

## SENSOR FAULT DETECTION AND DIAGNOSIS FOR VAV SYSTEM BASED ON PRINCIPAL COMPONENT ANALYSIS

Xiaowen Yi, Youming Chen

College of Civil Engineering, Hunan University,  
Changsha, Hunan 410082, China

### ABSTRACT

VAV system is a very complicated one in air-conditioning systems, thus automatic control become the key of such a system. As necessary components in automatic control system, sensor has failure risk. It is so expensive that detect sensor fault by hardware redundancy in comfortable air-conditioning system. This paper presents an approach, Principal Component Analysis (PCA), to detect and identify sensor fault in VAV system. The PCA model partitions the measurement space into a principal component subspace (PCS) where normal variation occurs, and a residual subspace (RS) that faults may occupy. When the actual fault is assumed, the maximum reduction in the squared prediction error (SPE) is achieved. A fault-identification index was defined in terms of SPE. Some examples were provided to prove this method is feasible. This paper also presents a fault reconstruction algorithm to reconstruct the identified faulty data.

### KEYWORDS

Sensor fault, Principal component analysis, Residual subspace, Squared prediction error, Fault reconstruction, VAV system

### INTRODUCTION

As VAV (Variable Air Volume) system becoming more and more popular in modern buildings, on-line monitoring of the process are getting important for comfortable air quality, lower energy consumption and fault detection. Studies have shown that twenty or thirty percent of energy consumption can be saved in small VAV system of business building after fault debugged and corrected [ANNEX34 Final Report 2001]. FDD (Fault Detection and Diagnosis) system can alarm the system operation failure and reconstruct sensor faults, thus sensor fault can be removed in time and then the unnecessary shutting down caused by sensor fault is reduced, so system energy consumption can be reduced. Meanwhile, FDD system is useful to maintain comfortable indoor air environment and improve indoor air quality [Youming Chen and Shengwei Wang 2001]. Consequently, it is beneficial to the service life and the maintenance expenses of equipment. But there are few of work reported in literature on use FDD tools in VAV system. This paper present an approach,

Principal Component Analysis (PCA), for fault detecting and diagnosing sensor fault in VAV system.

PCA is a traditional multivariate statistical analysis tool [K. Pearson 1901], which can reduce the dimensions of the source data by projecting the data onto a lower-dimensional space. In 1990s, PCA was applied to supervise the automatic process in industry by MacGregor [MacGregor J. F. 1996]. Subsequently, more and more research about the application of PCA in all kinds of process monitoring are published, such as Dunia [Ricardo Duina 1998] and Qin [S. Joe Qin 1997] applied PCA to supervise the boiler process and the air emission monitoring, Zhang [Haitao Zhang 1999] used PCA to monitor dynamic multivariate processes at different scales. Pranatyasto [Toto Nugroho Pranatyasto 2001] used PCA to validate sensors for FCC (Fluid Catalytic Cracker) system, José Camacho [José Camacho 2006] used PCA to online monitor the batch processes.

This paper will employ PCA for sensor fault detection and diagnosis in VAV system. And the remaining of this paper will be organized as follow: section 2 introduce the PCA method; section 3 discuss about how to use PCA to detect and identify sensor fault; section 4 discuss a method to reconstruct faulty sensor data and an approach to choosing the best number of principal components for PCA model; section 5 will apply PAC method in a simulation VAV system for sensor fault detection and identification; section 6 comes to conclusions and proposes the further work.

### PCA METHOD

PCA method is multivariate statistical analysis technique, this section will introduce PCA method in brief. First, Let's assume there is an sample vector  $x$  (can be a group of sensors), the variable number of  $x$  is  $m$  ( $x \in R^m$ ) and the sample number is  $n$ , so the data matrix  $X \in R^{n \times m}$  (is the set of the measurement data of  $x$ ). PCA method decomposes the matrix  $X$  as a bilinear product of two matrixes,  $T$  and  $P$ ,

$$X = TP^T + E \quad (1)$$

where,  $T$  represents the score matrix,  $T \in R^{n \times l}$ ,  $P$  represents the loading matrix,  $P \in R^{m \times l}$ ,  $l$  represents the number of principal components,  $E$  represents the

residual matrix (including mainly noise of the system under normal conditions).

