A LOW-ORDER THERMAL BRIDGE DYNAMIC MODEL FOR BUILDING SIMULATION

Y.Gao1, J.J. Roux2, L.H. Zhao3

1 Department of Building Science, Tsinghua University, Beijing 100084, China;
2Thermal Sciences Center – Building Physics (CETHIL), National Institute of Applied Sciences Lyon, Claude Bernard University UMR CNRS 5008,
Domaine Scientifique de la Doua- Bât FREYSSINET, 40 rue des arts, 69 100 Villeurbanne, France
3Department of Architecture, South China University of Technology, Guangzhou 510640, China

ABSTRACT

The heat transfer characteristics in building window frame have significant three-dimensional characteristics, but window U-value model are often used in most building codes for thermal analyses to simplify the calculations. State model reduction techniques were used to develop low-order three-dimensional heat transfer model for window frame, which is efficient and accuracy. The method was validated for a envelope designs by comparing the with complete 3-D models.

KEYWORDS

Thermal bridge, model reduction, window frame, thermal transfer

INTRODUCTION

Insulating materials are widely used in new construction and modern building envelope has more and more complex internal and external structures. All these bring more relative heat loss caused from thermal bridge (TB). Different research studies have shown that thermal bridges can significantly reduce the thermal resistance of walls and roof assemblies. Thermal bridges have influence on the local temperature distribution and heat flow rate thus they play an important role in heat loss calculation particularly for high insulated building envelope. So, thermal simulation with former heat flow assumptions may lead to significant deviations in heating or cooling loads calculation of buildings. Evidently, thermal bridge additional heat loss with three-dimensional (3D) heat transfers model need to be include in new simulation programs.

Window is an important envelope component, heat loss due to window takes a great part compared with well insulated opaque wall. Evidently, wall insulation becomes discontinuously for the insertion of window frame. So, thermal bridge appears between well insulated wall and window frame due to this discontinue of insulation. And the influence of thermal bridge varies with different position of window in the wall and thermal properties of wall and window.

In this context, most general building thermal analyses programs simplify the thermal conduction of wall by assuming 1D heat flows through envelope and the thermal conduction through the window with overall loss coefficient. The effects of "thermal bridges" are not taken into account.

The aim of this paper is to present the importance of thermal bridge caused by window in well insulated wall and proposes a simplified low-order model which can include heat loss from thermal bridge. This model is based on reduced linear state model for describing the dynamic thermal behaviour of window and its connection with wall. The first part of our paper reports on numerical model main characteristics. This is followed by the description of the importance of thermal bridge at the position of window with its connection with wall. The final part of our study is focused on the discussion of possibility of using low-order model.

NUMERICAL APPROACH

As we already mentioned, under steady-state conditions, we can simply make use of tabulated U-values for different kinds of windows in order to carry out heat loss calculations. This is extremely valuable for general thermal engineering applications. Consequently, our aim is to succeed in proposing a simplified model for dynamic and multi-dimensional heat transfer within window and its connection with wall.

To reach this goal, we need to initially pass through complete multi-dimensional dynamic representation of window thermal behaviour (differential equations system with very high matrix order) and to use model size reduction techniques for assembling reduced models suitable for computer implementation under dynamic specified boundary conditions. These two main steps of our methodology will be discussed below.

Linear state space model

It is not very difficult to set up the differential algebraic equations for a conduction heat transfer problem, particularly if the thermo-physical properties of the materials are supposed as independent from temperature (this hypothesis is commonly considered as legitimate in heat transfer studies within buildings, as it is consistent with the degree of accuracy usually looked for). After a spatial discretization of the problem in the use of finite volumes, it leads to the following matrix state-space equation:

$$\dot{T}(t) = C^{-1}\Lambda T(t) + C^{-1}\Pi U(t) \qquad (1)$$

where:

- T(t) is the state vector (dimension: n,1) : approximation of temperature field, representing the temperature Ti of each node i within the spatial discretization.

- C is the square capacitance matrix (dimension: n,n), diagonal definite positive and Cii elements are the calorific capacities of each control volume.

