ANALYSIS ON THE WELL-MIXING ASSUMPTIONS USED IN MULTIZONE AIRFLOW NETWORK MODELS

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ABSTRACT

The well-mixing assumptions, uniform distributions of air temperatures and contaminant concentrations and neglect of air momentum effects in a zone, used in multizone airflow network models could cause errors in some cases. Through dimensional analysis of the published data from the literature, this study has found correlation between the errors and mixing levels. Our study concluded that the assumption of uniform air temperatures is not appropriate if the vertical non-dimensional temperature gradient is higher than 0.03. The assumption of uniform air momentum effects could fail when the airflow with a strong momentum effect cannot be totally dissipated. The assumption of uniform contaminant concentrations can be invalid unless the contaminant source in the zone is also a heat source and the associated Archimedes number is greater than 400.

KEYWORDS

Multizone airflow models, Indoor air quality, Contaminant transport

INTRODUCTION

Multizone airflow network models (hereafter multizone models) have been widely used in designs of air distribution system, indoor air quality analyses, smoke controls, and building pressurization tests, etc. With a PC, multizone models can calculate the airflow and contaminant transport inside a building within minutes or even seconds.

However, the fast computing speed is achieved by using the well-mixing assumptions in each zone (room). The assumptions are uniform distributions of air temperature and contaminant concentration in each room and neglect of air momentum effect from an inflow opening. The well-mixing assumptions could be problematic for simulations with poorly mixed air and contaminants. Schaelin et al. (1994) and Upham (1997) pointed out that the well-mixing assumption neglected the impacts of the local variables near the flow paths on multizone model predictions. Consequently, the results of multizone models could be inaccurate, especially for calculations of contaminant dispersions. Clarke (2001) also noted that current building airflow modeling by network approach has significant limitations. Because momentum effects are neglected, intra-room airflow and temperature distribution cannot be determined. Gao and Chen (2003) found that multizone models produce incorrect results in a T-junction due to the neglect of preserved momentum within a zone. Although the previous studies recognized the problems caused by the well-mixing assumptions, no analyses have been conducted to provide quantitatively the errors caused by the assumptions.

This paper proposes to use a few dimensionless numbers to characterize mixing levels of air and contaminants in a zone. By using published data in the literature, this study tries to find the correlations between several common dimensionless numbers of air and the errors caused by the well-mixing assumptions.

ANALYSES AND RESULTS

The assumption of uniform distribution of air temperatures

Multizone models use a single temperature for each zone that implies the air temperature in the zone is uniform. This is not always the case because vertical temperature gradient exits in zones with stratified flow, such as room with displacement ventilation or under-floor air distribution systems and rooms with water heating systems. Zohrabian et al. (1990) measured the buoyancy-driven airflows through a two-compartment stairwell. If CONTAM (Walton and Dols 2003), a multizone program, is used, the predicted airflow rate can be 40% smaller than the measured one, when the vertical temperature difference was 17°C. When the vertical temperature gradient in a zone is moderate, however, the assumption of uniform temperatures could become tolerable. The experiment of Wang and Chen (2006) in a 4-zone chamber facility found that the difference between the multizone simulation and the experimental data was less than 10%, when the vertical temperature difference was 6 °C. Therefore, the validity of the assumption of uniform air temperatures depends on the magnitude of air temperature gradient in a zone.

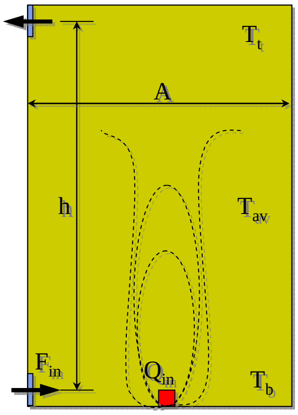

Figure 1 Illustration of non-uniform distribution of temperatures in a room

The temperature gradient relative to the average zone temperature can be defined as

$$\frac{\Delta T}{T_{av}} = \frac{|T_t - T_b|}{T_{av}} \qquad (1)$$

A smaller value of $\Delta T / T_{av}$ indicates a better mixing of air temperatures. A dimensionless number for temperature gradient, τ, can be further defined by the Reynolds number and the Stanton number (Reynolds 1986).

$$\tau = \mathrm{Re}^{1/3}\,\mathrm{St}^{2/3} \propto \frac{\Delta T}{T_{av}} \qquad (2)$$

with

$$\mathrm{Re} = \frac{F_{in}}{L\upsilon} \qquad (3)$$

$$\mathrm{St} = \frac{Q_{in}}{\rho C_p T_{av} A (gh)^{1/2}} \qquad (4)$$

where L equals to $V_{room}^{1/3}$ with V_{room} is the total volume of the room.