The columns of loading matrix P consist of eigenvectors of the correlation matrix R (denotes the covariance matrix of normal data) and just associate with the l largest eigenvalues, and the columns of matrix  $\tilde{P}$  are the remain m-l eigenvectors of R. Generally the correlation matrix R can be derived from normal process data. And the correlation matrix of matrix x can be approximately calculated as follows [Ricardo Dunia 1996].

$$R \approx \frac{X^T X}{n-1} \quad (2)$$

The residual E can be expressed as Eq.(3) [S.Joe Qin 1997],

$$E = \tilde{T}\tilde{P}^T \quad (3)$$

Through Eq.(1)-(3), a PCA model can be built. A new sample vector x can be decomposed into two parts by PCA model.

$$x = \hat{x} + \tilde{x} \quad (4)$$

Where:  $\hat{x} = PP^T x$ ,

$$\tilde{x} = \tilde{P}\tilde{P}^T x$$

$\hat{x}$  is the projection on the principal component subspace (PCS) and  $\tilde{x}$  is the projection on the residual subspace (RS), matrixes  $PP^T$  and  $\tilde{P}\tilde{P}^T$  are their projection matrixes, they can be rewritten as C and  $\tilde{C}$ , respectively.

From mentioned above, a PCA model was built. Through the PCA model, a sample matrix can be decomposed as the sum of the projection on the PCS and the projection on RS.

## FAULT DETECTION AND IDENTIFICATION

### **Fault detection**

After a PCA model is built for a system, the projection matrixes can be used for fault detection. Generally, under normal condition, measurement vector x should be mostly projected on PCS and simultaneously, the projection on RS is very small. But when the measurement vector x contains some kind of data under fault process, the projection on RS will be increased, which cause the squared prediction error (SPE) to increase remarkably in certain confidence limit  $\delta^2$ . SPE can be expressed as follows<sup>[5,6]</sup>.

$$SPE(x) = \|\tilde{x}\|^2 = \|\tilde{C}x\|^2 = x^T(I-C)x \quad (5)$$

Where:  $\|\cdot\|$  is Euclidean norm.

If  $SPE > \delta^2$ , fault may occur in the system. On the contrary, if  $SPE \leq \delta^2$ , the system is considered normal. The confidence limit  $\delta^2$  can be calculated by Eq.(6) [S. Joe Qin 1997] and more detail can be seen in [Jackson, J.E. 1979].

$$\delta_\alpha^2 = \theta_1 \left[ \frac{c_\alpha \sqrt{2\theta_2 h_0^2}}{\theta_1} + 1 + \frac{\theta_2 h_0 (h_0 - 1)}{\theta_1^2} \right]^{\frac{l}{h_0}} \quad (6)$$

Where:  $h_0 = 1 - \frac{2\theta_1\theta_3}{3\theta_2^2}$ ,

$$\theta_i = \sum_{j=l+1}^m \lambda_j^i, \quad i=1, 2, 3,$$

l is the number of the principal components of the model,  $c_\alpha$  is the confidence limit for the  $\alpha$  in a normal distribution,  $\lambda$  is the eigenvalue of the covariance matrix R.

### **Fault identification**

After a sensor fault is detected it is necessary to know which sensor becomes faulty, that is, to identify the faulty sensor. In this section, a fault identification approach is introduced. Firstly, it is assumed that only one sensor fault occurs in the system process simultaneously (the probability of two or more sensor faults occur in a system at the same time is very low). When sensor fault occurs in the measurement data, the sample vector x can be represented as follows [S. Joe Qin 1997].

$$x = x^* + f\xi_i \quad (7)$$

Where:  $x^*$  denotes the portion of normal data, f is the magnitude of the fault,  $\xi_i$  is the direction vector of unit length for the faulty sensor. For an example,  $\xi_2 = (0 \ 1 \ 0 \ \dots \ 0)$  represents a failure occurring in the second sensor.

For searching the fault direction, a sensor validity index (VI) should be introduced. VI can be defined as follows.

$$VI_j = \frac{SPE(x_j^*)}{SPE(x)} \quad (8)$$

Where:  $x_j^*$  is a vector of the measurement vector x reconstructed along the j th direction (In the next section, it will be discussed how to reconstruct  $x^*$ ).

Apparently,  $VI \in [0, 1]$  because of  $SPE(x) \cong SPE(x^*)$ . When  $VI_j$  is close to 0, it indicates that the

fault direction is  $j$  th. On the contrary, when  $VI_j$  is close to 1, it means that the  $j$  th is not the fault direction.

