IS IT REASONABLE TO CALCULATE THE THERMAL PEAK LOAD USING ACAUSAL MODELS AND SHORT TIME STEPS?

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ABSTRACT

Heating and cooling load estimation is based on the heat balance method. Although this method has strong physical basis, it yields an acausal system when the heat capacity of the indoor space is non-negligible. New emergent problems in low energy buildings require time steps significantly smaller than 1 hour, which is usually used by simulation software. Therefore, when the time step tends to zero, the calculated load tends to infinity when the indoor air temperature has an incremental variation. This paper demonstrates that the heat balance method results in an acausal system and analyses the variation of load estimation with the size of the simulation time step.

INTRODUCTION

Load estimation is an essential task for building design, especially for HVAC sizing. It is also used to check the conformity with the thermal regulations. The basis for load estimation is the heat balance. It is recommended by ASHRAE (Bellenger et al. 2001) and used by the CEN standards (e.g. CEN 2003, CEN 2007a, CEN 2007b) and simulation software (TRNSYS 2007, EnergyPlus 2012, Clarke 2001, Skeiker 2010). The method is based on the heat balance of the air-volume of a thermal zone (Wang and Chen 2000, Bellenger et al. 2001, Hernandez *et al.* 2003, Xu and Wang 2008, Li *et al.* 2009):

$$m_a c_a \frac{d\theta_a}{dt} = q_{ci} + q_v + \dot{Q}_g + \dot{Q}_{HVAC} \tag{1}$$

where:

θ_a is the air volume temperature considered homogeneous [°C or K];

$m_a c_a \dfrac{d\theta_a}{dt}$ is the heat accumulated in the air

volume of mass m_a and specific heat c_a;

$q_{ci} = \sum_i S_i h_i (\theta_{ci} - \theta_a)$ is the convective heat

flow from surface S_i having the surface temperature θ_{ci} and the inside heat convection coefficient h_i [W];

$q_v = \dot{m}_v c_a (T_o - \theta_a)$ is the heat flow due to mass

flow $\dot{m}_v$ of outdoor air at temperature T_o introduced into the space by infiltration and ventilation [W];

$\dot{Q}_g$ is the convective heat flow gained from the internal loads [W];

$\dot{Q}_{HVAC}$ is the heat flow from the HVAC system [W].

The load, $\dot{Q}_{HVAC}$, is determined from Equation (1). If the heat capacity of the room air is not neglected, then

$$\dot{Q}_{HVAC} = m_a c_a \frac{d\theta_a}{dt} - q_{ci} - q_v - \dot{Q}_g \tag{2}$$

If the heat capacity of the air is neglected, then

$$\dot{Q}_{HVAC} = -q_{ci} - q_v - \dot{Q}_g \tag{3}$$

The room air temperature, $\theta_a(t)$, is considered known in Equations (2) and (3) because it is the desired temperature of the indoor air. Finding indoor air temperature θ_a from Equation (1) is a simulation (or direct) problem. It models a physical system and since all physical systems are causal (Romagnoli and Palazoglu 2012), the model is causal (i.e. its output depends only on past inputs). Finding the load $\dot{Q}_{HVAC}$ from Equation (2) or (3) is a control (or inverse) problem. If the model for heat transfer is linear (or linearized), than equations (2) or (3) result in an improper transfer function which is an acausal model (i.e. its output depends also on future inputs).