To find a correlation of the dimensionless temperature gradient to the simulation errors caused by using the assumption of uniform air temperature, we define

$$\mathrm{Error} = \left| \frac{F_{CONTAM} - F_{exp}}{F_{exp}} \right| \qquad (5)$$

Three experimental studies from the literature were used to find a correlation of Error ~ τ. Besides the studies of Zohrabian et al. (1990) and Wang and Chen (2006), this paper also used the results from Kotani et al. (2003), who measured buoyancy-driven airflow rates through a light well model.

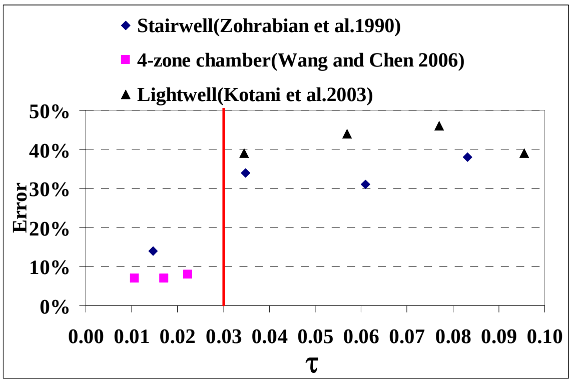

Figure 2 The simulation errors caused by the assumption of uniform air temperature vs. the dimensionless temperature gradient for different buoyancy-driven flows

Figure 2 shows the correlation of "Error ~ τ" for the three cases. When

$$\tau < 0.03 \qquad (6)$$

the error as defined by Eq. (5) is less than 20%, which implies that the uniform air temperature assumption is acceptable. For the rest of cases, the errors are very significant so that the air temperature gradient in a room should not be neglected.

The assumption of neglect of air momentum effects in a zone

Multizone models assume that the air in a zone is quiescent or still, and the airflow through a zone does not have an impact on zone pressure. The assumption is valid for the cases where air momentum effect through openings can be immediately dissipated or uniformly distributed after entering the zone. For example, this assumption is true for infiltration through small openings. However, a strong momentum effect may be preserved, contributing to spatial variations in zone pressure, such as airflows provided by large openings in buildings with natural ventilation.

This paper used the theory of indoor jet flows to study preservation and dissipation of airflow momentum effects. As shown in Figure 3, a jet flow comprises a jet expansion area and a recirculation area. The jet expansion area can be further divided into four zones (ASHRAE 2005), which describe how the jet is dissipated in a zone.

- Zone 1: initial zone
- Zone 2: transition zone
- Zone 3: fully developed turbulent flow
- Zone 4: degradation/dissipation zone

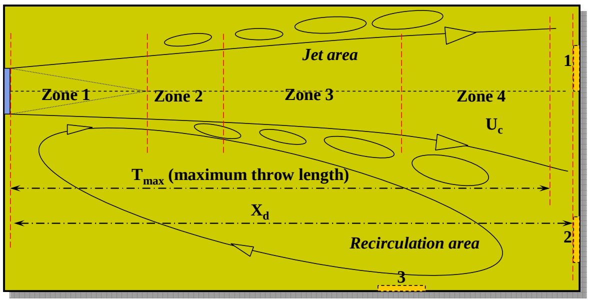

Figure 3 Illustration of a jet flow in a room

If an opening is located inside the recirculation area, such as openings 2 and 3, or nearby the end of the jet expansion area, such as opening 1 in Figure 3, the opening will be subject to minimum impact from the jet momentum effect. Therefore, this study recommends using the maximum jet throw length, T_{max}, to estimate how a jet dissipation affects airflow downwind. In Figure 3, an opening will be affected by the jet momentum effect (ASHRAE 2005) when

$$X_d < T_{max} \tag{7}$$

$$T_{max} = 1.13\frac{KQ}{U_c\sqrt{C_d R_{fa} A_c}} \tag{8}$$

where K, the jet centerline velocity constant, can be determined from ASHRAE Handbook for different supply velocities and openings.