- Λ is a square matrix (dimension: n,n), which translates the heat exchange between the different control volumes of the system. It is a symmetric matrix because of the reciprocity of the heat transfers.

- U(t) is a vector (dimension: p,1), which regroups the p solicitations acting on boundary of the studied domain (heat flows, surface or ambiance temperatures).

- Π is a rectangular matrix (dimension: n,p), which translates the thermal relations between the domain and its environment. It represents modes of action of solicitations among others on the system.

In addition, we are generally interested in the evolution of particular outputs or measures within the studied problem (e.g. “temperature at a specific point”, “heat flow leaving the domain”). All these outputs can be regrouped in one vector Y, expressed as linear combinations (with constant temperature coefficients) of T(t) and U(t) , according to Eq. (2).

$$Y(t) = JT(t) + KU(t) \qquad (2)$$

where:

-Y(t) is the output vector (dimension q: number of observed outputs).

-J is the rectangular observation matrix (dimension: q,n).

-K is a matrix of direct transmission (dimension: q,p) which translates the instantaneous actions of solicitations on outputs.

In this way, the dynamic behaviour of a thermal system can be clearly described by the matrixes set: C, Λ, Π, J, and K.

On the other hand, in the case of multi-dimensional thermal analysis of thermal bridge, the order of matrixes system is very high because of its numerous differential equations. Moreover, the CPU time for dynamic simulation processes of one multi-dimensional thermal system will be often intensive if we simply intend to apply the high-order complete matrix model. For this reason, it is necessary to find a simple, low-order model to replace the complete model. This reduced model will facilitate also its integration within building simulation codes.

Linear state space model reduction

The objective of model size reduction methods is to substitute the original complete state model including n ordinary differential equations (n is commonly referred to as complete model order) by a significantly smaller one (reduced model of order m<<n) without sacrificing vital characteristics of the physical system.

It is worthwhile to mention that model reduction has been the subject of numerous research endeavours and a great number of methods have already been represented over the last 30 years (J.J.Roux 1993**)**. The approaches widely used in the field of building thermal analysis are based on spectral methods (E Palomo et al. 2000). These methods represent the solution of problems as a linear combination of eigenfunctions on a particular basis. A low-order model is obtained by truncation of the equivalent high-order model coming from the previous state-space coordinates change. Many methods for models reduction are based on this principle: modal basis truncation methods (Marshall 1966) and aggregation methods (L.H.Zhao 2001, G.P.Michaïlesco 1979) or balanced transformation methods (B.C.Moore 1981). The interested reader can find thoroughly descriptions of these methods in (C. Menezo 1999).

Balanced realisation method (inner symmetrisation)

This method is based on the controllability and observability concepts used in automatic control (detailed descriptions of these notions are given in (B.C.Moore 1981)). We can simply define the controllability as the possibility to acquire and vary the model states by means of the system inputs, while the observability as the possibility to establish the model states using the outputs (F.Déqué et al 2001). As a result, the model size reduction can be fulfilled by eliminating the state variables characterized by either a powerless degree of controllability or a minor degree of observability. The controllable and observable state elements are defined by their controllability gramian W_C and observability gramian W_O. These matrixes (W_C and W_O) are solutions of the following Lyapunov equations:

$$\begin{cases}(C^{-1}\Lambda)W_C+W_C(C^{-1}\Lambda)^T=-(C^{-1}\Pi)(C^{-1}\Pi)^T\\(C^{-1}\Lambda)^T W_O+W_O(C^{-1}\Lambda)=-J^T J\end{cases} \quad (3)$$

Nevertheless, the controllability and the observability gramians strongly depend on the state-space coordinates chosen for system behaviour description. Therefore, they are definitely not system invariant. In addition, it occurs that there are state variables with a weak degree of observability and at the same time with a dominate grade of controllability, and vice versa. In order to prevent these difficulties, the initial system, corresponding to Eqs. (1) and (2), is put through a particular similar transformation to "balance" the degrees of controllability and observability. Hence, the resulting equivalent system includes merely state variables with the same degree of controllability and observability. This means that the concept of "balancing technique" is to be able to carry out the model reduction using identical controllability and observability matrixes and equal to a diagonal matrix:

$$W_C=W_O=\Sigma=\mathrm{diag}(\omega 1, \omega 2, ..., \omega n) \quad (4)$$

where $\omega 1 \geq \omega 2 \geq ... \geq \omega n > 0$ are the so-called Hankel singular values; they are the square roots of the product WCWO eigenvalues.

