

DEVELOPMENT OF SUPERVISORY CONTROL STRATEGY FOR ONLINE CONTROL OF CENTRAL COOLING WATER SYSTEMS

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ABSTRACT

This paper presents a model-based supervisory control strategy for online control of building central cooling water systems to enhance their energy efficiency. Simplified chiller and cooling tower models are developed and used to predict the system energy performance, environment quality as well as the system response to the changes of control settings. A novel and simple optimization technique, named as *PMES* method, is developed and used to seek the optimal control settings under the given working conditions. The performance of both simplified models is validated using the field measurement data and/or the factory performance test data while the performance of this model-based supervisory control strategy is evaluated by comparing with that of near optimal control strategies and other conventional control strategies for cooling water systems through simulations. The implementation issues of the supervisory control strategy for practical applications are also presented.

KEYWORDS

Simplified model, supervisory control, optimisation method, energy efficiency

INTRODUCTION

Reliable and intelligent control and operation of building HVAC&R (heating, ventilating, air-conditioning and refrigeration) systems are among the main achievable approaches to improve building energy efficiency and provide better performance. Model-based supervisory and optimal control aimed at seeking the minimum energy input to provide the satisfied indoor comfort and healthy environment taking into account the ever-changing indoor and outdoor conditions as well as the characteristics of HVAC&R systems, has received growing concern and more attention of building professionals over the past two decades (Koepfel et al. 1995, Wang and Jin 2000, Ahn and Mitchell 2001, Nassif et al. 2005, Sun and Reddy 2005). The essential issues related to the development and implementation of model-based supervisory and optimal control strategies are system performance prediction and optimization techniques. Accurate and reliable system and/or component

models, and simple and practical optimization techniques are essential for real-time control applications. Chillers and cooling towers are the critical components in building HVAC&R systems and they often consume substantial amounts of total energy consumption of the overall systems. It was estimated that as much as 35%-40% of the annual total energy consumption of an office building in Hong Kong was consumed by chillers (Chan and Yu 2002). The operating cost of cooling towers is relatively inexpensive, however, their operation significantly impacts the energy efficiency of other related sub-systems (Austin 1993, Crowther and Furlong 2004, Lu et al. 2004). Therefore, the development of supervisory and optimal control strategies for central cooling water systems is extremely important and essential to reduce the overall system operating cost and enhance their energy efficiency.

To develop the model-based supervisory and optimal control strategy for cooling water systems, the first issue is system performance prediction or modeling (i.e. chiller model, cooling tower model, etc.). In building HVAC&R field, much research has been paid on developing accurate and reliable chiller and cooling tower models over a few decades (Merkel 1925, Braun et al. 1989, Lebrun 1993, Bourdouxhe et al. 1998, Bendapudi and Braun 2002, Lebrun et al. 2004). All these models are primarily used to simulate the heat and mass transfer processes in chillers and cooling towers. Their performance of energy and environment evaluation was also demonstrated. They may be useful and helpful for the system design, optimal control strategy development and analysis, etc. However, these detailed/simplified physical models are rather complicated and beyond the experience of most practical engineers. Iteration process is always required in most of these models, which seriously prevent their online applications. Purely data-driven models cannot ensure stable performance prediction although they are simple. They are reliable only for operating points within the range of the training data covered, and extrapolation outside this range may lead to significant error. For practical applications, the models preferably have simplified structures with physical meaning to ensure stable performance prediction and acceptable

accuracy over a wide range of operation conditions. These models should also require less training or calibration efforts with readily or easily available operation data, less computational costs and memory demand.

The second issue related to develop the model-based supervisory and optimal control strategy is to select and/or develop appropriate optimization techniques. During the past two decades, a number of studies have been carried out on the development and application of various optimization techniques in supervisory and optimal control strategies for cooling water systems (Braun and Diderrich 1990, Austin 1993, Crowther and Furlong 2004, Lu et al. 2004). These studies primarily demonstrated substantial energy in buildings, particularly in cooling water systems, can be saved when these optimization techniques are used in the supervisory and optimal control strategies to optimal operation the cooling water systems. However, most of these optimization techniques cannot satisfy the requirements of online practical applications when computational cost and operating efficiency are of concern simultaneously. For real time control, several studies used near optimal control strategies (Braun and Diderrich 1990, Yao et al. 2004, Sun and Reddy 2005). These strategies are simple enough and are easy to be implemented in practice. However, they might deviate significantly from optimal and significant amounts of energy might be still consumed.