**FAULT RECONSTRUCTION AND THE OPTIMAL NUMBER OF PRINCIPAL COMPONENT**

**Fault reconstruction**

History data under normal conditions is used to build a PCA model. It is assumed that fault direction is known. An optimal estimation can be searched for the fault vector  $x$  by the PCA model and the fault direction. As mentioned before,  $\hat{x}$  can be considered as the projection of  $x$  on the PCS, thus  $\hat{x}$  can be considered as an estimation of  $x$  here. However,  $\hat{x}$  is not the optimal estimation of  $x$  because  $x$  contains some faulty data while it is used to estimate  $\hat{x}$ . If  $x$  is replaced by the estimated value  $x^{new}$  obtained last time,  $x^{new}$  will be close to the normal value  $x^*$  of the faulty sample vector  $x$ . Therefore, an iteration can be represented as follows.

$$\hat{x}_i^{new} = [x_{-i}^T \quad \hat{x}_i^{old} \quad x_{+i}^T] c_i = [c_{-i}^T \quad 0 \quad c_{+i}^T] x + c_{ii} \hat{x}_i^{old} \quad (9)$$

where,  $[c_{-i}^T \quad 0 \quad c_{+i}^T]$  is a vector of matrix  $C$  which the  $i$  th column of  $c_{ii}$  is substituted by 0.

It can be proved that iteration always converges to the following formula [Ricardo Duina 1996].

$$x_i^* = \frac{[c_{-i}^T \quad 0 \quad c_{+i}^T] x}{1 - c_{ii}} \quad (10)$$

Where:  $c_{ii} \neq 1$ . For  $c_{ii} = 1$  means that the variable can't be reconstructed by this method.

**The optimum principal component number**

It is very important to determine a proper number of principal components because it will affect the result of fault detection and diagnosis directly. If a fewer number of principal components (PCs) are used, the ability of detecting and diagnosing small faults is weakened because of  $\delta^2$  for SPE maybe so large. If more PCs are used, some faults maybe stay in the PCS and do not show enough effects in the RS. This will make it impossible to detect these faults. Furthermore, the number of PCs affects the accuracy of reconstruction. In this reserach, an optimal reconstruction method is introduced to determine the number of PCs [S. Joe Qin 1998].

The unreconstructed variance can be expressed in this method as Eq.(11).

$$u_j = \text{Var}(x_j - x_j^*) = \frac{\xi_j^T (I - C) R (I - C) \xi_j}{[\xi_j^T (I - C) \xi_j]^2} \quad (11)$$

Where:  $\xi_j$  is the fault direction of the sample vector.

The optimum number of PCs can be determined from minimizing the total value of the unreconstructed variance.

$$\text{Min}_l \left( \sum_{j=1}^m u_j \right) \quad (12)$$

Where,  $m$  is the number of measurement variables and  $l$  is the number of PCs.

So, the value of  $\sum u_j$  can be calculated with different number of PCs, and then the number of PCs which corresponding to the minimum  $\sum u_j$  is the optimum number of PCs.

**SIMULATION APPLICATION FOR USING PCA FOR SENSOR FAULT DETECTION AND DIAGNOSIS**

Since lots of sensors are used in VAV system, consequently, there is a risk of sensor fault in processing. The aim of this research work is to detect and diagnosis sensor fault in VAV system. The principal figure of simulation control for VAV system is provided in Figure 1. This system is verified through other documents, such as document [Shengwei Wang 1999]. Lots of sensors are used in VAV system to realize automatic control of the system. In this paper, seven relative sensors are selected for this research objective. There are respectively outdoor air flow sensor, total supply air flow sensor, return air flow sensor, outdoor air temperature sensor, VAV supply air temperature and CAV(constant air volume conditioner) supply air temperature sensor. Data derived from those seven sensors under normal conditions through simulation are used to build PCA model. Outdoor air flow sensor fault is used to demonstrate how to detect, identify and diagnose fault. The procedure for the remaining sensor is similar.

As aforementioned, the number of PCs is evidently very important while building a PCA model. Fig.2 shows that the optimum PCs number in the PCA model built by normal history data derived from simulation is three.

To affirming the number of PCs for the model is reasonable or not, data derived from normal conditions was used to test. The SPE value and the VI value can be seen as a test target here. Fig.3 is the value of SPE under normal conditions and Fig.3 shows that the value of SPE is stay in the confidence

limit  $\delta^2$ . It's means that the system is under normal conditions.

