# MODELING INTERCEPTION OF TREES' CANOPIES OF INDOOR DAYLIGHTING 

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#### Abstract

Developing a methodology of simulating tree shading on daylight performance in indoor spaces such as lux levels on a task plane is investigated. The findings will be useful in improving design of lighting environments in public buildings such as office buildings or classrooms. The method used a camera that takes hemispherical images, positioned in the centre of the task plane. Boundary conditions need to be defined at the outset include the type and age of the trees, the type of the sky and turbidity, the orientation of the window, the time of the day, and the time of the year. The methodology integrated measurement of tree canopy characteristics (gap fraction and Leaf area density) using hemispherical photography and development of 3D objects using parametric modelling to represent tree prototypes. It depended on expressions that predicted gap fractions of a theoretical canopy using a 'least squares' technique. The gap fractions results helped to model 2D object (screen) that represents the trees. The illuminance levels were simulated inside the treeshaded space and the output results were validated against actual measurements in a large physical model using MBE and RMSE techniques ( $\mathrm{MBE}=$ 0.13, RMSE $=0.02$ ).


## INTRODUCTION

Trees and other plants such as vines or vegetation grown on trellises can help to control sunlight and improve the quality and comfort levels of the luminous environment in the indoor spaces, when used for shading across the windows or farther away in the case of trees. The improvement in performance is caused by the combined effect of the vegetation to scatter direct sunlight and reduce its intensity while moderating glare coming from the bright sky. These natural methods (i.e., plants and trees) provide better quality of daylight than the use of light drapes or translucent glazing because of the problem of glare. Plants and vegetation can also provide other environmental benefits:

- create a better microclimate in courtyards and outdoor spaces with the modified thermal conditions as a result of shading and passive cooling;
- provide esthetics in the indoor and outdoor spaces;
- reduce noise levels;
- provide healthier spaces and filter air from dust and unhealthy particles; and
- improve thermal and visual comfort and improve wellbeing.
Accurate simulation of light passing through trees' canopies has been investigated previously using a method that relied on hemispherical image acquisition and computation of canopy gap fractions. The results helped to develop a 3D tree model that was used to simulate the effect of tree on illuminance levels and the output results were validated against actual measurements. So far this method produced rational results for only one case; i.e., interception of light right under the trees; a case that could be useful in some architectural applications such as effect of tree shading in the outdoor spaces or (green) roofs. Other important environmental settings related to more typical applications in the built environment need to be investigated especially the case when one needs to study tree shading on daylight performance in indoor spaces such as lux levels on a task plane. Such findings will be useful in improving design of lighting environments in public buildings such as office buildings or classrooms.


## BACKGROUND

Measuring the dissipation of light as it penetrates a tree canopy requires an estimation of the cumulative Leaf area index. LAI is expressed in $\mathrm{m}^{2}$ leaves area $/ \mathrm{m}^{2}$ of covered ground. It is widely used to describe the photosynthetic and transpirational surface of plant canopies and has broad applications in ecophysiology, water balance modelling, and characterisation of vegetation atmosphere interactions.
Several researchers discussed the potential of hemispherical photography. Bellow and Nair (2003) found that under conditions of low stand density and open canopies estimates from hemispherical photograph were better than other methods such as densitometer or visual index. Lin and Chiang (2002) stated that undercanopy hemispherical photographs provide permanent records of canopy structure, and as such have great potential for studies of forest
canopy dynamics. Rapid computerized image analysis has facilitated studies of undercanopy light environments and the canopy leaf area index (LAI).
Hemispherical photography depends on a hemispherical field of view, which, when looking directly upwards, corresponds to a viewshed (or skydome) of all sky directions (Rich et al. 1999). A self-leveling mount equipped with an LED (lightemitting diode) is used to level the hemispherical lens and to identify the range of the image. When hemispherical photographs are properly analysed using digital image analysis, they can literally provide a detailed map of sky visibility and obstruction. In the case of plant canopies, a hemispherical photograph can be interpreted as a map of the directions of canopy openings relative to the location from which the photograph is taken. Hence, this could help to study light penetration into tree canopies and the impact on nearby buildings. Each position in a hemispherical photograph represents a direction from the location where the photograph was acquired. The radial component of distance is proportional to zenith angle, and the distance in an arc at a fixed radius is proportional to azimuth angle. It is important to use high quality camera and lenses that help obtain images with high contrast as this would help to save time and increase the efficiency and accuracy of the image analysis.

