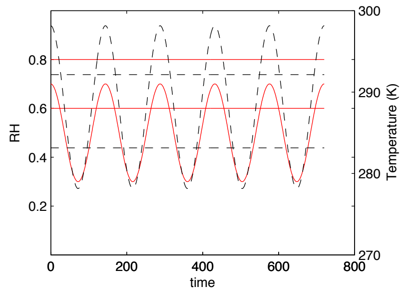

Figure 3: Boundary conditions for temperature (dash black) and relative humidity (red linear) (the time axis corresponds to the the numbers of iterations (720))

Eigenvalues problem (21) is solved and each problem (temperature and pressure) contains 2500 eigenvalues. The 10 largest eigenvalues for each problem are selected. With this choice, 99.90 % of the energy of the field of interest is captured. Matrix $\tilde{\Phi}$ is given by :

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}_{Pc} & 0_{(2500,10)} \\ 0_{(2500,10)} & \tilde{\Phi}_T \end{bmatrix} \qquad (29)$$

with $\tilde{\Phi}_{Pc}$ and $\tilde{\Phi}_T$ are the POD basis for respectively capillary pressure and temperature. The field of interest is obtained by the reduced model :

$$\tilde{U} = \tilde{\Phi} \cdot \zeta = \tilde{\Phi} \cdot \begin{bmatrix} \zeta_T \\ \zeta_{Pc} \end{bmatrix} \qquad (30)$$

with ζ, solution of the reduced order model following the explicit scheme (26). This scheme is also computed using the software *MATLAB*. The unknown of the reduced model problem is a (20×1) matrix instead of (5000×1) for the large original model. Following table gives matrix dimensions of the simulation case.

Table 2: Size of different matrices for the test case

matrix	**size**	**matrix**	**size**
U	(5000×1)	**Q**	(5000×721)
Φ	(5000×5000)	Λ	(5000×5000)
$\tilde{\Phi}$	(5000×20)	ζ	(20×1)

RESULTS

The solution of the reduced model is compared to the one of the complete problem. On figure 4 temperature and capillary pressure are plotted on the mesh at t=720 for the ROM and LOM.

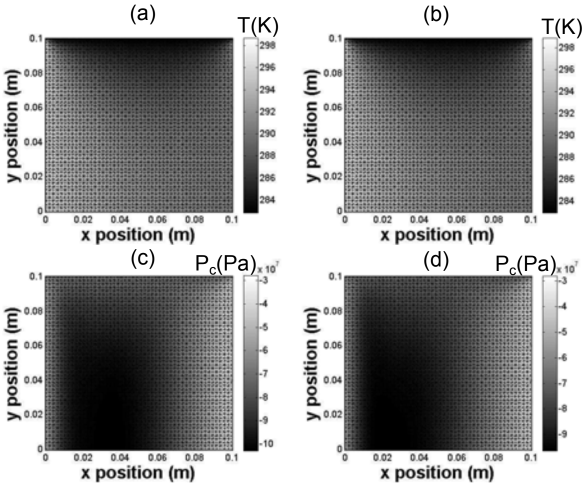

Figure 4: Temperature ((a) and (b)), capillary pressure ((c) and (d)) and grid nodes for the reduced order model ((a) and (c)) and large original model ((b) and (d)) at $t = 720$

To perform the analysis, the differences between two models were studied for the profile of temperature and capillary pressure at $y = 0.05$ m. Figures 5 and 6 represent results every 24 hours for the large original model and the reduced order model.