In this paper, a model-based supervisory control strategy is presented for central cooling water systems. Simplified semi-physical chiller and cooling tower models, and a novel and practical optimization technique (i.e. *PMES* method) are developed and used to construct and formulate the model-based supervisory control strategy. The performance of both simplified models and the model-based supervisory control strategy were evaluated. The implementation issues of this supervisory control strategy for practical applications are also presented.

MODELLING AND VALIDATION

CHILLER MODEL

A chiller model is developed based on the fundamental laws of heat and mass transfer phenomena in chillers. A fictitious refrigeration cycle (1'-2'-3'-4'), as shown in Figure 1, is assumed to simplify the complicated heat and mass transfer process occurred in the refrigeration systems. The major assumptions utilized to derive the basic modeling equations were summarized by Wang et al. (2000).

The chiller evaporator and condenser are simulated using the classical heat exchanger efficiency method. The overall heat transfer coefficients of the

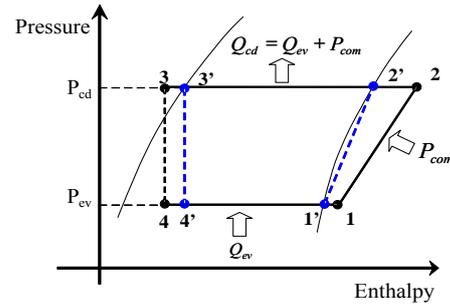


Figure 1 Schematic of pressure-enthalpy diagram (Actual cycle: 1-2-3-4; Fictitious cycle: 1'-2'-3'-4')

evaporator and condenser are represented empirically as Equation (1) and (2) respectively. The fictitious refrigerant mass flow rate ($M_{ref, fic}$) is calculated as Equation (3), which is based on the evaporator cooling energy and the enthalpy difference between the point 1' and point 3' in the fictitious cycle. The fictitious power (P_{fic}), as Equation (4), is the product of the fictitious refrigerant mass flow rate and the enthalpy difference between the point 2' and point 1' in the fictitious refrigeration cycle. The fictitious refrigerant mass flow rate and the enthalpy of each point are calculated according to the condensing temperature and evaporating temperature, which are determined by the inlet water temperatures and water mass flow rates in the condenser and evaporator together with the evaporator cooling energy by an iterative manner. A polynomial is used to characterize the relationship between the actual chiller power consumption (P_{com}) and the fictitious power (P_{fic}) as Equation (5).

$$C_1 M_{w, ev}^{-0.8} + C_2 Q_{ev}^{-0.745} + C_3 = \frac{1}{UA_{ev}} \quad (1)$$

$$C_4 M_{w, cd}^{-0.8} + C_5 (Q_{ev} + P_{com})^{1/3} + C_6 = \frac{1}{UA_{cd}} \quad (2)$$

$$M_{ref, fic} = \frac{Q_{ev}}{h'_1 - h'_3} \quad (3)$$

$$P_{fic} = M_{ref, fic} \cdot (h'_2 - h'_1) \quad (4)$$

$$P_{com} = a_0 P_{fic} + a_1 P_{fic}^2 + a_2 \quad (5)$$

Using chiller operation data, the model parameters can be identified easily. The evaporator and condenser overall heat transfer coefficients (UA_{ev} , UA_{cd}) are calculated using the measured evaporator cooling energy and compressor power consumption together with the calculated evaporator and condenser logarithm mean temperature differences. The coefficients of C_1 - C_6 can then be identified utilizing the measured water flow rates, measured evaporator cooling energy and compressor power consumption together with the calculated overall heat transfer coefficients. The coefficients of a_0 - a_2 can be regressed using the calculated fictitious power (P_{fic}) and measured actual chiller power consumption (P_{com}). After the model parameters are identified, the power consumption of the chiller at giving working condition can be predicted using the above equations.

COOLING TOWER MODEL

The cooling tower model is developed based on the simplification of the physical model developed by Lebrun (1993). The major assumptions used to derive the basic modeling equations were presented by Lebrun (1993). In order to simplify the model structure, the minimum capacity flow rate is assumed on the air side.