Fig.4 is the VI value of the outdoor air flow sensor. Fig.4 shows that the value of VI is very close to 1 which indicates that the outdoor air flow sensor is ....

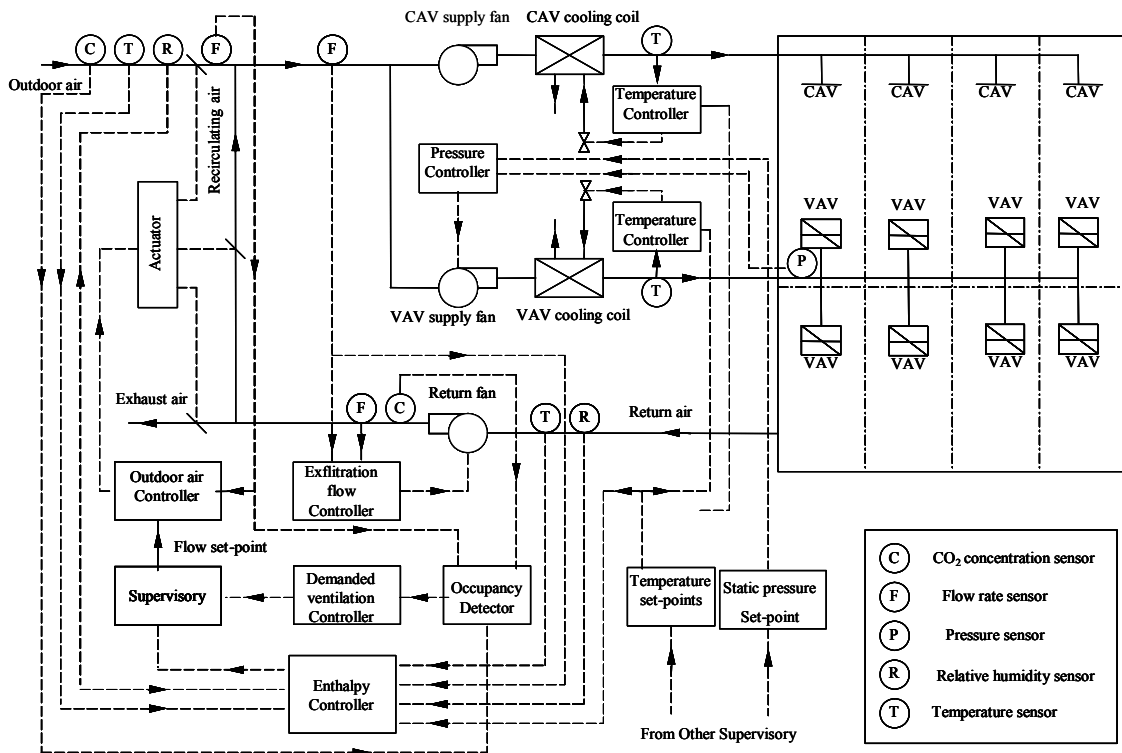


Fig.1 Simulation control sketch of VAV system

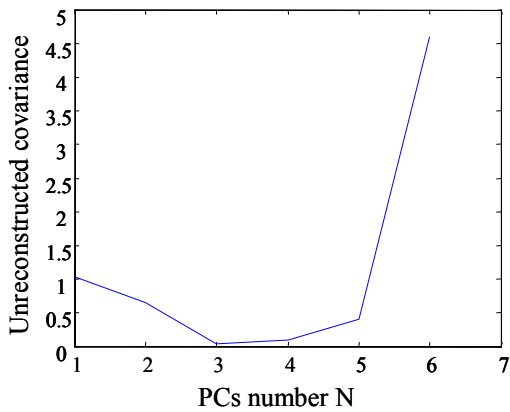


Fig.2. Optimum PCs number

normal, thus the PCs number is reasonable and effective.

For validating, a bias fault was added to the outdoor air flow sensor at certain time in simulation process; the SPE value and the VI value under the conditions are shown in Fig. 5 and Fig.6 respectively.

Fig.5 shows that the SPE value was increased and obviously beyond the value of the confidence limit while fault was added to the system. It's means that the fault can be detected by PCA method.

Fig.6 shows that the VI value of outdoor air flow sensor changes to close to zero while sensor fault occurred from close to one under normal conditions. It indicates that the outdoor air flow sensor is in failure while fault was added in the simulation process.

