PREPROCESSOR FOR THE GENERATION OF G-FUNCTIONS USED IN THE SIMULATION OF GEOTHERMAL SYSTEMS

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ABSTRACT

The objective of this paper is to present a general methodology for the calculation of thermal response factors, known as g-functions, of vertical borehole fields. The methodology accounts for the time variation of the heat extraction rates of individual boreholes. In addition, the original concept of the g-functions is extended to include variable borehole lengths and buried depths. Finally, the methodology is implemented into MATLAB with a convenient graphical user interface which pre-processes the hourly values of the thermal response factors for use in energy simulation programs.

INTRODUCTION

Ground source heat pump (GSHP) systems offer a significant potential for reducing the energy consumption associated with heating and cooling in buildings. In one of the most popular system, the heat carrier fluid from the heat pump circulates into an array of U-tube loops inserted into vertical boreholes. Heat is first transferred from the ground to the fluid and then to the building by the heat pump.

The simulation of GSHP systems relies on the prediction of the heat transfer in the bore field. The correct number of boreholes, their length and the spacing between boreholes need to be identified to ensure proper operation of the system. While simulation of GSHP systems is available in various building simulation software, the users are often limited to a finite amount of bore field layouts, usually in regular grid patterns. This limits the possibilities when deciding the positions of the boreholes in the bore field.

Long-term g-function

Simulation programs typically use thermal response factors, or g-functions (Eskilson, 1987), to model the transient heat transfer between the boreholes and the ground. g-functions give the time variation of the borehole wall temperature due to a constant total heat extraction rate in the bore field. The g-function is defined by the relation :

$$T_b = T_g - \frac{\bar{Q}}{2\pi k_s} \cdot g(t/t_s, r_b/H, B/H) \tag{1}$$

where T_b is the borehole wall temperature, T_g is the undisturbed ground temperature, $\bar{Q}$ is the total heat extraction rate per borehole length, k_s is the ground thermal conductivity, g is the g-function, t/t_s is the non-dimensional time, with $t_s = H^2/9\alpha_s$ the characteristic time of the bore field, α_s is the ground thermal diffusivity, r_b/H is the borehole radius to length ratio and B/H is the borehole spacing to length ratio.

The original g-functions were obtained by Eskilson (1987). They result from the simulation of the ground heat transfer around the boreholes using an explicit finite difference method. Each borehole is modelled in a separate 2D radial-axial mesh. The temperature distributions around the boreholes are superposed spatially to obtain the total temperature variation at the borehole walls. The heat extraction rate of each cell along the lengths of the boreholes is calculated at each time step to obtain a uniform temperature at the borehole walls, and equal for all boreholes.

Each g-function is unique to one bore field layout and one combination of the non-dimensional parameters r_b/H, B/H. A third parameter D/H, the buried depth to length ratio, was not included in Eskilson's original g-functions. The g-functions are documented in a series of non-dimensional graphs such as the one shown on Figure 1 (for a 3×2 bore field). g-functions are included within databases in simulation programs such as EnergyPlus (Fisher, 2006).
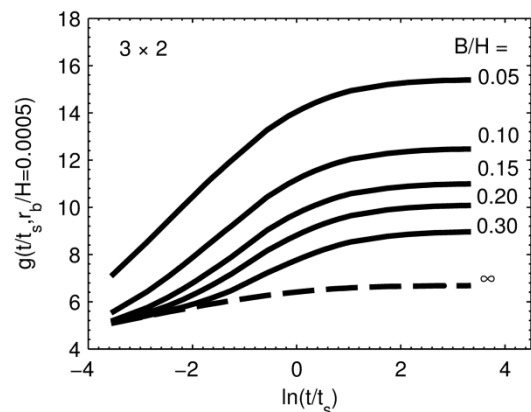

Figure 1 g-function of a 3 × 2 bore field

Analytical solutions may be preferred over numerically generated g-functions since they are often more easily obtained than numerical solutions. The infinite line source (Ingersoll and Plass, 1948) and the cylindrical heat source (Carslaw and Jaeger, 1959; Bernier, 2000) were originally used to model heat transfer around boreholes. However, such solutions do not account for axial heat transfer and overestimate the temperature response for large values of time ($t \gtrsim 3$ years).