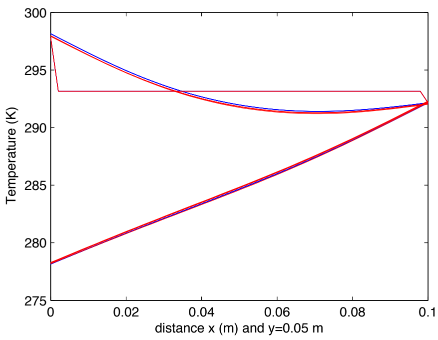

Figure 5: Large original model (blue) versus reduced order model (red) for temperature profile at $y = 0.05$ m every 24 hours

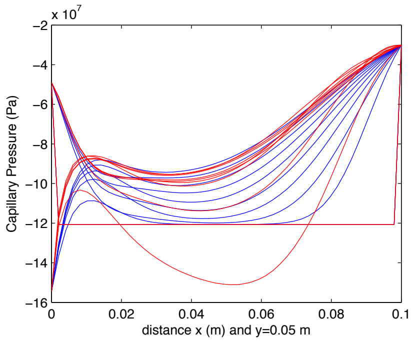

Figure 6: Large original model (blue) versus reduced order model (red) for capillary pressure profile at $y = 0.05$ m every 24 hours

Figures 7 and 8 give the mean, maximum and minimum values of each model for temperature and capillary pressure profile at $y = 0.05$ m.

The squared mean quadratic errors (SMQE) for both field of interest, as function of time, are plotted on figures 10 and 9.

And Table 3 gives the total SMQE for this simulation.

Table 3: Total SMQE for temperature and capillary pressure

	SMQE
Temperature (K)	0.0075
Capillary Pressure (Pa)	$1.14 \cdot 10^6$

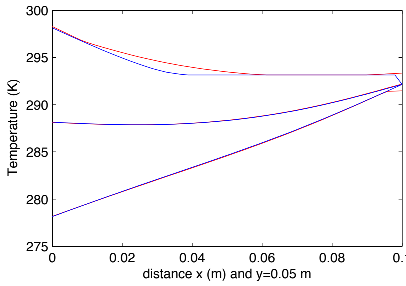

Figure 7: Mean, maximum and minimum values of each model (LOM=blue, ROM=red) of temperature profile at $y = 0.05m$

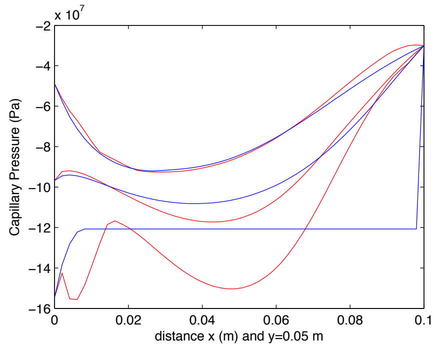

Figure 8: Mean, maximum and minimum values of each model (LOM=blue, ROM=red) of capillary pressure profile at $y = 0.05m$

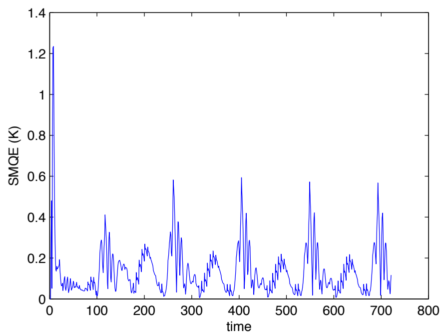

Figure 9: Temperature SMQE for the total grid as function of time

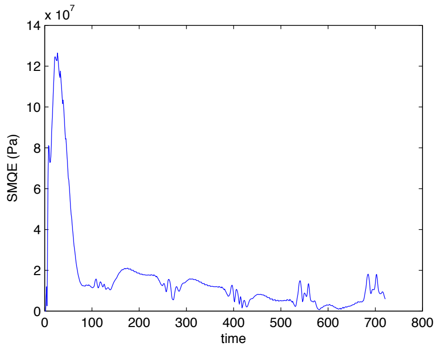

Figure 10: Capillary pressure SMQE for the total grid as function of time

DISCUSSION AND RESULTS ANALYSIS

The final temperature and capillary pressure for reduced order model and for large original model are very close (figures 7 and 8). Furthermore, the reduced model succeeds in representing temperature inside the material. Results of SMQE correspond to the one of (Chinesta et al., 2011). On the contrary the capillary pressure at the beginning of the simulation is not well modelled by the reduced model at the beginning of the simulation. The approximation error is first high (figure 10) and then decreases. The issue of our work is to carry simulation during a large time interval. Therefore, for this issue, the reduced order model yields in an accurate representation of temperature and capillary pressure. The results presented above illustrate the significant contribution of model reduction technics for precise simulations of coupled HAM transfers in 2D geometries.