The moist air enthalpy is assumed to be fully dominated by its wet-bulb temperature (Lebrun 1993) and is expressed as Equation (6). Since the outlet status in cooling towers is strong dependent on the four inlet parameters (M_a , M_w , $T_{w,in}$, $T_{wb,in}$), the air average fictitious specific heat can be simplified as Equation (7), which is a function of X, Y and Z. X, Y and Z are defined as Equation (8).

$$h_a = d_0 + d_1 T_{wb} + d_2 T_{wb}^2 + d_3 T_{wb}^3 \quad (6)$$

$$c_{p,a,fc} = \frac{h_{a,out} - h_{a,in}}{T_{wb,out} - T_{wb,in}} \quad (7)$$

$$= f_0(M_a, M_w, T_{w,in} - T_{wb,in}) = f_0(X, Y, Z)$$

$$X = M_a ; Y = M_w ; Z = T_{w,in} - T_{wb,in} \quad (8)$$

The fictitious NTU can be derived as a function of X and Y as Equation (9) since the values of AU_{des} , $M_{w,des}$ and $M_{a,des}$ are constant at design working condition and the moist air specific heat ($c_{p,a}$) can be considered as constant. Since $c_{p,a,fc}$ is the function of X, Y and Z, and the water specific heat ($c_{p,w}$) can be considered as constant, mass flow rate capacity ratio (ω) is also the function of X, Y and Z as Equation (10).

$$NTU_{fc} = \frac{AU_{fc}}{C_{min}} \quad (9)$$

$$= \frac{AU_{des}}{M_a \cdot c_{p,a}} \cdot \left(\frac{M_w}{M_{w,des}} \right)^m \cdot \left(\frac{M_a}{M_{a,des}} \right)^n = f_1(X, Y)$$

$$\omega = \frac{C_{min}}{C_{max}} = \frac{M_a \cdot c_{p,a,fc}}{M_w \cdot c_{p,w}} = f_2(X, Y, Z) \quad (10)$$

The heat transfer effectiveness of the counter flow cooling tower and the crossover flow cooling tower are defined as Equation (11) and (12) respectively. They can be considered as a function of NTU_{fc} and ω . Since ω is the function of X, Y and Z, and NTU_{fc} is the function of X and Y, the fictitious heat transfer effectiveness (ε_{fc}) is still the function of X, Y and Z, as shown in Equation (13). The heat transfer between water and air can be then expressed as Equation (14) since ε_{fc} and $c_{p,a,fc}$ both are the functions of X, Y and Z.

$$\varepsilon_{fc} = \frac{1 - e^{(-NTU_{fc} \cdot (1-\omega))}}{1 - \omega \cdot e^{(-NTU_{fc} \cdot (1-\omega))}} \quad (11)$$

$$\varepsilon_{fc} = \frac{1}{\omega} \cdot \left(1 - e^{(-\omega(1 - e^{(-NTU_{fc})})} \right) \quad (12)$$

$$\varepsilon_{fc} = f_3(NTU_{fc}, \omega) = f_4(X, Y, Z) \quad (13)$$

$$Q = \varepsilon_{fc} \cdot C_{min} \cdot (T_{w,in} - T_{wb,in}) = \varepsilon_{fc} \cdot M_a \cdot c_{p,a,fc} \cdot (T_{w,in} - T_{wb,in}) = f_5(X, Y, Z) \quad (14)$$

Under the normal operation of the cooling tower, X, Y and Z can be considered as continuous variables since air and water flow rates as well as inlet water temperature and ambient air wet-bulb temperature can vary continuously. For practical applications, the heat rejection as Equation (14) can be rewritten in the form as Equation (15) approximately, which is the final form of the cooling tower model. This model has high flexibility since it is very easy to be reformed and used to predict the required air mass flow rate for online predictive control applications.

$$Q = b_0 \cdot X^{b_1} \cdot Y^{b_2} \cdot Z^{b_3} \quad (15)$$

For the application of this model, the critical issue is to identify these four parameters (b_0 , b_1 , b_2 , b_3). They can be regressed using the measured heat rejection, air and water mass flow rates together with the inlet water temperature and ambient air wet-bulb temperature.