Eskilson (1987) proposed the use of the finite line source (FLS) solution to approximate the g-function. Eskilson obtained the solution by integrating the point heat source solution over the length of the borehole and of a mirror source of opposite sign. The solution was evaluated at the borehole mid-length and at a radius $r = \sqrt{1.5}r_b$.

Zeng (2002) used the FLS solution to calculate the integral mean temperature over the length of the borehole. The solution is presented as a double integral. The use of the integral mean temperature results in a lower temperature response when compared to the FLS evaluated at mid-length. The solution is however more difficult to obtain since it requires the evaluation of a double integral.

Lamarche and Beauchamp (2007a) simplified the double integral formulation of the FLS solution and obtained a solution involving a single integral for the specific case $D = 0$. The thermal response of bore fields of 2 and 4 boreholes were presented and compared to Eskilson's g-functions for a borehole spacing-to-length ratio $B/H = 0.1$. Claesson and Javed (2011) later obtained a solution for the case $D \geq 0$.

Fossa (2011) compared the thermal response factors obtained using the FLS solution to Eskilson's g-functions for fields of 3×3 and 8×2 boreholes. It was shown that the FLS tends to overestimate the g-functions for small values of B/H and large values of time.

Malayappan and Spitler (2013) studied the impact of the overestimation of the g-functions on the sizing of bore fields. The results indicate that the overestimation of the g-functions leads to an oversizing of the bore fields. The oversizing increases with the number of boreholes and the sizing period, and decreases when the length of the boreholes and the spacing between boreholes increase.

Cimmino et al. (2013) used the FLS solution to obtain thermal response factors of bore fields while accounting for the time variation of the heat extraction rates of individual boreholes. Results were presented for fields of 3×2, 6×4 and 10×10 boreholes. The new methodology was shown to reduce the differences between the FLS solution and Eskilson's g-functions. The remaining differences are attributed to the boundary condition at the borehole wall, which differs from the condition used by Eskilson.

Temporal superposition

Simulation of GSHP systems consists in calculating the time variation of the borehole wall and fluid temperatures in the bore field due to a varying total heat extraction rate. The varying borehole wall temperature is obtained from the temporal superposition of the thermal response factor :

$$T_b(t_k) - T_g = \sum_{p=1}^{k} \frac{-\bar{q}(t_p)}{2\pi k_s} \cdot g(t_k - t_p) \tag{2}$$

where $\bar{q}(t_p) = \bar{Q}(t_p) - \bar{Q}(t_{p-1})$ is the total heat extraction rate increment per borehole length.

As the number of time steps increases, the temporal superposition of the thermal response factor is increasingly difficult to calculate (i.e. the number of terms in the sum increases) and the simulations become increasingly time consuming. The calculation time can be decreased using load aggregation (Yavuzturk and Spitler, 1999; Bernier, 2004; Liu, 2005). Load aggregation consists in averaging parts of the heat extraction rate history to reduce the number of terms to be superposed. As a result, the calculation time can be significantly reduced at the cost of small errors in the predicted borehole wall temperatures.

Lamarche and Beauchamp (2007b) and Lamarche (2009) introduced a new algorithm for the temporal superposition of the thermal response factor. The new algorithm is able to further reduce the calculation time while increasing the precision of the solution when compared to load aggregation.

Marcotte and Pasquier (2008) used fast Fourier transforms (FFT) to calculate the temporal superposition of the thermal response factors. This method does not cause any error in the calculation of the temporal superposition since it does not modify the heat extraction rates or the thermal response factor.

The objective of this paper is to present a simplified methodology for the approximation of g-functions based on the FLS solution as explained by Cimmino et al. (2013). The FLS solution is expanded to cover boreholes of unequal lengths. The new methodology is simpler and more efficient than the methodology presented by Cimmino et al. (2013) while giving equivalent results.

The methodology is implemented into MATLAB with a convenient graphical user interface (GUI). The GUI generates the hourly g-function for the bore field specified by the user. The boreholes can be placed into any position and can also have different individual lengths.