## THEORY

Calculation of LAI involves use of Beer's Law, which can be expressed as follows:
$G(\theta)=\exp (-K(\theta) . L)$
Where $G$ is gap fraction, $K(\theta)$ is the extinction coefficient at angle $\theta$, and L is LAI, $\theta$ is zenith angle. Gap fraction is the proportion of visible sky within a given sky sector, where a sky sector is defined by a range of zenith and azimuth angles. A gap fraction of zero ( 0 ) means that the sky is completely blocked (obscured) in that sky sector. A gap fraction of one (1) means that the sky is completely visible (not obscured) in that sky sector.
Campbell (1986) derived a straightforward value for the extinction coefficient for an infinite uniform canopy of randomly oriented leaf elements, of given LAI and an ellipsoidal leaf angle distribution. This allowed the leaf angle distribution to be described by a single parameter, the Ellipsoidal Leaf Angle Distribution Parameter or ELADP. In this model, the leaf elements are distributed in the same proportions as the surface of an ellipsoid of revolution, where ELADP is the ratio of the horizontal to vertical axes of the ellipsoid. An ellipsoidal canopy shape is chosen because of its adaptability to various other shapes (Abraha and Savage, 2010).

For a canopy of LAI L and ELADP x, the extinction coefficient for a light ray passing through the canopy at zenith angle $\theta$ is given by the equation:
$K(x, \theta)=\frac{\sqrt{x^{2}+\tan (\theta)^{2}}}{x+1.702(x+1.12)^{-0.708}}$
Where x is the ELADP and $\theta$ is the zenith angle of the direct beam. The probability of the light ray passing through the canopy, or the gap fraction in that direction, can be expressed as follows:
$\tau(x, \theta)=\exp (-K(x, \theta) . L)$
where $\tau$ is the gap fraction, $L$ is the Leaf Area Index, and $K(x, \theta)$ is the extinction coefficient. The equation can be inverted to give an estimate of LAI; this would require finding the gap fraction using one of the measurement methods such as hemispherical photography or light interception. Calculation of LAI involves a kind of iterative "inversion" model, whereby LAI is inferred from the observed distribution of gap fraction as a function of zenith angle. The model finds the values of LAI and ELADP for an ellipsoidally distributed theoretical canopy that give the best fit to the measured gap fraction values. The model is completely independent of scale, so that the actual thickness of the canopy does not make any difference, only the total LAI of the whole canopy matters.
The probability of a ray of light passing through the canopy depends on the distance it has to travel through the crown. The extinction coefficient $K(x, \theta)$ defined above gives the extinction of a ray of light passing completely through a horizontal canopy. The extinction coefficient for a ray of light passing through unit canopy distance is given by:
$G(x, \theta)=K(x, \theta) \cdot \cos (\theta)$
and the gap fraction is now given by:

$$
\begin{equation*}
\tau(x, \theta)=\exp (-G(x, \theta) \cdot L D \cdot d) \tag{5}
\end{equation*}
$$

where $d$ is the distance through the canopy and LD is the leaf area density, which is defined as the total area of (single sided) leaf surface per unit canopy volume. This equation can now be inverted to find LD (and ELADP) from the gap fraction values if we can estimate d, the distance through the canopy (tree crown) at the angle of each light ray (or hemispherical photo sector). One approach to finding the distances through the crown is to measure them. An easier approach is to assume the crown has a simple geometric shape and calculate the distances using trigonometry. Moreover, the model calculates
the canopy path length (d) through which radiation has to travel in order to reach a certain point on a horizontal plane, which could be at or above the ground level (Abraha and Savage, 2010).


Figure 1 Hemispherical photograph of the real tree with the frame used to represent the window before and after image masking.