MODEL VALIDATION

The thermal performance of the simplified chiller model is validated using the site measurement data. The chiller data was collected during 75 days from 1 July 2005 to 16 September 2005 in an office building in Hong Kong. The sampling interval is one hour. These operation data is divided into two groups. One group is as training data for model training (i.e., parameter identification). The other group is used for model validation. Figure 2 shows the validation results. It can be found that the model predicted power consumptions excellently agreed with the measurements for all operating points. The deviations for most of operating points are within $\pm 10\%$, which is acceptable for online applications.

A direct contact, crossover flow cooling tower, as shown in Figure 3, was tested at the cooling tower thermal test laboratory within manufacturer's factory. This cooling tower is an in-house type, and consists of six important parts, i.e., water distribution basin, cold water collecting basin, fill packing, axial fan, air plenum, and silencer. Part of the experimental data was utilized to identify the model parameters, and the others are utilized to model validation.

Figure 4 presents the model predicted heat rejections of the simplified cooling tower model and the "measured" heat rejections calculated by the product of measured water flow rates and the temperature differences between inlet and outlet water. The result shows that the model predicted heat rejections agree very well with the measurements for all operating points. The deviations due to the model prediction are within $\pm 5.9\%$, which is also tolerable for practical applications.

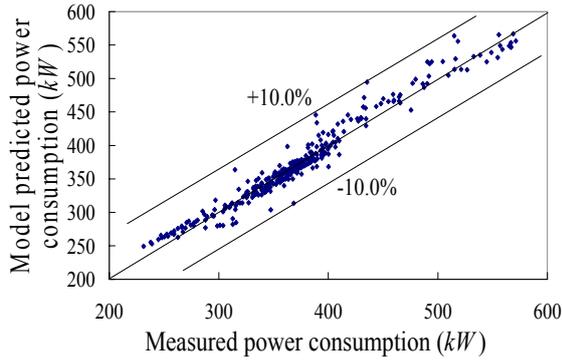


Figure 2 Comparison between measurements and predicted power consumptions using the simplified chiller model

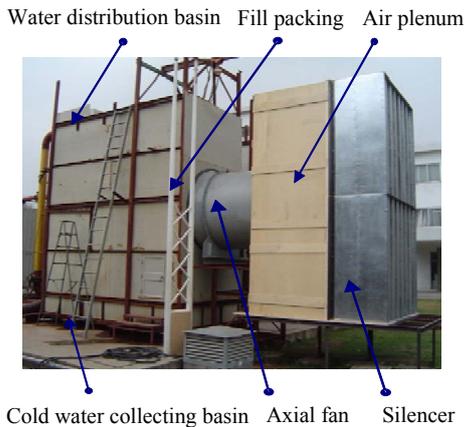


Figure 3 Profile of the experimental cooling tower

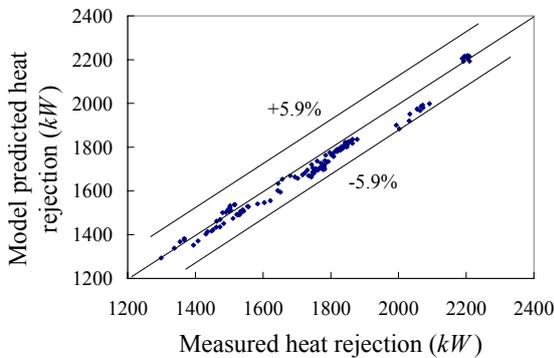


Figure 4 Comparison between measurements and predicted heat rejections using the simplified cooling tower model

Computational cost and memory demand are among the critical issues of online application of both models in model-based supervisory control strategies. Therefore, it is essentially necessary to evaluate the computational cost and memory demand when using both models for online control applications. In this study, the computational costs of both models were tested under Matlab environment. The test results were evaluated by comparing with the results obtained from other physical models. Computer configuration used in the test is described as follows. The operation system is Microsoft windows XP professional, the processor is Intel(R) Pentium(R) 4

CPU 3.20GHz (2 CPUs), and the memory is 2040MB RAM. Table 1 summarizes the average computational costs when using simplified models and physical models for performance prediction to achieve a set of results. It is obvious that the simplified models have less computational costs as compared to the physical models. Although the computational cost for a single model is relative small, there are always many models involved in control strategies and optimization algorithms are also integrated together, which may result in large computational requirement.