METHODOLOGY

The calculation of the g-function is separated into three steps. First, the FLS solution gives the temperature distribution around individual boreholes and thus the temperature variation at the borehole walls due to heat extraction from any borehole. Then, spatial superposition is used to calculate the total temperature variation at the borehole walls by superposing the contribution of all boreholes. Finally, temporal superposition is used to account for the time variation of the heat extraction rates of individual boreholes. A system of equations is built at each time step to calculate the heat extraction rates of the boreholes and the borehole wall temperature common to all boreholes.

The ground thermal properties are assumed to be uniform, isotropic and constant. The ground is initially at a temperature T_g and the ground surface is maintained at the initial ground temperature. Each borehole i has a length H_i, is buried at a depth D_i from the ground surface and is positioned at coordinates (x_i, y_i). All boreholes have the same radius r_b and the same borehole wall temperature T_b. As an example, a field of 3 arbitrarily sized and positioned boreholes is shown on Figure 2.
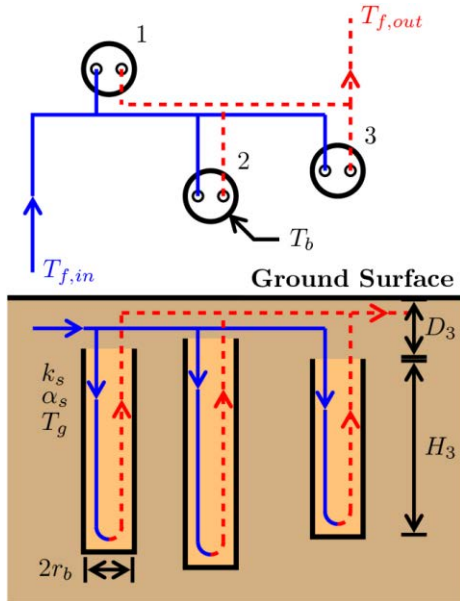

Figure 2 Field of 3 arbitrarily sized and positioned boreholes

Finite line source

The temperature variation $\Delta T_{i \to j}$ at the wall of the jth borehole due to the extraction of heat by the ith borehole, at a rate per unit length Q_i uniform along its length, is obtained from the FLS solution. The FLS solution is obtained from the integration of the point heat source solution over the length of the extracting borehole, superposed with a mirror line source of opposing heat extraction rate above the ground surface as shown on Figure 3.
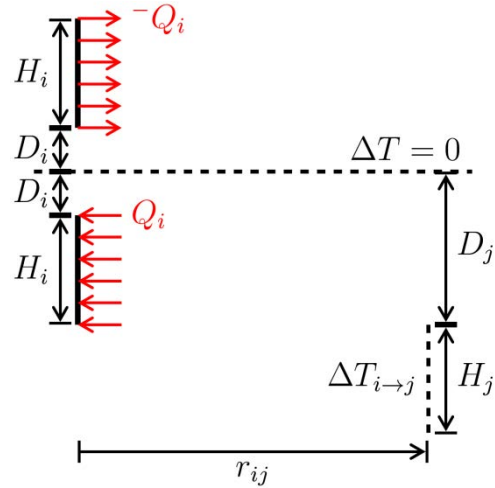

Figure 3 Illustration of the finite line source solution

The temperature variation $\Delta T_{i \to j}$ is obtained by averaging the FLS solution over the length of the jth borehole.

$$\Delta T_{i \to j}(t) = \frac{-Q_i}{2\pi k_s} \cdot h_{i \to j}(t) \tag{3}$$

$$h_{i \to j}(t) = \frac{1}{2} \int_{1/\sqrt{4\alpha_s t}}^{\infty} exp(-r_{ij}^2 s^2) \tag{4}$$
$$\cdot \frac{1}{H_j} \int_{D_j}^{D_j + H_j} \int_{D_i}^{D_i + H_i} \left[exp\left(-(z_j - z_i)^2 s^2 \right) \right.$$
$$\left. - exp\left(-(z_j - z_i)^2 s^2 \right) \right] dz_i \, dz_j \, ds$$

where $h_{i \to j}(t)$ is the borehole-to-borehole response factor, k_s is the ground thermal conductivity, α_s is the ground thermal diffusivity, $r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ is the radial distance between borehole i and borehole j and $r_{ii} = r_b$.