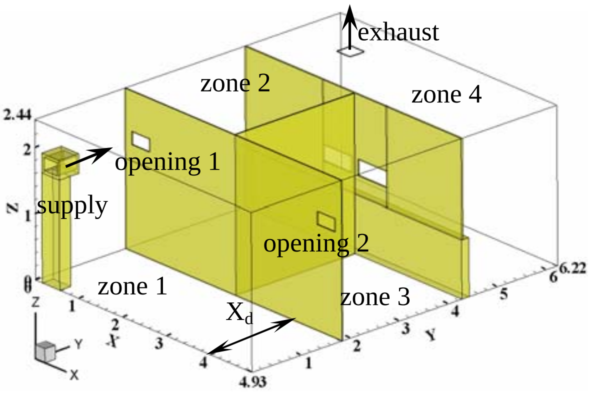

Figure 4 The layout of the 4-zone chamber used in the study of indoor airflows with strong momentum effects

To verify Inequality (7), we used the experimental results obtained from a 4-zone chamber in Figure 4 (Wang and Chen 2006). The experiment used mechanical ventilation through an air supply in zone 1 to create a strong momentum effect. Since opening 1 is directly subjected to the momentum effect from the supply, more air was brought to opening 1 than to opening 2. A multizone model would calculate the same airflow rate through the two openings no matter how strong the air momentum effect was.

Table 1 shows that the errors caused by neglecting the momentum effects by CONTAM. When the airflow rate was 0.011 m^3/s, X_d was greater than T_{max} so that the multizone model works fine. For the rest of cases, the air momentum effects cannot be totally dissipated in zone 1 and the simulation error could reach as high as 30%, since T_{max} became greater than X_d. Therefore, Inequality (7) is a good evaluation criterion.

Table 1 Comparison of multizone calculations with the experimental data for airflows with momentum effects in a 4-zone chamber

Supply (m^3/s)	**Flow ratio (Open1/Open2)**	**X_d (m)**	**T_{max} (m)**	**Error**
0.011	1.1	1.83	1.8	3%
0.034	1.6	1.83	5.2	19%
0.053	2.2	1.83	8.1	27%
0.105	2.4	1.83	15.9	29%
0.140	2.5	1.83	21.1	30%
0.215	2.5	1.83	32.5	30%

The assumption of uniform distribution of contaminant concentrations

Multizone models assume that a contaminant is perfectly mixed in a zone. The assumption can cause large errors for a zone containing the contaminant source. The experimental studies by Yuan and Srebric (2002) and Wang and Chen (2006) have confirmed the errors. In fact, the mixing of contaminant in a zone is dependent on locations of contaminant source, air inflow and outflow, local airflow pattern, air velocity direction and magnitude, and strength of turbulence effect. Finlayson (2004) and Thatcher (2004) found that contaminant concentrations were hardly well mixed for mechanically ventilated spaces, especially when the space had physical obstructions (Gadgil et al. 2003).

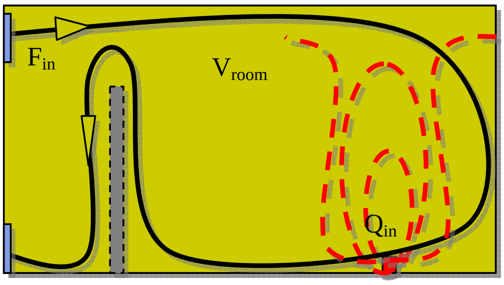

Figure 5 Sketch of contaminant transport enhanced by buoyancy effect in a room with an internal partition

However, if the contaminant source is also a heat source, the buoyancy effect may be strong enough to improve contaminant mixing in the space. By using Figure 5 as an example, the strength of buoyancy force, compared with inertial force, can be characterized by the Archimedes number (Incropera and DeWitt 2007)

$$\mathrm{Ar} = \frac{\mathrm{Gr}}{\mathrm{Re}^n} \qquad (9)$$

The exponent of the Reynolds number is normally “2”, which was however considered to underestimate the inertial force for indoor air simulations (Zhang et al. 1993). Based on the studies of temperature stratification in a ventilated tunnel, Xue et al. (1994) suggested that the exponent could be 2.5. Reynolds (1986) found that the air motion driven by buoyancy force could be better represented by a relation of $\mathrm{Gr} \propto \mathrm{Re}^3$. Based on these previous studies, this study uses the exponent of “3”.

$$\mathrm{Ar} = \frac{\mathrm{Gr}}{\mathrm{Re}^3} \qquad (10)$$

$$\mathrm{Gr} = \frac{g\beta Q_{in} L^3}{\rho C_P F_{in} \upsilon^2} \qquad (11)$$

To characterize the mixing level of contaminant, we also defined an index of non-uniformity of contaminant, “m” as

$$m = \sqrt{\frac{\sum_{i=1}^{N} (C_i - \bar{C})^2 V_i}{\sum_{i=1}^{N} V_i}} \qquad (12)$$

$$\bar{C} = \frac{\sum_{i=1}^{N} C_i V_i}{\sum_{i=1}^{N} V_i} \qquad (13)$$

The greater the non-uniformity index is, the less uniform the contaminant concentration will be.