## METHODOLOGY

The methodology depends on hemispherical photography using a camera with a fish-eye lens. Since the design goal is to study daylight performance in indoor spaces such as classrooms and how this will affect lux levels on a horizontal task plane, the camera needs to be positioned on the centre of the task plane (e.g.; a desk). From this position, the camera can capture the sky portions (or gaps) visible within a tree foliage located in the outdoors through a window. Logically, the set-up for such conditions would require to take the required images inside a room with a window shaded from outside by a tree. However, this set-up has many limitations and was not available. Another more flexible and less costly way is just to use a window frame in front of trees and take the required hemispherical images from the right position. Then later in the image analysis, one can mask all portions of the image that are located outside the window frame (Figure 1). To do this, the experimental set-up was designed to consider the following:

- The center of the camera and the center of the tree are both aligned on the window perpendicular axis (i.e.; the indoor/outdoor axis). The window frame is imagined in a centered position on the wall.
- The vertical level of the camera is positioned at 0.80 m height from the floor level to represent a height of a desk surface (i.e.; the task plane).
- The horizontal distance from the tree canopy to the camera $=\mathrm{a}+\mathrm{b}+\mathrm{c}=2.0 \mathrm{~m}+0.30 \mathrm{~m}$ $+1.0 \mathrm{~m}=3.30 \mathrm{~m}$, where: $\mathrm{a}=$ distance from the tree centre to outside wall surface, $\mathrm{b}=$ the wall thickness, and $c=$ the distance from the interior wall surface to the center of the task plane.
Other condition parameters related to trees shading effect needed to be defined: the tree type, age and
foliage as these determine canopy size and density. The decision taken was to investigate mature Neem trees based on previous research that showed its high potential as shade trees in hot arid climates.


## Image Acquisition

Six trees were selected to represent mature Neem trees. To analyse the trees, each tree was photographed and the images of all trees were recorded and organized in digital files. Image acquisition involved taking hemispherical photographs using a digital camera looking upward from a location of interest. The basic goal was to obtain high quality photographs. This was achieved by using a high quality camera (Nikon model \# COOLPIX-8400) with high quality hemispherical lens (Nikon Fish-eye lens model \# SCL6 fitted to SLM6 frame); and the photographs were taken after sunset when even back-lighting conditions were available.

## Image Analysis

The images of the trees were classified to distinguish visible and obscured sky directions. . All portions of the image that are located outside the window frame were masked. The sky visibility and obstruction as a function of sky direction was calculated. The information about sky visibility and obstruction helped to calculate the canopy indices. The gap fractions were calculated from the hemispherical photo and their values were used as basis to develop a 3D pervious surface with openings that correspond to the sky gap fractions seen through the tree.

## Development of Geometrical Tree

With the crown geometry and the predicted gap fractions data being available now, developing a a simple object to resemble the measured tree is possible. The developed geometrical model can be used in lighting simulation software to simulate its shading effect on lighting performance. The method depended on creating a pervious surface that look like a window screen with openings that correspond to the sky gap fractions seen through the tree. Based on the information taken from the image analysis, one can see how the window limits the view angle range of the captured data along the azimuth direction (1-2 sectors of the sky map in the azimuth direction are only visible). However, on the zenith direction, the view angle range of the captured data is larger (4-5 sectors of the sky map in the azimuth direction are visible) and the data in this direction varies noticeably. The sky map sectors that are visible along the zenith direction (from horizontal to vertical) are $85^{\circ}, 75^{\circ}, 65^{\circ}, 55^{\circ}$, and $45^{\circ}$. Based on this, a value for each of these sectors was calculated by taking the average of all measured trees data for that sector. To validate the modelled tree, one needs to import it into radiance, build a full radiance scene, simulate the required lighting conditions and compare the results to actual measurements. This process is described below.


Figure 2 The experimental set up for measuring lux levels inside a real model in front of a tree.

Table 1
Gap fractions of the real tree; shaded cells represent masked area of the image.

|  | Azimuth |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Zenith | $0^{\circ}$ | $45^{\circ}$ | $90^{\circ}$ | $135^{\circ}$ | $180^{\circ}$ | $225^{\circ}$ | $270^{\circ}$ | $315^{\circ}$ |
| $5^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $15^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $25^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $35^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $45^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $55^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.039 | 0.001 |
| $65^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.158 | 0.012 |
| $75^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.221 | 0.038 |
| $85^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.008 | 0.006 |

Table 2
The average results of the predicted gap fractions; shaded cells represent masked area of the image.