In a balancing model, the state components corresponding to small Hankel singular values require much energy to be reached, while at the same time producing little energy on the output. Therefore, if there is a $s \in \{1, ..., n-1\}$ for which $\omega s >> \omega s+1$, the modal elements corresponding to ωm, $m=s+1, ..., n$ can be truncated from the system description, preserving only those features of the dynamics that are most relevant to observable and controllable modes. According to this manner of system representation, the model reduction is performed by simple removal of the model elements related to the state variables showing weak controllability and observability grades.

The system representation in the "balanced" base (after the calculation of controllability and observability matrixes) can be written as follows:

$$\begin{Bmatrix}\dot{X}_1\\\dot{X}_2\end{Bmatrix}=\begin{bmatrix}\Omega_{11} & \Omega_{12}\\\Omega_{21} & \Omega_{22}\end{bmatrix}\begin{Bmatrix}X_1\\X_2\end{Bmatrix}+\begin{bmatrix}B_1\\B_2\end{bmatrix}U \quad (5)$$

The reduction model, taking into account the m modes the most controllable and observable, is expressed by the following equation:

$$\begin{cases}\dot{X}_{MO}=A_{MO}X_{MO}+B_{MO}U\\Y=H_{MO}X_{MO}+D_{MO}U\end{cases} \quad (6)$$

where A_{MO}, B_{MO}, H_{MO}, D_{MO} are reduced system mode matrixes given by the balanced method:

$$A_{MO}=\Omega_{11}-\Omega_{12}\cdot\Omega_{22}^{-1}\cdot\Omega_{21}$$

$$\begin{aligned}B_{MO}&=B_1-\Omega_{12}\cdot\Omega_{22}^{-1}\cdot B_2\\H_{MO}&=H_1-H_2\cdot\Omega_{22}^{-1}\cdot\Omega_{21}\\D_{MO}&=D-H_2\cdot\Omega_{22}^{-1}B_2\end{aligned} \quad (7)$$

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NUMERICAL SIMULATION

In most general building thermal analyses programs, window and the wall connected with this window are separated for simplifying modeling description. Heat flow of thermal conduction through window is the product of interior and exterior air temperature difference and U-value. Heat conduction through the wall connected with window is modeled in one-dimensional. The boundary between window and wall is supposed as adiabatic. Evidently, heat conduction between window and wall is ignored. For taking account of heat flow between window and wall, multi-dimensional heat model is necessary. Generally speaking, heat flow between window and wall is not very great for no-isolation wall, but for well isolated wall, this heat flow is important owing to its heat bridge effect. So, the necessity for well isolated wall using multi-dimensional heat model is firstly revealed by two cases example.

The non-linearity due to the coupling between heat transfers by conduction, natural convection (between air and surfaces of cavities) and long wave radiation (among surfaces of cavities) within cavities of window frame and cavities in glazing requires thorough studies of the heat transfer mechanisms. To deal in a practical manner with this combined heat transfer phenomena inside cavities, most of the available studies are based on the equivalent thermal conductivity. Consequently, this method is also taken into account within our study in order to simplify the heat transfer analysis in blocks cavities. The equivalent thermal conductivity of an unventilated space between glass panes in glazing and air cavities in frame shall be determined according to EN 673 (2003) and prEN ISO 10077-2(2000). So, Thermal properties of different compounds are shown in Table1

Two cases are studied for comparing the difference for evaluating the affects of thermal bridge. The first one is window with its connection of well insulated wall, Figure1. Heat conduction model is in two-dimensional, which can take account of heat flow between window and wall. The second one is the case which includes two parts separately, the part of window including frame and glazing and the other part of wall. In this case, U-value and one-dimensional model are used for window and wall respectively and heat flow between window and wall is considered as zero.