The practical consequence is that in the case of non-negligible thermal capacity of the indoor air, when the indoor temperature varies, the value of the calculated thermal load will depend on the value of the time step used in simulation. The physical
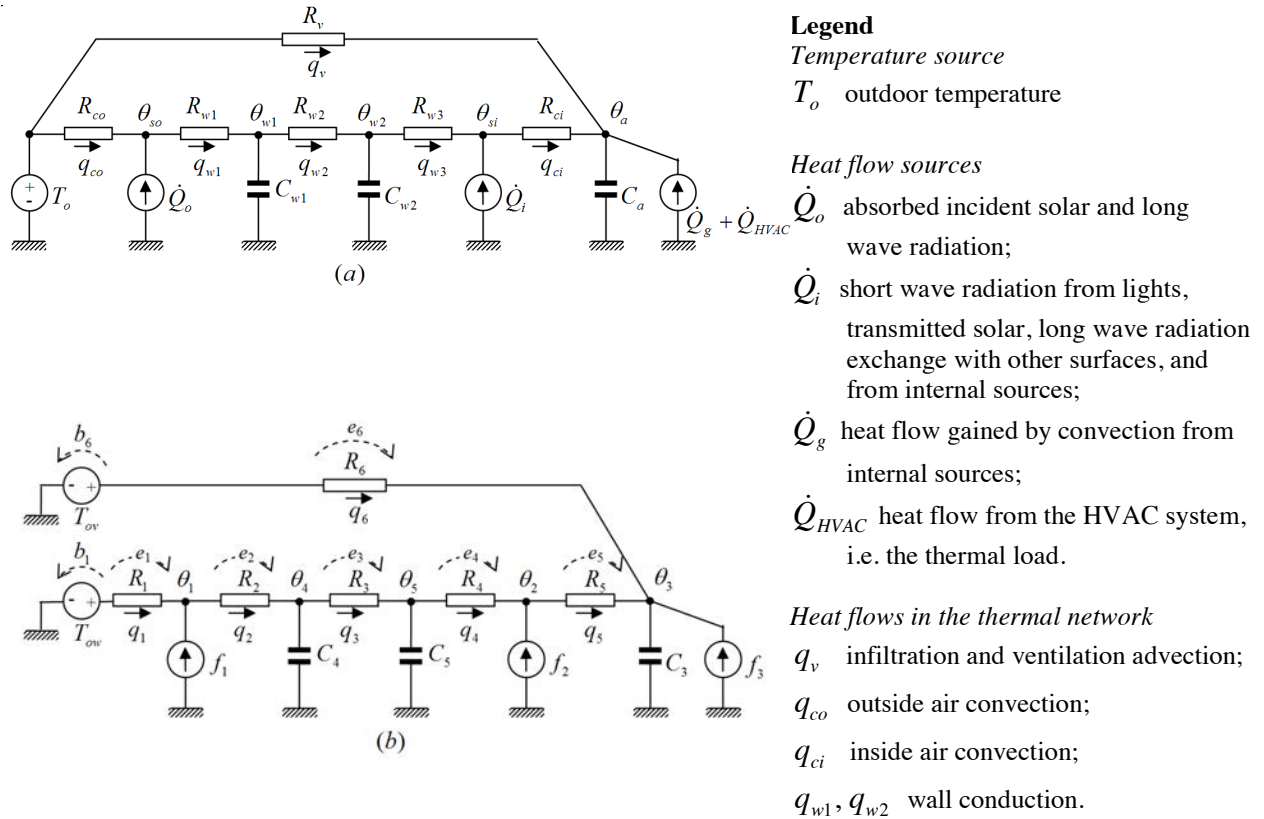
Legend

Temperature source

T_o outdoor temperature

Heat flow sources

$\dot{Q}_o$ absorbed incident solar and long wave radiation;

$\dot{Q}_i$ short wave radiation from lights, transmitted solar, long wave radiation exchange with other surfaces, and from internal sources;

$\dot{Q}_g$ heat flow gained by convection from internal sources;

$\dot{Q}_{HVAC}$ heat flow from the HVAC system, i.e. the thermal load.

Heat flows in the thermal network

q_v infiltration and ventilation advection;

q_{co} outside air convection;

q_{ci} inside air convection;

q_{w1}, q_{w2} wall conduction.

Figure 1 Epitome thermal network of heat balance method:

a) usual representation; b) representation in the form of typical branches

explanation is that the power needed to change the indoor temperature in exactly a time-step depends on the size of the time-step. Therefore, the size the HVAC system, which is chosen in function of the calculated load, will depend on the time-step used in simulation. Practitioners noticed this problem and introduced load limitations in the estimation of the load (e.g. CEN 2007b).

An epitome example will show how the transfer function can be obtained from the linear thermal model of a building, how this model is causal for the calculation of the indoor temperature and acausal for the thermal load, and, finally, the consequences on the results of load estimation.