The triple integral in Equation 4 was simplified to a single integral by Claesson and Javed (2011) for the case $D_i = D_j$, $H_i = H_j$. The same methodology is used here to simplify Equation 4 and obtain a new solution for the general case $D_i \neq D_j$, $H_i \neq H_j$.

$$h_{i \to j}(t) \tag{5}$$
$$= \frac{1}{2} \int_{1/\sqrt{4\alpha_s t}}^{\infty} exp(-r_{ij}^2 s^2) \frac{Y(H_i s, D_i s, H_j s, D_j s)}{H_j s^2} ds$$

$$Y(h_1, d_1, h_2, d_2) \tag{6}$$
$$= ierf(d_2 - d_1 + h_2) - ierf(d_2 - d_1)$$
$$+ ierf(d_2 - d_1 - h_1)$$
$$- ierf(d_2 - d_1 + h_2 - h_1)$$
$$+ ierf(d_2 + d_1 + h_2) - ierf(d_2 + d_1)$$
$$+ ierf(d_2 + d_1 + h_1)$$
$$- ierf(d_2 + d_1 + h_2 + h_1)$$

$$ierf(X) = X \cdot erf(X) - \frac{1}{\sqrt{\pi}}(1 - exp(-X^2)) \tag{7}$$

where *erf* is the error function.

The temperature variation and the heat extraction rate per unit length are expressed as non-dimensional parameters:

$$\Delta\Theta_{i \to j}(t) = \frac{\Delta T_{i \to j}}{-\bar{Q}/2\pi k_s} = \tilde{Q}_i \cdot h_{i \to j}(t) \tag{8}$$

where $\Delta\Theta_{i \to j}$ is the non-dimensional temperature variation at the wall of the jth borehole due to the extraction of heat by the ith borehole, $\tilde{Q}_i = \frac{Q_i}{\bar{Q}}$ is the normalized heat extraction rate per unit length of the ith borehole.

Spatial superposition

The total non-dimensional temperature variation at the borehole wall Θ_b, which corresponds to the g-function of the bore field, is obtained from the spatial superposition of the non-dimensional temperature variations due to heat extraction from all boreholes:

$$\Theta_b(t) = \sum_{i=1}^{n_b} \tilde{Q}_i \cdot h_{i \to j}(t) \tag{9}$$

Since the borehole wall temperature is equal for all boreholes, Equation 9 can be evaluated for any borehole j.

Temporal superposition

The heat extraction rate of individual boreholes varies in time. For a succession of normalized heat extraction rates $\tilde{Q}_i(t_p)$ starting at time t_{p-1} up to time t_p, the borehole wall temperature at time t_k is obtained from the temporal superposition of the normalized heat extraction rate increments:

$$\Theta_b(t_k) = \sum_{i=1}^{n_b} \sum_{p=1}^{k} \tilde{q}_i(t_p) \cdot h_{i \to j}(t_k - t_p) \tag{10}$$

where $\tilde{q}_i(t_p) = \tilde{Q}_i(t_p) - \tilde{Q}_i(t_{p-1})$ is the normalized heat extraction rate increment of the ith borehole.

The non-dimensional temperature at the borehole wall is evaluated one time step at a time. As such, only the normalized heat extraction rates at time t_k are unknown and are therefore separated from the rest of the sum in Equation 10:

$$\Theta_b(t_k) = \sum_{i=1}^{n_b} \tilde{Q}_i(t_k) \cdot h_{i \to j}(t_k - t_{k-1}) \\ + \Theta_{b,j}^*(t_k) \tag{11}$$

$$\Theta_{b,j}^*(t_k) \tag{12} \\ = \sum_{i=1}^{n_b} \sum_{p=1}^{k-1} \tilde{q}_i(t_p) \cdot h_{i \to j}(t_k - t_p) \\ - \tilde{Q}_i(t_{k-1}) \cdot h_{i \to j}(t_k - t_{k-1})$$

Equation 11 can be evaluated for any borehole j and therefore forms a set of n_b equations with n_b+1 unknowns, i.e. all Θ_b and $\tilde{Q}_i$. One last equation is required to complete the set, which sets the total heat extraction rate in the field as constant.

$$\bar{Q} \cdot \bar{H} \cdot n_b = \sum_{i=1}^{n_b} Q_i(t_k) \cdot H_i \tag{13}$$

where $\bar{H}$ is the average length of the boreholes in the field.