In Case1, heat flow through window and wall is Q1. For the Case2, heat flow through window and wall are calculated respectively, and the sum of the two parts heat flow is marked as Q2. The difference between Q1 and Q2 can be considered as heat flow

Q3 resulting from thermal bridge, which is ignored by Case2.

Under static state, interior surface heat flows of two cases are shown in Table2. Results represent heat flow from thermal bridge is important portion and cannot be ignored in simulation.

Clearly, multi-dimensional heat transfer model is necessary for reaching simulation results including Q3 in dynamic simulation codes. However complete multi-dimensional heat transfer model (Eq. 1 and 2) is not practical for its high-order. Order reduced low-order model is induced as being mentioned before (Eq. 8) for a window frame shown as Cas1. The model obtained after reduction for the Cas1 configurations has only 10 modes (their complete models had 1218 modes), shown in Table3.

Table1 Thermal Properties of wall and window

	Wall	Insulation	Window Frame (equivalent)	Glazing (equivalent)
Thermal conductivity (W/m^2·K)	1.75	0.05	0.17	0.12

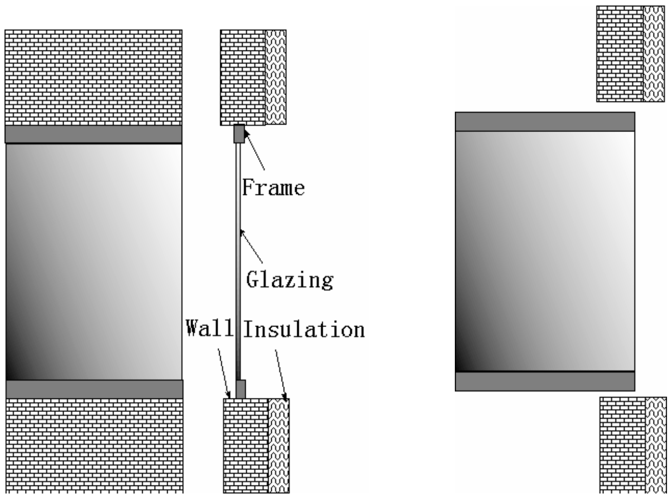

Figure 1 Window configuration

Table 2 Comparison of surface heat flow

Interior air Temperature (°C)	Exterior air Temperature (°C)	Interior Surface Resistance (m^2 K/W)	Exterior Surface Resistance (m^2 K/W)	Q1 (W)	Q2 (W)	Q3 (W)
20	0	0.13	0.04	140.1	104.3	35.8

Table3 Simulation parameters

	Step amplitude of linear temperature excitation (°C)	Interior Surface Resistance (m^{2}K/ W)	Exterior Surface Resistance (m^{2}K/ W)	Complete Modes	Reduced Modes
Cas1	20.0	0.13	0.04	1218	10

RESULTS AND DISCUSSION

Since the main objective of our study is the capacity and the precision of our reduced model to predict the thermal behaviour of window with its connection of wall, we present the full-order model / low-order model confrontations regarding the results of the example of Cas1 configurations.

The first comparison is under exterior air temperature step excitation, shown in Table3.

We present in Fig. 2 numerical data (heat flow outputs of Case1) from the complete and the reduced model for Cas1 configuration. This figure show the Reduced model and Complete model nearly have the same output values(Heat flow through window with its connection of wall). The reduced model (defined by 10 modals) has high efficiency as the relative errors between the full-order models and low-order models are constantly less than 0.01%.

The second comparison, exterior air temperature is provided by January weather data in Beijing and heating set-point temperature is 20°C.

The results from the complete and the reduced model are nearly the same as shown in Fig.3, the difference between these two models are less than 0.1W.