DIFFERENTIAL-ALGEBRAIC EQUATIONS

We will consider the model shown in Figure 1 with the values of the parameters given in Table 1. This network has the main characteristics of a thermal model. More elements may be added but the procedure and the results presented hereafter remain the same. The model of thermal zone is described by a set of differential algebraic equations that can be obtained from the way in which the components are

Table 1 Parameter of the thermal network shown in Figure 1

Parameter	Value
Indoor air capacity, C_a	$82 \cdot 10^3$ J/K
Walls capacity, $C_{w1} = C_{w2}$	$2 \cdot 10^6$ J/K
Thermal conductance of the walls, R_w^{-1}	1.45 W/K
Thermal conductance of one third of the wall, $R_{w1}^{-1} = R_{w2}^{-1} = R_{w3}^{-1}$	4.35 W/K
Thermal conductance of the windows and due to losses by ventilation, R_v^{-1}	38.3 W/K
Outdoor convection conductance, R_{co}^{-1}	250.0 W/K
Indoor convection conductance, R_{ci}^{-1}	125.0 W/K

connected in the circuit and the values of the circuit elements (Strang 1986, Ghiaus 2013). The thermal network from Figure 1 is described by the matrices and the vectors given in Figure 2. The temperature drops over the resistances are:

$$\mathbf{e} = -\mathbf{A}\boldsymbol{\theta} + \mathbf{b} \qquad (4)$$

The heat flow rates are related to these temperature drops by the thermal conductivities:

$$\mathbf{q} = \mathbf{G}\mathbf{e} \qquad (5)$$

$$
\begin{array}{ccccc}
\theta_{so} & \theta_{si} & \theta_a & \theta_{w1} & \theta_{w2}
\end{array}
$$

$$
\begin{array}{c}
q_{co} \\ q_{w1} \\ q_{w2} \\ \mathbf{A} = \\ q_{w3} \\ q_{ci} \\ q_v
\end{array}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & 1 \\
0 & 1 & 0 & 0 & -1 \\
0 & -1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\quad
\mathbf{G} =
\begin{bmatrix}
R_{co}^{-1} & 0 & 0 & 0 & 0 & 0 \\
0 & R_{w1}^{-1} & 0 & 0 & 0 & 0 \\
0 & 0 & R_{w2}^{-1} & 0 & 0 & 0 \\
0 & 0 & 0 & R_{w3}^{-1} & 0 & 0 \\
0 & 0 & 0 & 0 & R_{ci}^{-1} & 0 \\
0 & 0 & 0 & 0 & 0 & R_v^{-1}
\end{bmatrix}
\quad
\mathbf{b} =
\begin{bmatrix}
T_{ow} \\ 0 \\ 0 \\ 0 \\ 0 \\ T_{ov}
\end{bmatrix}
$$

$$
\mathbf{C} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_a & 0 & 0 \\
0 & 0 & 0 & C_{w1} & 0 \\
0 & 0 & 0 & 0 & C_{w2}
\end{bmatrix}
\qquad
\begin{cases}
\mathbf{e} = -\mathbf{A}\boldsymbol{\theta} + \mathbf{b} \\
\mathbf{q} = \mathbf{G}\mathbf{e} \\
\mathbf{C}\dot{\boldsymbol{\theta}} = \mathbf{A}^T\mathbf{q} + \mathbf{f}
\end{cases}
$$

$$
\mathbf{f} = \begin{bmatrix} \dot{Q}_o & \dot{Q}_i & \dot{Q}_g + \dot{Q}_{HVAC} & 0 & 0 \end{bmatrix}^T
$$

$$
\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f}
$$

Figure 2 Differential-algebraic equations model of the circuit from Figure 1

The heat balance for the indoor temperature, expressed by Equation (1), is the 3rd row in this set of equations. The heat balance for each node gives:

$$
\mathbf{C}\dot{\boldsymbol{\theta}} = \mathbf{A}^T\mathbf{q} + \mathbf{f} \tag{6}
$$

By eliminating $\mathbf{e}$ and $\mathbf{q}$ from the set of equations (4)-(6) we obtain a system of differential – algebraic equations:

$$
\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f} \tag{7}
$$

that can be written as:

$$
\mathbf{C}\dot{\boldsymbol{\theta}} = \mathbf{K}\boldsymbol{\theta} + \mathbf{K}_b\mathbf{b} + \mathbf{f} \tag{8}
$$

where $\mathbf{K} = -\mathbf{A}^T\mathbf{G}\mathbf{A}$ and $\mathbf{K}_b = \mathbf{A}^T\mathbf{G}$.