The choice of order of the reduced model, i.e. number of eigenvectors for the POD basis, is a relevant question. To illustrate this, a reduced model of order 3 was compared with the reduced model obtained previously (order 10). Figure 11 and 12 give the comparison between the large original model and the reduced order 3 model.

The reduced order model has difficulties to represent the fields of interest. For temperature, profiles are really different between large original model and reduced one. In addition, table 4 shows that errors is 10 times more important than for the reduced order 10 model (see figures 9 and 10).

On figure 13 and 14, comparison of mean temperature and capillary pressure is done between reduced order 3 model, reduced order 10 model and large original model. The reduced order 3 model does not succeed representing the large original model. Reduced order 10 model yields in a better accurate representation of large original model.

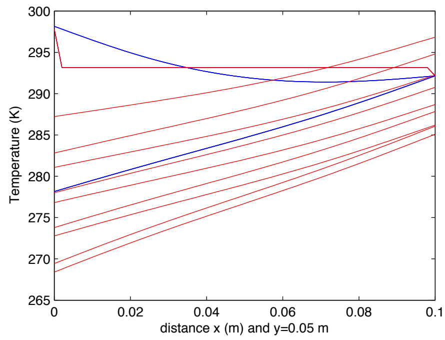

Figure 11: Large original model (blue) versus reduced order 3 model (red) for temperature profile at $y = 0.05$ m each 24 hours

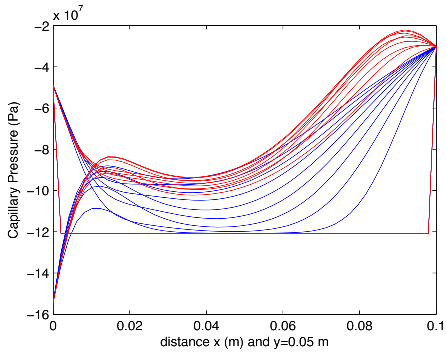

Figure 12: Large original model (blue) versus reduced order 3 model (red) for capillary profile at $y = 0.05$ m each 24 hours

Table 4: Total SMQE for temperature and capillary pressure for reduced order 3 model

	SMQE
Temperature (K)	0.353
Capillary Pressure (Pa)	$9.8 \cdot 10^6$

The choice of the order of the reduced model is important. A small order will reduce computational time but less accurate information will be available on hygrothermal fields inside materials. It can be noticed that, following 22, $f(3) = 0.9845$ and $f(10) = 0.9999$. It explains why the reduced order 10 better represents the large original model.

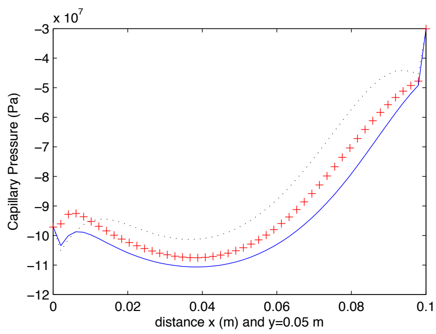

Figure 13: Mean values of each model (LOM=continuous, ROM_{10}=cross, ROM_3=dots) of capillary pressure profile at $y = 0.05m$

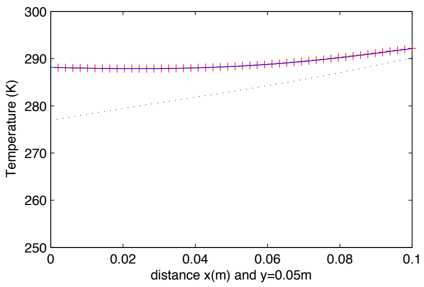

Figure 14: Mean values of each model (LOM=continuous, ROM_{10}=cross, ROM_3=dots) of temperature profile at $y = 0.05m$

As already mentioned, disadvantage of POD is that this method requires the full solution of the large original model. Others authors, as (Liberge and Hamdouni, 2010), observe that the use of the reduced order model is restricted to similar conditions (boundary and material properties) used for the construction of the large original model. This purposes will be studied in future works.