In non-dimensional form:

$$n_b = \sum_{i=1}^{n_b} \tilde{Q}_i(t_k) \cdot \frac{H_i}{\bar{H}} \tag{14}$$

Solution of the system of equations

The system of equations is solved for all times t_k, starting at time t_1. As shown by Marcotte and Pasquier (2008), the temperature response to a constant heat extraction is a smooth and monotonically increasing function. It can therefore be calculated at a few selected times and later interpolated using a cubic spline.

A time step of 1 h is used for the first 48 time steps t_1-t_{48}, the time step is doubled for each subsequent time step after t_{48} (e.g. $t_{49} = 50$ h, $t_{50} = 54$ h, $t_{51} = 62$ h) up to a time of 1000 years for a total of 71 time steps.

The borehole-to-borehole response factors $h_{i \to j}$ are pre-calculated for all times t_k prior to the calculation of the g-function. The values of the borehole-to-borehole response factors for times in between times t_k are obtained through spline interpolation for use in Equation 11.

IMPLEMENTATION AS A MATLAB GUI

The methodology was implemented in a graphical user interface (GUI) using Matlab's GUI development environment. The GUI is compiled into an executable application and requires only the installation of Matlab Component Runtime 2012a (available for free[1]). The GUI is shown on Figure 4 at the end of the paper.

The ground thermal diffusivity and borehole radius are specified on the Parameters panel (A). The borehole positions, length and buried depth are specified in the Borehole positions panel (C). The field can be visualized on the bottom right hand side graph (E). The g-function is generated and visualized on the top right hand side graph (D). The g-function is exported into a text file at a time step of one hour up to a time set in the File export panel (B).

The g-function is exported in a two-column text file. The first column lists the hourly values of the non-dimensional time $\ln(t/t_s)$ and the second column lists the hourly values of the calculated g-function. The g-function can later be used for the simulation of the bore field. Knowing the hourly ground heat extraction rate in the bore field and the g-function, the borehole wall temperature is obtained through temporal superposition (Equation 2).

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[1] http://www.mathworks.com/products/compiler/mcr/

RESULTS

The g-function of a field of 7×3 equally spaced boreholes generated with the preprocessor is compared to the g-function generated using the methodology of Cimmino et al. (2013) and with Eskilson's g-function. All boreholes have a length $H = 140$ m, a radius $r_b = 0.075$ m and are buried at a distance $D = 2$ m from the ground surface. The spacing between boreholes is $B = 7$ m. The ground thermal diffusivity is $\alpha_s = 1 \times 10^{-6}$ m^2/s.

As shown on Figure 5, the g-function obtained with the preprocessor is equivalent to the g-function obtained with the methodology of Cimmino et al. (2013). However, the preprocessor overestimates Eskilson's g-function for greater values of time. These differences were also observed by Cimmino et al. (2013) for fields with numerous boreholes and are attributed to the differences in the boundary condition at the borehole walls used by each model : Eskilson's model uses a condition of uniform temperature along the length of the boreholes while the methodology presented in this work uses a condition of uniform heat extraction rate along the length of the boreholes.

The difference between the g-function calculated by the preprocessor and the g-function obtained using Eskilson's model increases with the value of $\ln(t/t_s)$. The differences at times $t = 5, 10, 20, 30, 40$ and 50 years are presented in Table 1.
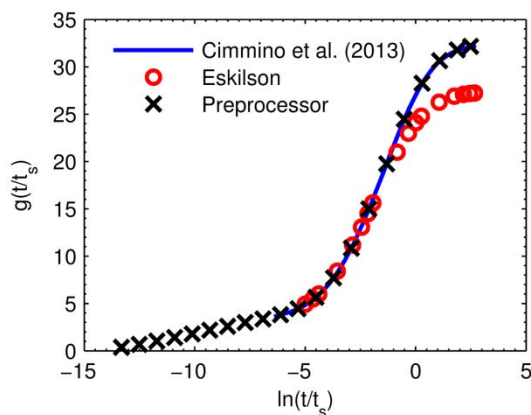
APPLICATION