This investigation used the CFD results from Schaelin et al. (1994), Wang and Chen (2006), and the experimental data from Srebric et al. (1999) to find the errors caused by the assumption of uniform contaminant concentration in a zone. Schaelin et al. simulated the contaminant gradient in a mechanically-ventilated house with a CFD program. Wang and Chen studied the impact of partitions on contaminant distribution in a 4-zone chamber. Srebric et al. measured the distribution of contaminant as simulated by SF_6 in an office room with displacement ventilation. Figure 6 shows that the non-uniformity index is 9.7 and contaminant concentration is highly non-uniform when Ar is zero for the case of 4-zone chamber. With the increase of Ar, the buoyancy effect improved the contaminant mixing so that the non-uniformity index was less than 0.2 when Ar was greater than 400. Compared to the case of 4-zone chamber, the non-uniformity indices of the rest two cases were about 0.3 when Ar was less than 100.

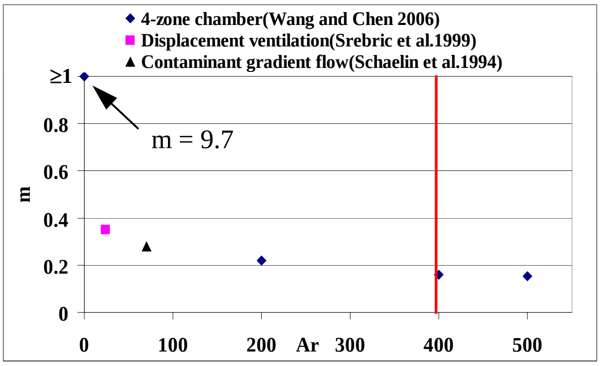

Figure 6 The correlation of Ar ~ m for the three cases

Since the non-uniformity index in Eq. (12) is defined as the Root Mean Square (RMS) of the difference of local contaminant concentrations from the averaged concentration, it is equivalent to the error if the assumption of uniform contaminant concentration is used. Since an error of 20% is normally considered acceptable for multizone simulations (Emmerich 2001), this study suggests that the assumption of uniform distribution of contaminant concentration is acceptable when

$$\mathrm{Ar} \geq 400 \qquad (14)$$

When Ar is less than 400, the buoyancy effect is not strong enough to enhance the mixing of contaminant. As a result, the assumption of uniform contaminant concentration becomes unacceptable.

CONCLUSIONS

This investigation analyzed the impact of non-uniform distributions of air momentum effects, air temperatures, and contaminant concentrations used in multizone airflow and contaminant simulations. The study concluded that the assumptions can cause significant errors in the following scenarios.

For airflows with air temperature gradient in a zone, the non-dimensional temperature gradient τ is greater than 0.03.

For airflows with strong momentum effect preserved in a zone, the distance between the upstream and downstream openings is smaller than the maximum jet throw length from the upstream opening.

For airflows with contaminant concentration gradient in a zone, the Ar is smaller than 400.

ACKNOWLEDGMENT

This research was supported by U.S. National Institute of Standards and Technology (NIST) through contract SB1341-04-Q-0771. The authors would like to thank Dr. A. K. Persily, Dr. G. N. Walton, Mr. W. S. Dols, and Mr. S. J. Emmerich of NIST for their valuable contributions to the research.