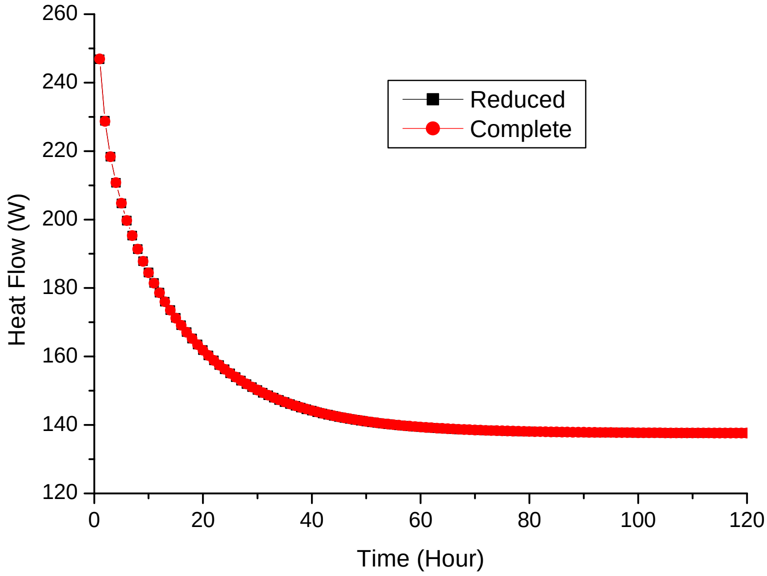

Figure 2 Heat flow results from two models under step amplitude

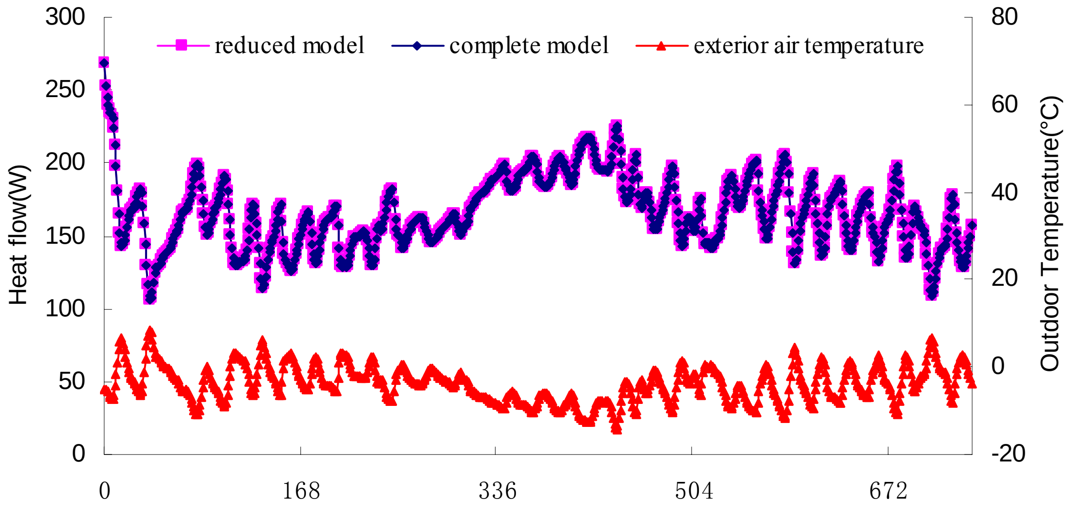

Figure 3 Heat flow results from two models under weather condition

CONCLUSION

In conclusion, thermal bridge appears between well insulated wall and window frame due to this discontinue of insulation. Heat flow from thermal bridge is important portion and cannot be ignored in simulation. Based on reduced linear state model, it is possible for describing the dynamic thermal behaviour of window and its connection with wall efficiently. The simulation results by reduced model have little difference from complete model, and it is possible replace complete model in simulation. The interest of this methodology also lies in the fact which is general and can apply to all the components of envelope, such as hollow block (Y.Gao 2004). In addition its numerical implementation in the computer codes of simulation of the thermal behavior of the building is also very easy.

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