From Fig.5 and Fig.6, fault in outdoor air flow sensor was detected and identified. Obviously PCA method can detect and identify such kind of sensor fault.

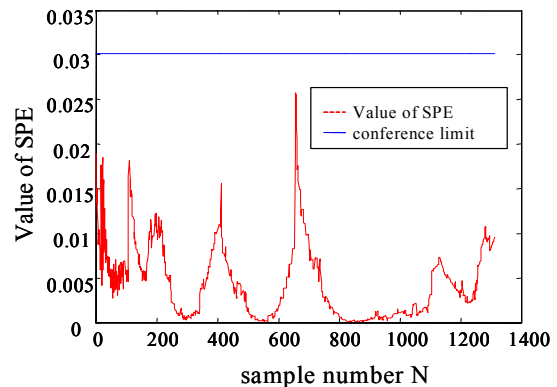


Fig.3. SPE under normal conditions

The fault reconstruction algorithm is used and the SPE value before and after fault reconstruction is

compared in Fig.7. Obviously, the value of SPE after reconstruction stays in the confidence limit while outdoor air flow sensor fault was added in the simulation process. It indicates that the data of failure sensor can be reconstructed by normal history data. Consequently, sensor fault online diagnosis can be realized and HVAC system break caused by sensor fault can be decreased.

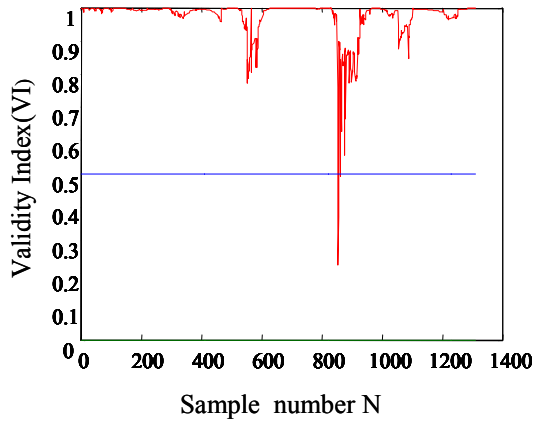


Fig.4. VI under normal conditions

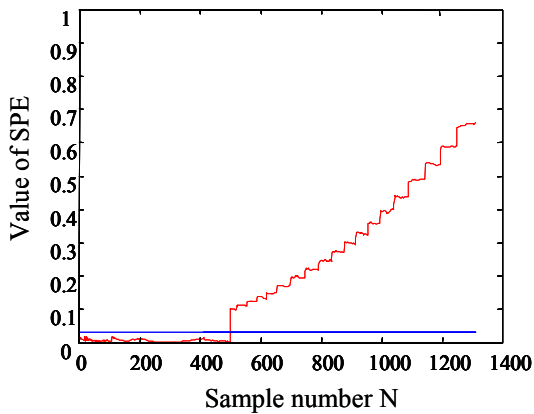


Fig.5. SPE under fault conditions

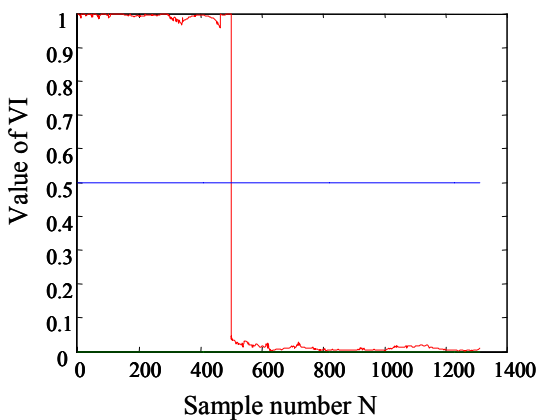


Fig.6. VI under fault conditions

## CONCLUSION

Sensor fault detection, identification and reconstruction for VAV system based on PCA method was proposed in this paper. PCA method partitions the measurement vector space into PCS and RS. If sensor fault occurred, the sample vector projection on RS...