|  | Azimuth |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Zenith | $0^{\circ}$ | $45^{\circ}$ | $90^{\circ}$ | $135^{\circ}$ | $180^{\circ}$ | $225^{\circ}$ | $270^{\circ}$ | $315^{\circ}$ |
| $5^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $15^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $25^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $35^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $45^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $55^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 |
| $65^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.040 |
| $75^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.098 | 0.098 |
| $85^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.143 | 0.143 |

Table 3
Mean bias errors (MBE) and root mean bias errors (RMSE) for all studied trees.

| Time | MBE | RMSE |
| :--- | :--- | :--- |
| Dec 21, 10 AM | 0.10 | 0.04 |
| Dec 21, 12 PM | 0.11 | 0.04 |
| Mar 21, 10 AM | 0.13 | 0.02 |
| Mar 21, 12 PM | 0.17 | 0.03 |
| Overall | 0.13 | 0.02 |

## Simulation

The surface model created to simulate the tree canopy along with other objects in the lighting environment is imported to Radiance to create the required simulation scene. Radiance is a computer software package developed by the Lighting Systems Research group at Lawrence Berkeley National Laboratory, University of California, under the direction of Greg Ward (Ward, 1994; Ward and Shakespeare, 1998). It is widely used in lighting research for accurately simulating and predicting the distribution of visible radiation. Radiance uses a combination of ray-tracing and radiosity techniques to calculate and predict the distribution of visible radiation. The program generates spectral radiance values in the form of photo realistic images and numerical tables. Another part of the scene description is the definition and application of materials for the objects in the scene. Material spectral properties include their color, reflective properties, and transmittance properties. It can be acquired from field measurement. The third part of the scene is the sky description, created by the 'gensky' generator. Gensky produces a Radiance scene description for the CIE (Commission Internationale de L`eclairage) standard sky distribution at the given month, day and time for any given geographical location.

## Validation

To validate the results and to check to what extent the model is a suitable representative of the actual trees, the best way is to simulate the illuminance level on the task plane and compare the results to actual field measurements. This would require having an actual space with a mature Neem tree located outside at a distance of 2-3 meters from the axis of the space's window. Yet, such situation was not available and instead the investigator used a large physical model in front of Neem trees and took actual measurements using photometric sensors (Figure 2). The sensors' locations were selected at four different depth points from the window: $\mathrm{D} 1=0.75 \mathrm{~m}, \mathrm{D} 2=$ $2.25 \mathrm{~m}, \mathrm{D} 3=3.75 \mathrm{~m}$, and D4 $=5.25 \mathrm{~m}$. The relative mean bias error (MBE) and the relative root mean square error (RMSE) were calculated to estimate the difference between actual measurements and simulation values.

## RESULTS AND DISCUSSION

The gap fractions from a hemispherical photograph for each tree were calculated. Their values are associated with the sky map, based on division of the sky dome into eight segments in the azimuth direction (i.e.; every $45^{\circ}$ azimuth angle) and nine segments in the zenith direction (i.e.; every $10^{\circ}$ zenith angle); as described in Table 1 for one of the trees. The average value for the gap fractions were also calculated, throughout all trees to produce representative values for the whole group. The crown of the tree is assumed to be as half ellipse shape with 4 meters height and 2 meters radius. The leaf area density (LD) and Ellipsoidal Leaf Angle Distribution Parameter (ELADP) are calculated based on the tree geometry and the best fit to the measured gap fraction values using equations 4 and 5 . Based on the calculated values of ELADP, Leaf Area Density, and the assumed crown geometry, the gap fractions for an ellipsoidally distributed theoretical canopy are predicted. This procedure was repeated and the results were produced for each tree. The average results of the predicted sky gap fractions are shown in Table 2. These values helped to develop a threedimensional object to resemble the measured trees using parametric object modelling.