CONCLUSION

In this paper, a reduced order model for coupled heat and mass transfers in porous material has been presented. After analysis of existing methods, a proper orthogonal decomposition method has been chosen as it was already applied for non linear problems. The appropriateness of the proposed method has been illustrated on a 2-dimensional case. A large original model of 5000 unknowns has been successfully replaced by a ROM of 20 unknowns. Temperature field was very precisely represented by the reduced order model, slightly larger differences were observed for capillary pressure. The choice of the order of the reduced model has been highlighted as a relevant question and will be further investigated in future works.

REFERENCES

Chinesta, F., Ammar, A., Leygue, A., and Keunings, R. 2011. *An overview of the proper generalized decomposition with applications in computational rheology*. *Journal of Non-Newtonian Fluid Mechanics*, 166(11):578–592.

Dauvergne, J.-L. and Palomo del Barrio, E. 2010. *Toward a simulation-free pod approach for low-dimensional description of phase-change problems*. *International Journal of Thermal Sciences*, 49(8):1369–1382.

Dumon, A., Allery, C., and Ammar, A. 2011. Proper general decomposition (PGD) for the resolution of NavierStokes equations. *Journal of Computational Physics*, 230(4):1387–1407.

Gao, Y., Roux, J., Zhao, L., and Jiang, Y. 2008. *Dynamical building simulation: A low order model for thermal bridges losses*. *Energy and Buildings*, 40(12):2236–2243.

Girault, M., Derouineau, S., Salat, J., and Petit, D. 2004. *Model reduction for natural convection by identification method (in French)*. *Comptes Rendus Mecanique*, 332(10):811–818.

Girault, M. and Petit, D. 2005. *Identification methods in nonlinear heat conduction. Part I: Model reduction*. *International Journal of Heat and Mass Transfer*, 48(1):105–118.

Hagentoft, C.-E., Sasic Kalagasidis, A., and Adl-Zarrabi, A. 2004. *Assessment method of numerical prediction models for combined Heat, Air and moisture transfer in building components. Benchmarks for one-dimensional cases*. *Journal of thermal envelope and building science*, (27:327-52).

KIM, E.-J. 2011. *Development of numerical models of vertical ground heat exchangers and experimental verification : domain decomposition and state model reduction approach*. PhD thesis, Institut National des Sciences Appliques de Lyon, Lyon.

Laffay, P., Quemener, O., and Neveu, A. 2009. Developing a method for coupling branch modal models. *International Journal of Thermal Sciences*, 48(6):1060–1067.

Liberge, E. and Hamdouni, A. 2010. Reduced order modelling method via proper orthogonal decomposition (POD) for flow around an oscillating cylinder. *Journal of Fluids and Structures*, 26(2):292–311.

Neveu, A. and Khoury, K. 2000. *Reduction of a nonlinear thermal model by the branch modes : test case on heating cable (in French)*. Societe Francaise de Thermique.

Palomo Del Barrio, E. 2011. *Solving high dimension thermal problems : reduction method (in French)*. Editions Universitaires Europeennes.

Rouchier, S., Janssen, H., Rode, C., Woloszyn, M., Foray, G., and Roux, J.-J. 2012. Characterization of fracture patterns and hygric properties for moisture flow modelling in cracked concrete. *Construction and Building Materials*, 34:54–62.

Verdon, N. 2007. *A low-order dynamical system based on a APR-POD approach for studying turbulent flow particles interaction (in French)*. PhD thesis, Universite de La Rochelle.

Woloszyn, M. and Rode, C. 2008. Tools for performance simulation of heat, air and moisture conditions of whole buildings. *Building Simulation*, 1(1):5–24.