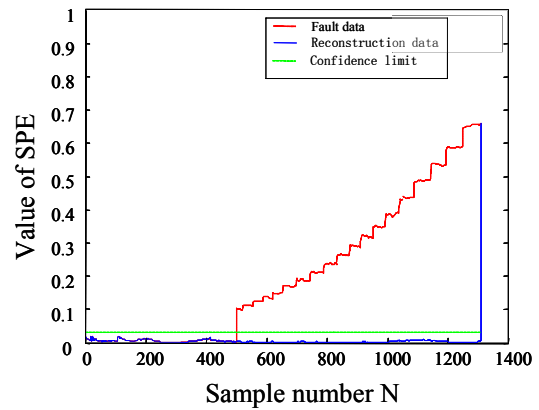


Fig.7 Value of SPE before and after reconstruction

will increase remarkably and consequently the value of SPE will increase and beyond the confidence limit, herein fault will be detected. A fault reconstruction method is introduced, which minimizes the value of total unreconstructed variance to determine the optimum PCs number of the PCA model. Whereafter, a sensor VI was introduced to identify faulty sensors.

A simulation case about how to use PCA method to detect and diagnosis sensor fault in VAV system was described and demonstrated. Results show that PCA method can detect and identify sensor fault well, and then reconstruct sensor faulty data by PCA model of VAV system. With PCA method, online sensor fault detect and diagnosis may be realized.

Further research shows that PCA method can detect collinear sensor fault in VAV system but can not identify it. Further research should be done to identify collinear sensor fault in VAV system.

## NOTATION

- C model projection matrix
- X sample matrix
- T score matrix
- P loading matrix
- E residual matrix
- R correlation matrix
- f fault magnitude
- l number of principal component
- m number of sensors
- n number of sample
- u unreconstructed variance
- x sample vector
- $x_j$  reconstructed vector

## GREEK LETTERS AND SYMBOLS

$\lambda$	eigenvalue of the covariance matrix R
$\delta$	confidence limit
$\xi_i$	fault direction
$\in$	belongs to
$\  \ $	Euclidean norm

## SUPERSCRIPTS AND SUBSCRIPTS

$\sim$	projected to the residual space
$\wedge$	projected to the principal component space
*	normal portion(uncorrupted portion)
i	subscript for the actual fault
j	subscript for the assume fault

## ABBREVIATION

Min	minimum
PC	principal component
PCA	principal component analysis
PCS	principal component subspace
RS	residual subspace
SPE	square prediction error
VI	validity index
VAV	variable air volume conditioner
CAV	constant air volume conditioner

## REFERENCES

- Annex34. "Computer-aided Evaluation of HVAC System Performance," final report, 2001.
- Haitao Zhang 1999. "Dynamic process monitoring using multiscale PCA," Proceeding of the IEEE Canadian conference on electrical and computer engineering, Alberta Canadian 1999.
- José Camacho 2006. "Online monitoring of batch processes using multi-phase principal component analysis," Journal of process controll . 2006(16): 1021-1035.
- Jackson J. E. and Mudholkarg 1979 "Control procedures for residuals associated with principal componentanalysis," Technometrics. 1979(21): 341-349.
- K. Pearson 1901. "On lines and planes of closest fit to systems of points in space," Philosophical Magazine. 1901(2): 559-572.
- MacGregor J. F. 1996. "Analysis, monitoring and fault diagnosis of industrial processes using multivariate statistical projection methods," Proceedings of IFAC 1996 Congress, San Francisco: Vol M, pp145-150.
- Ricardo Duina 1996. "Identification of Faulty Sensors Using Principal Component Analysis," AICHE Journal. 1996(42): 2797-2812.
- Ricardo Duina 1998. "Joint diagnosis of process and sensor faults using principal component analysis," Control Engineering Practice. 1998(6): 457-469.
- S. Joe Qin 1997. "Self-Validating Inferential Sensors with Application to Air Emission Monitoring," Ind. Eng. Chem. Res. 1997(36): 1675-1685.
- S. Joe Qin 1998. "Determining the Number of Principal Components for Best Reconstruction," IFAC Dynamic and Control of Process Systems. 1998: 357-362.
- Shengwei Wang 1999. "Dynamic simulation of building VAV air-conditioning system and evaluation of EMCS on-line control strategies," Building and Environment. 1999(34): 681-705.
- Toto Nugroho Pranatyasto 2001. "Sensor validation and Process fault diagnosis for FCC units under MPC feedback," Control Engineering Practice. 2001(9): 877-888.
- Youming Chen and Shengwei Wang 2001. "Sensor Fault Tolerant Control of Outdoor Air Flow Based on Neural Networks," Indoor Air Quality, Ventilation and Energy Conservation in Buildings. 2001(2): 231-242.