The preprocessor allows for fields of boreholes of different lengths and uneven spacings between boreholes. The g-function of a field of 9 boreholes with lengths and positions presented in Table 2 was calculated using the preprocessor. The radius of the boreholes is $r_b = 0.05$ m, the buried depth of the boreholes is $D = 2$ m, the ground thermal diffusivity is $\alpha_s = 1 \times 10^{-6}$ m^2/s and the ground thermal conductivity is $k_s = 2$ W/m-K. The undisturbed ground temperature is $T_g = 10°$C. The positions of the boreholes are illustrated on Figure 6 and the resulting g-function is presented on Figure 7. The g-function is presented as a function of the non-dimensional time $\ln(t/t_s)$, with $t_s = \frac{\bar{H}^2}{9\alpha_s}$ the characteristic time of the bore field calculated based on the average borehole length $\bar{H} = 91.11$ m.

The g-function is linear up to a time $\ln(t/t_s) = -5$ (72 days), when thermal interaction becomes significant and the slope of the g-function increases. The g-function starts to stabilize towards its steady-state value near time $\ln(t/t_s) = 0$ (29 years), when axial effects (i.e. thermal interaction with the ground surface and the ground below the bore field) become significant.

Table 2
Borehole lengths and positions

i	H$_i$ (m)	x$_i$ (m)	y$_i$ (m)
1	100	0	12
2	95	5	9
3	90	10	6
4	85	15	3
5	80	20	0
6	85	25	3
7	90	30	6
8	95	35	9
9	100	40	12

Figure 5 g-function of a 7 × 3 bore field

Table 1
Difference with Eskilson's g-function

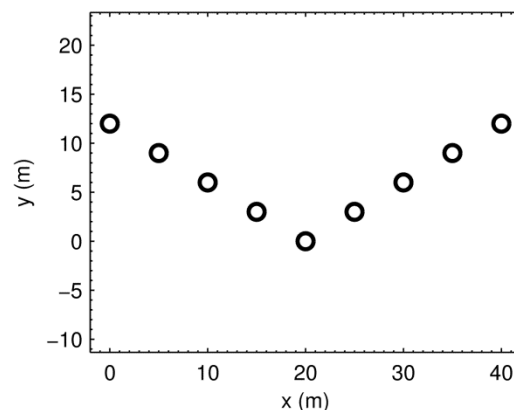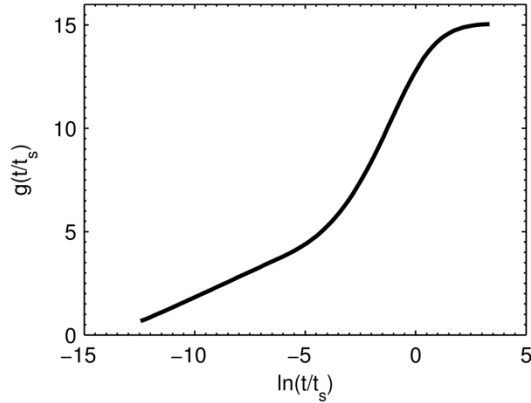| t (years)	ln(t/t$_s$)	Difference (%)
5	-2.625	0.25
10	-1.932	2.6
20	-1.239	5.6
30	-0.834	7.7
40	-0.546	9.4
50	-0.323	10.6

Figure 6 Positions of the 9 boreholes

Figure 7 g-function of the field of 9 boreholes

The g-function exported by the preprocessor can be used to simulate the bore field and obtain the hourly variation of the borehole wall temperature due to heat extraction in the bore field. For a variable total heat extraction rate per borehole length, as shown in the example on Figure 8, the borehole wall temperature is calculated by temporal superposition of the g-function (Equation 2).
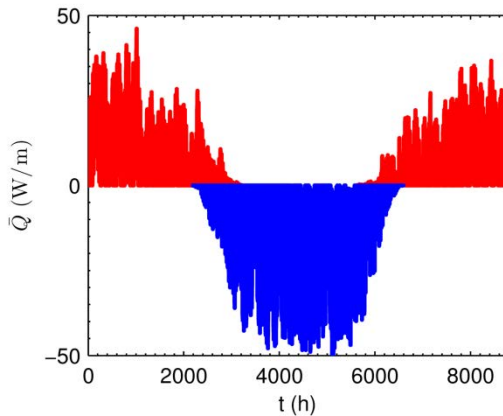