STATE-SPACE REPRESENTATION

A more suitable form for system analysis is the state-space representation. In order to obtain it, the temperatures corresponding to nodes without thermal capacity need to be eliminated from the set of Equations (8). Partitioning the set of Equations (8),

$$
\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_C \end{bmatrix}
\begin{bmatrix} \dot{\boldsymbol{\theta}}_0 \\ \dot{\boldsymbol{\theta}}_C \end{bmatrix}
=
\begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix}
\begin{bmatrix} \boldsymbol{\theta}_0 \\ \boldsymbol{\theta}_C \end{bmatrix}
+
\begin{bmatrix} \mathbf{K}_{b1} \\ \mathbf{K}_{b2} \end{bmatrix}\mathbf{b} \tag{9}
$$
$$
+
\begin{bmatrix} \mathbf{I}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{22} \end{bmatrix}
\begin{bmatrix} \mathbf{f}_0 \\ \mathbf{f}_C \end{bmatrix}
$$

where

$\boldsymbol{\theta}_0$ and $\mathbf{f}_0$ correspond to the nodes without thermal capacity;

$\boldsymbol{\theta}_C$ and $\mathbf{f}_C$ correspond to the nodes with thermal capacity;

$\mathbf{C}_C$ is the bloc of the partitioned matrix $\mathbf{C}$ for which the diagonal elements are non-zero;

$\mathbf{K}_{11}$, $\mathbf{K}_{12}$, $\mathbf{K}_{21}$, $\mathbf{K}_{22}$ are blocs of the partitioned matrix $\mathbf{K}$ obtained according to the partitioning of the matrix $\mathbf{C}$;

$\mathbf{K}_{b1}$, $\mathbf{K}_{b2}$ are blocs of the partitioned matrix $\mathbf{K}_b$ obtained according to the partitioning of the matrix $\mathbf{C}$;

$\mathbf{I}_{11}$, $\mathbf{I}_{22}$ are identity matrices,

we obtain the state-space representation:

$$
\dot{\boldsymbol{\theta}}_C = \mathbf{A}_S\boldsymbol{\theta}_C + \mathbf{B}_S\mathbf{u} \tag{10}
$$
$$
\theta_a = \mathbf{C}_S\boldsymbol{\theta}_C + \mathbf{D}_S\mathbf{u}
$$

where the state matrix is

$$
\mathbf{A}_S = \mathbf{C}_C^{-1}(-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12} + \mathbf{K}_{22}) \tag{11}
$$

the input matrix is

$$
\mathbf{B}_S = \mathbf{C}_C^{-1}\begin{bmatrix} -\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{b1} + \mathbf{K}_{b2} & -\mathbf{K}_{21}\mathbf{K}_{11}^{-1} & \mathbf{I}_{22} \end{bmatrix} \tag{12}
$$

and the input vector is:

$$\mathbf{u} = \begin{bmatrix} \mathbf{b} & \mathbf{f}_0 & \mathbf{f}_C \end{bmatrix}^T \qquad (13)$$

The vector of temperatures $\boldsymbol{\theta}_0$ can be obtained from the first row of the equation (9):

$$\boldsymbol{\theta}_0 = \mathbf{C}_S \boldsymbol{\theta}_C + \mathbf{D}_S \mathbf{u} \qquad (14)$$

where

$$\mathbf{C}_S = -\mathbf{K}_{11}^{-1}\mathbf{K}_{12} \qquad (15)$$

and

$$\mathbf{D}_S = \begin{bmatrix} \mathbf{K}_{b1} & \mathbf{I}_{11} & \mathbf{0} \end{bmatrix} \qquad (16)$$

TRANSFER FUNCTION

The relation between the inputs, $\mathbf{u}$, and the output, θ_a, can be expressed as a transfer function. Applying Laplace transform for zero initial conditions, the set of equations (10) become:

$$\theta_a = [\mathbf{C}_S(s\mathbf{I} - \mathbf{A}_S)^{-1}\mathbf{B}_S + \mathbf{D}_S]\mathbf{u} \qquad (17)$$

where

$$\mathbf{H} = \mathbf{C}_S(s\mathbf{I} - \mathbf{A}_S)^{-1}\mathbf{B}_S + \mathbf{D}_S \qquad (18)$$

is the transfer function the indoor air temperature θ_a.