Table 1 Testing results of computational costs

Models		Computational cost
Chiller model	Simplified	3.766ms
	Wang et al.(2000)	16.668ms
Cooling tower model	Simplified	0.141ms
	Toolkit	0.199ms

Training or calibration effort is another critical issue when using these models for online applications. Since parameter identification of the detailed chiller model developed by Wang et al. (2000) requires a considerable amount of chiller performance data at full load working condition, it is not easy to obtain these performance data in the site field. However, the parameter identification of the simplified chiller model only requires the performance data of the chiller at normal operating conditions. The calibration effort is reduced greatly. The calibration process of the simplified cooling tower model is straightforward and simple. No iterations are required. The parameter identification and performance prediction of the Toolkit cooling tower require a lot of iterations. For the online applications, the adaptive models are always required. The model parameters need to be updated according to the historical date and current working condition to achieve the better prediction performance. Iteration may result in instability and divergence, which is the main weakness of the models requiring a lot of iterations, particularly for real time online applications. Therefore, the simplified chillers and cooling tower models developed in this study are more feasible for practical online applications.

OPTIMIZATION TECHNIQUE

A novel and simple optimization technique, named as *PMES* (performance map and exhaustive search)-method, is developed based on the combination of the performance map-based control strategy and the exhaustive search method to seek the optimal solutions. Performance map-based control strategy is an exactly method that often uses the results generated from the detailed simulation of the targeted system over the range of expected operation conditions to draw a performance map, and then

utilizes this performance map to control the operation of HVAC systems. For instance, for an electric-driven chiller plant without significant thermal energy storage, using the component models, various combinations of cooling loads, ambient air temperatures, the numbers of operating chillers, the numbers of operating pumps as well as the numbers of operating cooling towers and their individual fan speeds can be used as inputs to the simulation platform. At each operating condition, the power consumptions or performance data for all combinations are computed, and the control settings giving minimum energy value or best performance are identified. A performance map can then be drawn using those combinations with minimum energy values or best performance identified from over the full operating range of a system and further can be used as a supervisory controller to optimal operation of the HVAC system. It is worthwhile to notice that the performance map is not necessarily obtained by simulations. For example, it could be obtained by testing the system over significant range of settings and operating conditions, although simulation is an effective tool. For real time control applications, simple and near optimal strategies, as one shown in Equation (16), can be generated based on these performance-maps and these strategies can be named as performance map-based control strategies. The PMES-based method presented in this study is developed based on the consideration of the requirements and constraints of practical applications, i.e. computational cost, memory demand, control accuracy and control robustness. The PMES method uses the exhaustive search method to find the global minimum within a relative narrow search range for the condenser inlet water temperature set-point as bounded in Equation (16). The search center $T_{w,cd,in}^{n,o}$ in Equation (16) is generated using a linear model as Equation (17), which is regressed from a performance map. The search center is actually a near optimal solution for current working condition. The exhaustive search method may seek the global optimal solution within the limited search range with proper increment, i.e. 0.1°C.

$$T_{w,cd,in}^{n,o} - \Delta t \leq T_{w,cd,in} \leq T_{w,cd,in}^{n,o} + \Delta t \quad (16)$$

$$T_{w,cd,in}^{n,o} = e_0 T_{wb}^2 + e_1 T_{wb} + e_2 \quad (17)$$

FORMULATION AND PERFORMANCE EVALUATION OF MODEL-BASED SUPERVISORY CONTROL STRATEGY

Based on the developed simplified chiller and cooling tower models, and the PMES optimization

method, a model-based supervisory strategy for cooling water systems can be formulated easily.