Figure 8 Hourly heat extraction rate

Cimmino et al. (2012) demonstrated how to use fast Fourier transforms (FFT) to simulate bore fields with g-functions, as proposed by Marcotte and Pasquier (2008). Temporal superposition (Equation 2) is expressed in the Fourier domain:

$$\mathrm{T}_b(t) - T_g = \mathcal{F}^{-1}\left(\mathcal{F}\left(\frac{-\bar{q}(t)}{2\pi k_s} \right) \cdot \mathcal{F}(g(t)) \right) \tag{15}$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ are the direct and inverse Fourier transforms, calculated using a FFT algorithm.

The calculation of the temporal superposition using Fourier transforms is done in 5 steps. (1) The g-function is evaluated at each time-step of the simulation and (2) the total heat extraction rate increment per borehole length $\bar{q}$ is calculated from the total heat extraction rate per borehole length $\bar{Q}$. (3) The Fourier transforms of the g-function and the ratio $\frac{-\bar{q}(t)}{2\pi k_s}$ are calculated using a FFT algorithm. (4) The two Fourier transforms are then multiplied for each term in the Fourier domain. Finally, (5) the inverse transform of the result of the multiplication is calculated using an inverse FFT algorithm and added to the undisturbed ground temperature to obtain the borehole wall temperature. The variation of the borehole wall temperature in the bore field described in Table 2, due to a variable heat extraction rate shown on Figure 8 for 1 year of simulation, is shown on Figure 9.
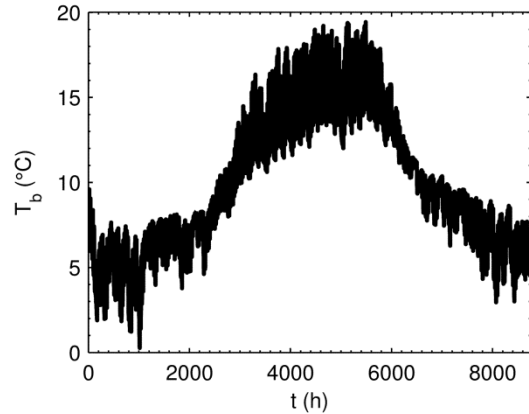

Figure 9 Hourly borehole wall temperature

CONCLUSION

A new finite line source (FLS) solution is presented and used to calculate thermal response factors, called g-functions, for fields of arbitrarily sized and positioned boreholes. Heat extraction rates vary from borehole to borehole as well as in time in order to obtain a borehole wall temperature equal for all boreholes. Spline interpolation is used to reduce the number of evaluations of the analytical solution and the number of time steps in the temporal superposition.

The methodology is implemented into a graphical user interface (GUI) in Matlab. Thermal response factors calculated using the presented methodology are equivalent to those obtained using the methodology of Cimmino et al. (2013). The g-function of a 7×3 bore field calculated by the preprocessor was also compared to Eskilson's g-function. The differences between the two g-functions was inferior to 2.6 % for times $t \leq 10$ years. The differences are attributed to the boundary condition at the borehole walls, which is different for each method. Future work will introduce a new solution for the calculation of the g-function using the same boundary condition as Eskilson.

NOMENCLATURE

α_s = ground thermal diffusivity
D_i = buried depth of the ith borehole
g = g-function
H_i = length of the ith borehole
$\bar{H}$ = average borehole length in the bore field
$h_{i \to j}$ = borehole-to-borehole response factor
k_s = ground thermal conductivity
n_b = number of boreholes

Q_i = heat extraction rate per unit length of the ith borehole

$\tilde{Q}_i$ = normalized heat extraction rate per unit length of the ith borehole

$\tilde{q}_i$ = normalized heat extraction rate increment per unit length of the ith borehole

$\bar{Q}$ = total heat extraction rate per unit length in the bore field

$\bar{q}$ = total heat extraction rate increment per unit length in the bore field

r_b = borehole radius

r_{ij} = radial distance between the ith and j^{th} boreholes

T_b = borehole wall temperature

T_g = undisturbed ground temperature

$\Delta T_{i \rightarrow j}$ = temperature variation at the wall of the jth borehole caused by the ith borehole