For the vector of inputs:

$$\mathbf{u} = \begin{bmatrix} T_{ow} & T_{ov} & \dot{Q}_o & \dot{Q}_i & \dot{Q}_g + \dot{Q}_{HVAC} \end{bmatrix}^T \qquad (19)$$

and for the values given in Table 1, the transfer function

$$\mathbf{H} = \begin{bmatrix} H_1 & H_2 & H_3 & H_4 & H_5 \end{bmatrix} \qquad (20)$$

for non-negligible indoor air heat capacity is

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix} = \begin{bmatrix} \dfrac{3.59\cdot10^{-2}}{1.50\cdot10^{14}s^3 + 7.93\cdot10^{10}s^2 + 6.56\cdot10^5 s + 1} \\[2ex] \dfrac{9.64\cdot10^{-1}\cdot(7.29\cdot10^{10}s^2 + 6.26\cdot10^5 s + 1)}{1.50\cdot10^{14}s^3 + 7.93\cdot10^{10}s^2 + 6.56\cdot10^5 s + 1} \\[2ex] \dfrac{1.43\cdot10^{-4}\cdot}{1.50\cdot10^{14}s^3 + 7.93\cdot10^{10}s^2 + 6.56\cdot10^5 s + 1} \\[2ex] \dfrac{2.49\cdot10^{-2}\cdot(7.13\cdot10^{10}s^2 + 6.17\cdot10^5 s + 1)}{1.50\cdot10^{14}s^3 + 7.93\cdot10^{10}s^2 + 6.56\cdot10^5 s + 1} \\[2ex] \dfrac{2.52\cdot10^{-2}\cdot(7.29\cdot10^{10}s^2 + 6.26\cdot10^5 s + 1)}{1.50\cdot10^{14}s^3 + 7.93\cdot10^{10}s^2 + 6.56\cdot10^5 s + 1} \end{bmatrix}$$

and for negligible indoor air heat capacity is:

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix} = \begin{bmatrix} 3.59\cdot10^{-2}\dfrac{1}{7.80\cdot10^{10}s^2 + 6.53\cdot10^5 s + 1} \\[2ex] 9.64\cdot10^{-1}\dfrac{7.29\cdot10^{10}s^2 + 6.26\cdot10^5 s + 1}{7.80\cdot10^{10}s^2 + 6.53\cdot10^5 s + 1} \\[2ex] 1.43\cdot10^{-4}\dfrac{1}{7.80\cdot10^{10}s^2 + 6.53\cdot10^5 s + 1} \\[2ex] 2.49\cdot10^{-2}\dfrac{7.12\cdot10^{10}s^2 + 6.17\cdot10^5 + 1}{7.80\cdot10^{10}s^2 + 6.53\cdot10^5 s + 1} \\[2ex] 2.51\cdot10^{-2}\dfrac{7.29\cdot10^{10}s^2 + 6.26\cdot10^5 s + 1}{7.80\cdot10^{10}s^2 + 6.53\cdot10^5 s + 1} \end{bmatrix}^T$$

By using the transfer functions, the output is

$$\theta_a = H_1 T_{ow} + H_2 T_{ov} + H_3 \dot{Q}_o + H_4 \dot{Q}_i \qquad (21)$$
$$+ H_5(\dot{Q}_g + \dot{Q}_{HVAC})$$

Equation (21) is a simulation problem: transfer functions are proper or strictly proper.