In cooling water systems, chiller and cooling tower performance are highly interacted and they are affected in different directions by the condenser inlet water temperature. The lower condenser inlet water temperature can improve the COP of chillers resulting in less electricity consumption while the lower temperature requires more air flow rate to increase the heat rejection capacity of cooling towers, and hence, more power is consumed by fans. Although higher condenser inlet water temperature can save electricity consumption of cooling tower fans, it deteriorates the efficiency of chillers, which results in more electricity consumption to obtain the same cooling load. Therefore, the condenser inlet water temperature ($T_{wcd,in}$) must be optimized to minimize the electricity consumption of chillers and cooling tower fans and the objective function for systems in which each chiller is associated with one constant condenser water pump can be expressed as Equation (18).

$$\min_{T_{wcd,in}} P_{tot} = P_{ch,tot} + P_{ct,tot} \quad (18)$$

The operation of cooling water systems has to obey a lot of constraints, i.e. the lowest condenser inlet water temperature is bound to 18°C, the heat rejected in cooling towers is equal to the heat absorbed by the cooling water from the chiller condensers, etc.

The performance of this model-based supervisory control strategy is evaluated by comparing to that of the performance map-based control strategy in terms of condenser inlet water temperature and power consumptions. In this study, the performance of the fixed approach (i.e. fixed approach is to vary the cooling tower air flow to maintain the constant temperature difference between the cooling tower outlet water temperature and ambient air wet-bulb temperature) control method (approach temperature is 5°C) is used as the benchmark. Figure 5 presents the profiles of condenser inlet water temperatures using proposed model-based supervisory control strategy and the performance map-based control strategy. The condenser inlet water temperature resulted from the performance map-based control strategy is employed to determine the search range for the proposed PMES strategy by bounded with 2.5°C. It is obvious that the optimal condenser inlet water temperatures using the proposed strategy are different from the near optimal values using the performance map-based control strategy. It also can be found the optimal condenser inlet water temperatures are within the defined search ranges.

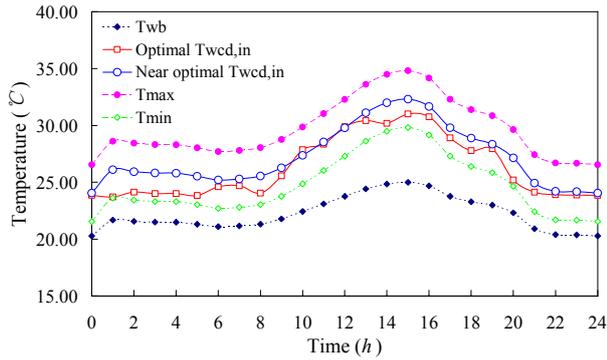


Figure 5 Profiles of condenser inlet water temperature

The performance of this supervisory control strategy is further validated concerning system power consumptions. Figure 6 presents the difference between the power consumptions using the proposed strategy and fixed approach method, and the difference between the power consumptions using the performance map-based control strategy and fixed approach method. It can be seen clearly that substantial energy was saved when this model-based supervisory control strategy is used. It also can be found that performance map-based control strategy is not always better than the fixed approach control method. For instance, at some working conditions, the power consumptions using performance map-based control strategy is less than that of using fixed approach method, however, at other cases, the power consumptions using performance map-based control strategy is larger than that of using fixed approach method, which further reveals that proposed supervisory control strategy is much better than the performance map-based control strategy.

Table 2 presents the power consumptions of the cooling water system (chillers+cooling towers) in a typical summer sunny day when using different control methods. Compared with the fixed approach control method, the *PMES* method can save about 1.221% energy. It also can be found that near optimal control method can save about 0.239% energy as compared with the fixed approach control method although the hourly power consumption of near optimal control method is not always less than that of fixed approach control method.

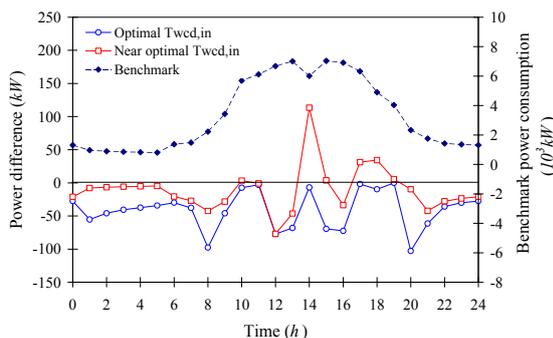


Figure 6 Difference of power consumptions

Table 2 Power consumption of cooling water system using different methods in a typical summer day

Methods	Power (kWh)	Difference (kWh)	Difference (%)
Fix approach method	81760.79	-	-
Near optimal method	81521.05	-239.74	-0.239
<i>PMES</i> method	80762.22	-998.57	-1.221

ISSUES OF IMPLEMENTATION FOR PRACTICAL APPLICATIONS

This model-based supervisory control strategy is being implemented in “International Commerce Center”, a super high rise commercial office building at construction stage in Hong Kong. Figure 7 shows the outlook of the building.