Θ_b = non-dimensional borehole wall temperature

$\Delta\Theta_{i \rightarrow j}$ = non-dimensional temperature variation at the wall of the jth borehole caused by the ith borehole

ACKNOWLEDGEMENT

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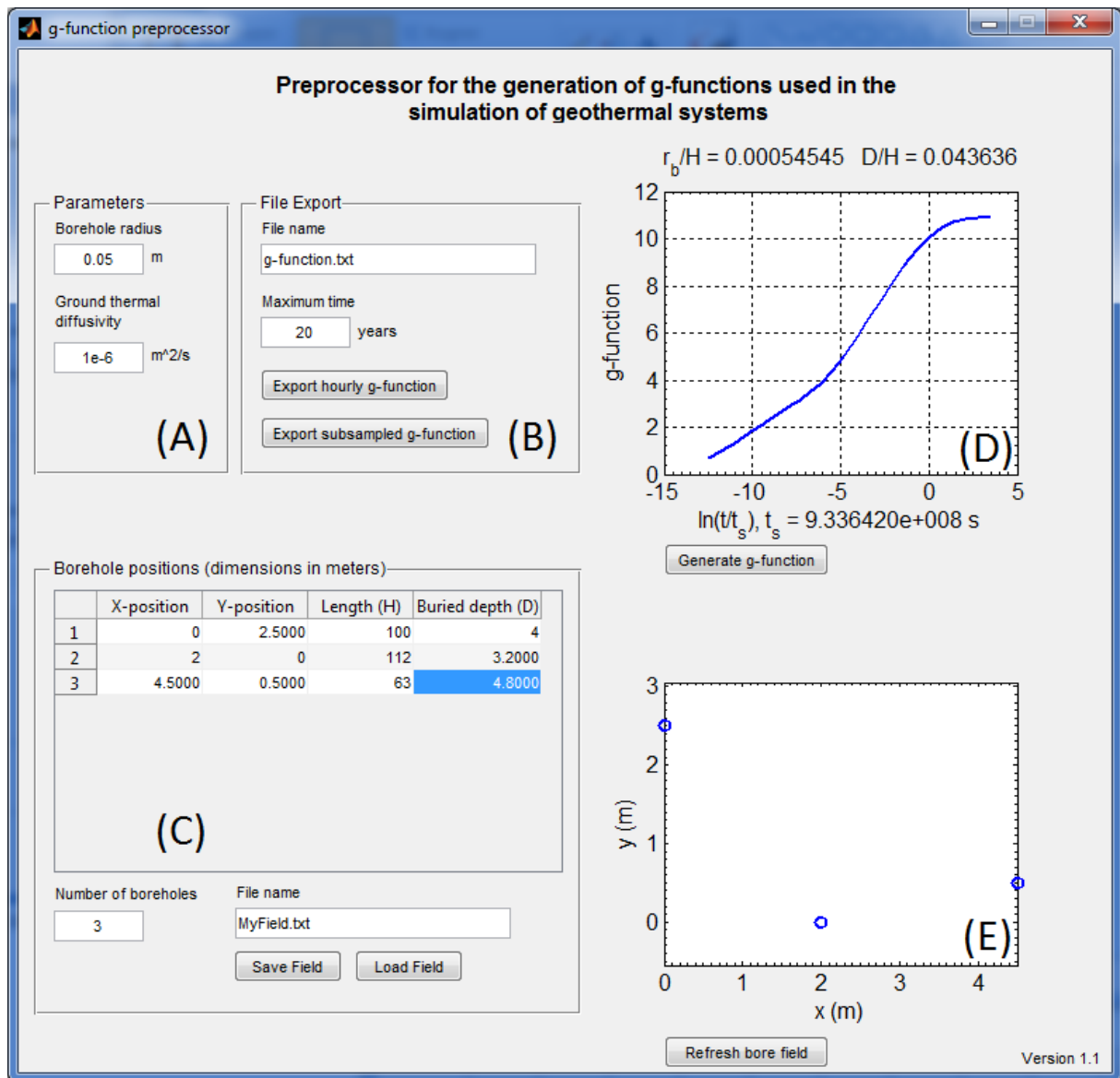
Figure 4 Graphical user interface (GUI) of the preprocessor