TRANSFER FUNCTION OF THE THERMAL LOAD

In heat balance method, the thermal load results from Equation (21), according to Equations (2) and (3):

$$\dot{Q}_{HVAC} = H_5^{-1}\theta_a - H_5^{-1}(H_1 + H_2)T_o \qquad (22)$$
$$- H_5^{-1}H_3\dot{Q}_o - H_5^{-1}H_4\dot{Q}_i - \dot{Q}_g$$

If the thermal capacity of the indoor air is non-negligible, the transfer function H_5^{-1}, corresponding to input θ_a, is improper. The transfer functions $H_5^{-1}H_2$, relating $\dot{Q}_{HVAC}$ to the outdoor temperature T_{ov} through ventilation, and $H_5^{-1}H_4$ relating $\dot{Q}_{HVAC}$ to the heat flow rate on the indoor walls $\dot{Q}_i$, are proper. They correspond to inputs connected directly to the node θ_a, i.e. not through a node containing a thermal capacity. The functions $H_5^{-1}H_1$ corresponding to the influence of the outdoor temperature, T_{ow}, through walls and $H_5^{-1}H_3$ corresponding to influence of the heat flux on the outdoor wall, $\dot{Q}_o$, are strictly proper. They correspond to inputs connected to the node θ_a through nodes containing at least a thermal capacity.
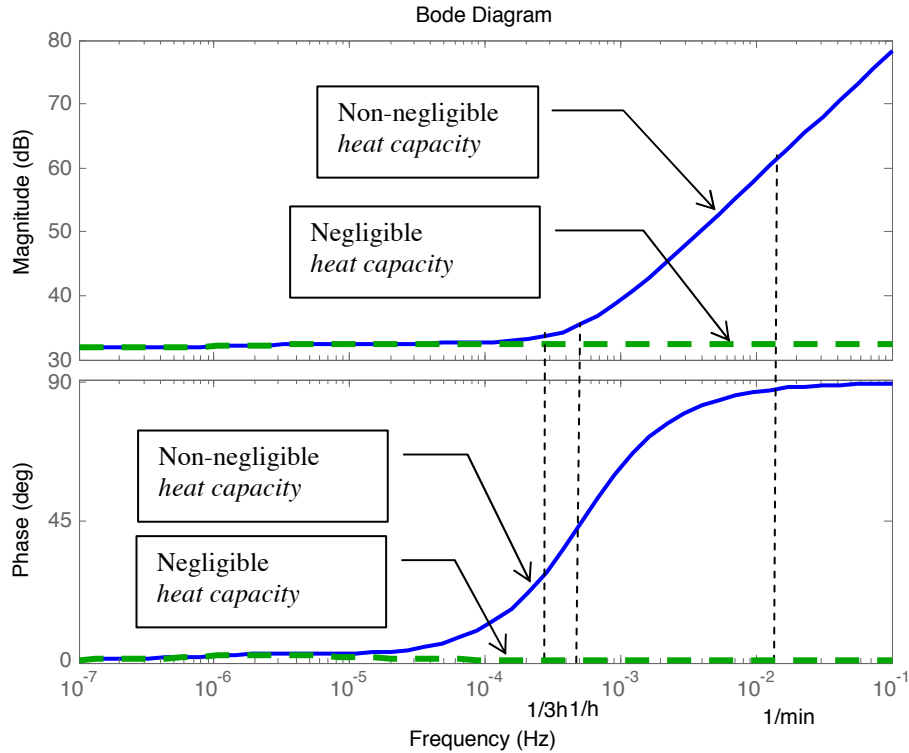
Figure 3 Bode diagrams of the transfer function $H_5^{-1} = \dfrac{\dot{Q}_{HVAC}}{\theta_a}$

The method for load calculation based on Equation (2) results in an improper function for input θ_a if the heat capacity of the air is non-negligible. This is true not only for the epitome example but for any model constructed on this method. For any thermal model constructed according to the heat balance method for load estimation:

- the transfer function relating the load $\dot{Q}_{HVAC}$ to the temperature in node θ_a will be always improper (the degree of the numerator is larger than the degree of the denominator),
- the transfer functions relating the inputs to the output $\dot{Q}_{HVAC}$ through branches which do not contain at least a thermal capacity will be strictly proper and
- the transfer functions connecting the inputs to the output $\dot{Q}_{HVAC}$ by branches containing at least a thermal capacity will be proper.