The practical implementation architecture of the optimal control strategy is illustrated in Figure 8. Data exchanger between the real building and cooling water system control optimizer is achieved through the communication platform. This communication platform is developed based on the IBmanager. IBmanager employs standard middleware technologies to realize data and services integration and interoperation among distributed building automation systems (BASs) on the Intranet/Internet. The Web Services methods of

IBmanager can provide a convenient platform for various building and facility management applications and enterprise applications. The computation of cooling water system control optimizer is achieved by application programs in the environment of commercial software–Matlab. The Matlab application programs are compiled as dynamic link library (DLL) to be invoked by IBmanager. IBmanager reads status data from real building systems and transfers them to the control optimizer. The control optimizer will decide the optimal control parameters (e.g. set-points) for the BA system based on the received status data. The control parameters are then transferred to real building systems by IBmanager to achieve energy efficient control and operation. All these control parameters and online operating status are monitored and recorded by IBmanager. The software for control optimization is wrapped as a component running on the IBmanager platform.



Figure 7 Profile of International Commerce Center

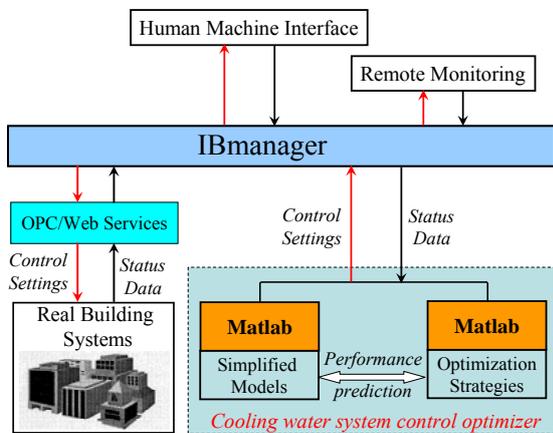


Figure 8 Implementation architecture of control optimizer

CONCLUSION

A model-based supervisory control strategy for optimal control and operation of building central cooling water systems is presented. Simplified semi-physical chiller and cooling tower models are developed and used to predict system energy performance and environment quality as well as the system response to the change of control settings. The *PMES* optimization technique is developed taking into account the requirements of practical applications, and is used to seek the energy efficient control settings under the given working conditions.

The performance validation of simplified semi-physical chiller and cooling tower models showed that both models can provide reliable and accurate performance prediction over a wide range of operation conditions. Computational costs and calibration efforts of both models are reduced greatly at compared with that of other models. All these characteristics make them feasible for online control applications. The performance evaluation of the proposed model-based supervisory control strategy showed that *PMES* method can always find the

optimal solutions, and hence, substantial energy can be saved as compared to that of using the performance map-based control strategy and the fixed approach control method. The proposed strategy is very simple and easy to be understood by application engineers. The implementation issues of this strategy for online control are presented and the performance for practical applications will be reported in the future.

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NOMENCLATURE

a_0-a_2	coefficients
b_0-b_3	coefficients
d_0-d_3	polynomial coefficient, depending on value of atmospheric pressure
e_0-e_2	coefficients
C_1-C_6	coefficients
f	function
h	enthalpy, kJ/kg/K
c	specific heat, kJ/ kg/K
C	mass flow rate capacity, W/K
M	mass flow rate, kg/s
NTU	number of transfer units
P	power consumption, kW
Q	heat transfer rate, kW
T	temperature (°C)
UA	heat transfer coefficient, W/K
ϵ	heat transfer effectiveness
ω	mass flow rate capacity ratio

SUBSCRIPTS

a	air
cd	condenser
ch	chiller
com	compressor
ct	cooling tower
des	design condition
ev	evaporator
fic	fictitious
in	inlet
max	maximum
min	minimum
out	outlet
p	pressure
ref	refrigerant
tot	total
w	water
wb	wet-bulb

SUPERSCRIPTS

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