NOMENCLATURE

A	Cross-sectional area of a room; Gross area of an opening [m^2]
Ar	Archimedes number
C	Discharge coefficient of an opening; Heat capacity [J/kg ·˚C]; Contaminant concentration
$\bar{C}$	Volumetric averaged contaminant concentration
F	Volumetric flow rate [m^3/s]
g	Acceleration of gravity [m/s^2]
Gr	Grashof number
h	Height from the inflow opening to the outflow opening [m]
K	Jet centerline velocity constant
L	Characteristic length of the room [m]
m	Index of non-uniformity
n	Exponent
N	Total number of the sample cells
Q	Heat [W]; Volumetric inflow rate [m^3/s]
R	Ratio of free area to gross area of an opening
Re	Reynolds number
St	Stanton number
T	Temperature [Kelvin]; Jet throw length [m]
ΔT	Temperature gradient [Kelvin]
U	Velocity [m/s]
V	Volume [m^3]
X	Distance from an inflow to an outflow opening [m]

Greek

β	Volumetric thermal expansion coefficient
ρ	Air density
τ	Dimensionless temperature gradient
υ	Kinematic viscosity of the air [m^2/s]

Subscript

av	average
b	bottom of the zone
c	characteristic
d	distance; discharge
exp	experiment
fa	free area
i	cell i
in	inflow; input
max	maximum
p	pressure
t	top of the zone

REFERENCES

ASHRAE. 2005. ASHRAE Handbook of Fundamentals: American Society of Heating, Refrigerating and Air-Conditioning Engineers.

Clarke J. A. 2001. "Domain integration in building simulation," Energy and Buildings 33(4): 303-308.

Emmerich S. J. 2001. "Validation of multizone IAQ modeling of residential-scale buildings: a review," ASHRAE Transactions 107(2): 619-628.

Finlayson, U. E., Gadgil, J. A., Thatcher, L. T., Sextro and G. R. 2004. "Pollutant dispersion in a large indoor space. Part 2: Computational fluid dynamics predictions and comparison with a scale model experiment for isothermal flow," Indoor Air 14(4): 272-283.

Gadgil A., Lobscheid C., Abadie M. O. and Finlayson E. U. 2003. "Indoor pollutant mixing time in an isothermal closed room: an investigation using CFD," Atmospheric Environment 37(39-40): 5577-5586.

Gao Y. and Chen Q. 2003. "Coupling of a multi-zone airflow analysis program with a computational fluid dynamics program for indoor air quality studies," The 4th International Symposium on HVAC, Symposium on HVAC, Tsinghua University, Beijing, China.

Incropera F. P. and DeWitt D. P. 2007. Fundamentals of heat and mass transfer. Hoboken, NJ: John Wiley.

Kotani H., Satoh R. and Yamanaka T. 2003. "Natural ventilation of light well in high-rise apartment building," Energy and Buildings 35(4): 427-434(8).

Reynolds A. J. 1986. "The scaling of flows of energy and mass through stairwells," Building and Environment 21(3-4): 149-153.

Schaelin A., Dorer V., Maas J. v. d. and Moser A. 1994. "Improvement of multizone model predictions by detailed flow path values from CFD calculations," ASHRAE Transactions 100(Part 2): 709-720.

Srebric J., Chen Q. and Glicksman L. R. 1999. "Validation of a zero-equation turbulence model for complex indoor airflows," ASHRAE Transactions 105(2): 414-427.

Thatcher, L. T., Wilson, J. D., Wood, E. E., Craig, J. M., Sextro and G. R. 2004. "Pollutant dispersion in a large indoor space: Part 1 - Scaled experiments using a water-filled model with occupants and furniture," Indoor Air 14(4): 258-271.

Upham R. 1997. A validation study of the airflow and contaminant migration computer model CONTAM as applied to tall buildings. University Park, Pennsylvania, The Pennsylvania State University.

Walton G. N. and Dols W. S. 2003. CONTAMW 2.1 user manual. Gaithersburg, MD, USA: National Institute of Standards and Technology.

Wang L. and Chen Q. 2006. "Validation of a coupled multizone and CFD program for building airflow and contaminant transport simulations " Accepted by HVAC&R Research.

Xue, H., Hihara, E., Saito and T. 1994. "Temperature stratification of heated airflow in a fire tunnel," JSME International Journal Series B-Fluids and Thermal Engineering 37(1): 187-194.

Yuan J. and Srebric J. 2002. "Improved prediction of indoor contaminant distribution for entire buildings," The 2002 ASME International Mechanical Engineering Congress and Exposition, New Orleans, Louisiana, USA.

Zhang J. S., Wu G. J. and Christianson L. L. 1993. "A new similitude modeling technique for studies of non-isothermal room ventilation flows," ASHRAE Transactions 99(1): 129-138.

Zohrabian A. S., Mokhtarzadehdehghan M. R. and Reynolds A. J. 1990. "Buoyancy-driven airflow in a stairwell model with through-flow," Energy and Buildings 14(2): 133-142.