CONSEQUENCES OF ACAUSALITY

The Bode diagrams of the acausal transfer function $H_5^{-1} = \dot{Q}_{HVAC} / \theta_a$ show the problem induced by a simulation time step lower than the cut-off frequency. The magnitude of the transfer function increases with the frequency. This means that the calculated thermal load tends to infinity if the

simulation time-step tends to zero. In our example, for simulation time step larger than 3h, the high frequencies will be cut and the amplification will be around 32 dB (i.e. for $u = 1$, it results $y = 10^{32/20} = 39.8$). For simulation time step smaller than 3h, the output will depend on the time step. For example, the amplification will be around 61 dB (i.e. for $u = 1$, it results $y = 10^{61/20} = 1122$) for the time step around 1 min,

In order to see the consequences of acausality in time-domain, let us simulate the response of the transfer function $H_5^{-1} = \dot{Q}_{HVAC} / \theta_a$ to a step input. The integration can be done by using any method. For the sake of simplicity, let us consider the Euler method. In this case

$$s \leftrightarrow (1 - q^{-1}) / \Delta t \qquad (23)$$

where $s = \sigma + j\omega$ is the complex variable and q^{-1} is the one-step delay operator. The transfer function H_5^{-1} is

$$\frac{\dot{Q}_{HVAC}}{\theta_a} = \frac{-22.1q^{-3} + 108.2q^{-2} + -152.3q^{-1} + 65.2}{0.969q^{-2} - 1.969q^{-1} + 1}$$
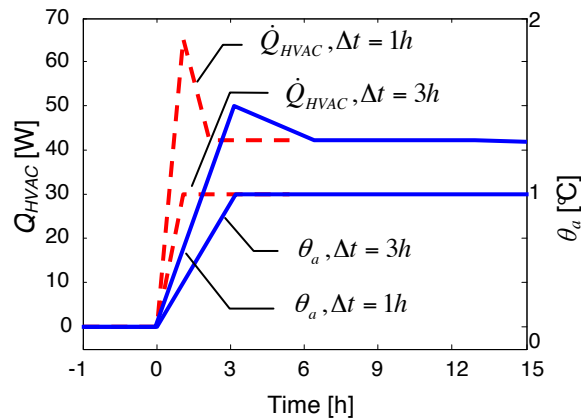
for the simulation time-step $\Delta t = 3600s$ and

Figure 4 Influence of time step on the estimation of thermal load in the case of non-negligible indoor air thermal capacity

$$\frac{\dot{Q}_{HVAC}}{\theta_a} = \frac{-6.94q^{-3} + 60.3q^{-2} + -103.3q^{-1} + 50.0}{0.914q^{-2} - 1.912q^{-1} + 1}$$

for the simulation time-step $\Delta t = 3 \times 3600s$.

Using these transfer functions, the load $\dot{Q}_{HVAC}$ can be estimated for a step variation of the indoor temperature, θ_a. The results are given in Figure 4. We can see that the value of the maximum load depends on the size of the simulation time-step. After one time-step the load gets its maximum value and in two time-steps it gets its final value.

CONCLUSIONS

Using the heat balance method for estimating the variation of the indoor temperature as a function of environmental variables and internal gains is a simulation problem. It follows the physical causality and results in a causal model, i.e. the output depends only on present and past values of the inputs. The thermal load is an input in this causal model. Finding the load corresponds to inverting the model. In the case of non-negligible thermal capacity of the indoor air, the model relating the load to the indoor temperature becomes acausal (the output, i.e. the load, depends on future values of the input, i.e. the indoor temperature).

The practical consequence of the acausal model of thermal load calculation appears when the thermal capacity of the indoor air is non-negligible and when the value of the indoor air temperature varies. The formulation of the problem requires that the load needs to change the indoor temperature in a few time steps of simulation (depending on the numerical method used, e.g. precisely one time step for Euler method). Consequently, the maximum load calculated with this method will depend on the size of the time step use in simulation.

The solution to this problem is to transform the acausal model into a causal model. This can be done by introducing a filter that achieves the regularization. In fact, the problem of finding the thermal load is a control problem and can be treated with the methods used in control systems theory.

ACKNOWLEDGEMENT

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