## Error Analysis

Actual measured illuminance levels were compared against corresponding results produced by Radiance and a holistic analysis of the differences was carried out. The MBE and RMSE were calculated for all the depth points (sensors' locations) in the space on the winter solstice and spring equinox at 10:00-AM and 12:00-PM. These two statistical methods have been used in previous daylighting studies to validate daylight modeling results (e.g.; Reinhart 2009; Reinhart and Walkenhorst 2001; Reinhart and Anderson 2006). Equations 6 and 7 show the calculation method for MBE and RMSE as follows:
$M B E=\frac{1}{N} \sum_{i=1}^{N}\left(\frac{\left(x_{\text {test }, i}-x_{r e f, i}\right)}{x_{r e f, i}}\right)$
$R M S E=\frac{1}{N} \sqrt{\sum_{i=1}^{N}\left(\frac{\left(x_{t e s t, i}-x_{r e f, i}\right)^{2}}{x_{r e f, i}^{2}}\right)}$
Where $N$ is the population number, $x$ test is the measured illuminance value in lux, $x$ ref is the simulated illuminance value in lux.
The statistical quantities MBE and RMSE characterize the similarities and/or differences between two data sets, where MBE indicates the tendency of one data set to be larger or smaller than the other; and RMSE indicates how far one data set fluctuates around the other. The smaller the RMSE/MBE numbers the better, where it indicates
less variation and smaller difference. Table 3 shows the calculated MBE and RMSE for all the points at the different tested times. The results showed relatively smaller numbers for March 21 and with an overall value of $\mathrm{MBE}=0.13$ and $\mathrm{RMSE}=0.02$, calculated for the 16 pairs. It should be noted that there is no a previous standard or a common reference that suggests how high or low typical MBEs and/or RMSEs should be for reliable modeling of trees, therefore; the author suggests error margin not to exceed $20 \%$ with a statistical confidence level of $80 \%$. According to Reinhart (2011), some recent simulation studies have yielded that dynamic daylight simulations using different methods can simulate indoor illuminances with a relative error below $25 \%$ compared to measurements (Mardaljevic 2000a; Mardaljevic 2000b; Reinhart and Anderson 2006; Reinhart and Breton 2009; Reinhart and Walkenhorst 2001). This margin could be acceptable when one considers that the human eye adapts quickly to the fluctuations in the illuminance levels and it can barely notice illuminance changes over time in that order of magnitude (Reinhart, 2011). Looking at the obtained MBEs and RMSEs values, the simulated data are in a good agreement with the measured ones and can be considered reliable.

## CONCLUSION

The study investigated how to simulate natural light passing through tree canopies and the effect of tree shading on daylight performance in indoor spaces. It uses a methodology that integrates measurement of tree canopy characteristics (gap fraction and leaf area density) using hemispherical photography, development of 3D objects using parametric modeling to represent tree prototypes, and the use of trees in lighting simulation programs. The method relied on a simple derivative 2D object (screen surface) to represent the visible sky segments through the window from the tree 3D model based on the calculated gap fraction values. The lighting level results were validated using MBE and RMSE techniques; which showed close agreement between the measured and simulated illuminance levels (MBE $=0.13$, $\mathrm{RMSE}=0.02$ ).

## NOMENCLATURE

| $G, \tau$ | Gap fraction |
| ---: | :--- |
| $K(\theta)$ | Extinction coefficient at angle $\theta$ |
| $L, L A I$ | Leaf Area Index $\left(\mathrm{m}^{2} \mathrm{~m}^{-2}\right)$ |
| $\theta$ | zenith angle |
| $E L A D P$ | Ellipsoidal Leaf Angle Distribution Parameter |
| $L D$ | Leaf area density $\left(\mathrm{m}^{2} \mathrm{~m}^{-3}\right)$ |
| $D$ | Distance through the canopy (m) |
| $M B E$ | Mean Bias Error |
| $R M S E$ | Root Mean Square Error |
| $N$ | population number |
| $x_{\text {test }}$ | measured illuminance (lux) |
| $x_{\text {ref }}$ | simulated illuminance (lux) |

$K(\theta) \quad$ Extinction coefficient at angle $\theta$
$L, L A I \quad$ Leaf Area Index $\left(\mathrm{m}^{2} \mathrm{~m}^{-2}\right)$
$\theta$ zenith angle
ELADP Ellipsoidal Leaf Angle Distribution Parameter
$L D \quad$ Leaf area density $\left(\mathrm{m}^{2} \mathrm{~m}^{-3}\right)$
$D$ Distance through the canopy (m)
MBE Mean Bias Error
RMSE Root Mean Square Error
$N$ population number
$x_{\text {test }}$ measured illuminance (lux)
$x_{\text {ref }}$ simulated illuminance (lux)

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