RELATIONSHIP BETWEEN ACCURACY OF HEAT CONDUCTION CALCULATION AND MATERIAL PROPERTIES OF BUILDING SLABS

Xiangqian Li[1], Youming Chen[1, 2], Jeffrey D. Spitler[2], Dan Fisher[2]

[1]College of Civil Engineering, Hunan University, Changsha, Hunan 410082, China
[2]School of Mechanical and Aerospace Engineering, Oklahoma State University, Stillwater, Oklahoma 74078-5016, USA

ABSTRACT

Conduction transfer function (CTF) is widely used to calculate conduction heat transfer in building cooling loads and energy calculations. It can conveniently fit into any loads and energy calculation techniques to perform conduction calculations. There are three methods, Laplace transform (LP) method, state-space (SS) method and frequency-domain regression (FDR) method to calculate CTF coefficients (CTFs). The limitation of the methodology possibly results in imprecise or false CTFs. This paper desires to seek an accurate and robust calculation method through the relationship between the calculation accuracy of the three methods and material properties of building slabs. Results show that the FDR method is more accurate than the LP and SS methods. The FDR method may be a better choice to calculate the building cooling/heating loads.

KEYWORDS

CTF coefficients; Laplace transform method; state-space method; frequency-domain regression method

INTRODUCTION

Conduction heat transfer through the building envelope is one of the principal components of space cooling/heating loads and energy requirements (Chen et al. 2006). In current building simulation programs such as DOE-2 (LBNL 1982), TRNSYS (2000), EnergyPlus (Crawley et al. 2001), as well as space cooling load calculations ASHRAE Loads Toolkits (Strand et al. 2001), the dynamic thermal behavior data of building slabs including thermal response factors, CTF coefficients (CTFs) or periodic response factors are calculated by various algorithms, and then utilized in conjunction with weather data to calculate the heat flow through the slabs (Falconer and Sowell 1993). The accuracy of the dynamic thermal behavior data directly affects the accuracy of the building loads and/or energy calculations. However, there are various potential factors such as the limitation of calculation methods, too big of an iteration step, low calculation precision, unconverged computational results, etc., which may lead to incorrect results in calculating the dynamic thermal behavior data. Pointed out by Spitler and Fisher (1999), computational inaccuracy sometimes occurs in calculating the dynamic thermal behavior data. Therefore, it is necessary to seek a precise calculation dynamic thermal behavior data solution for obtaining accurate space cooling/heating loads.

In cooling load and energy calculation, building simulation and energy analysis, conduction heat transfer is usually modeled as a one-dimensional, transient process with constant material properties (McQuiston et al. 2000). The simplified heat diffusion equation in Cartesian coordinates is shown in equation (1) (Incropera and DeWitt 1996).

$$\frac{\partial^2 T(x,\tau)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,\tau)}{\partial \tau} \tag{1}$$

where T, x, α and τ are the temperature, heat flow direction, thermal diffusivity and time, respectively. Fourier's law, equation (2), specifies the conduction heat flux in terms of the thermal conductivity of the material and temperature gradient across a differential thickness.

$$q = -k\frac{\partial T(x,\tau)}{\partial x} \tag{2}$$

where q and k are heat flux, thermal conductivity, respectively.

Since equation (1) is a partial differential equation, the system is usually solved numerically, often by means of conduction transfer function methods. CTFs represent the material's thermal response as determined by its material properties. The method results in a simple linear equation that expresses the current heat flux in terms of the current temperature and temperature and heat flux histories as shown in equations (3) and (4) (McQuiston et al. 2000).

$$q_{o,\theta} = -\sum_{n=0}^{N_y} Y_n T_{is,\theta-n\delta} + \sum_{n=0}^{N_x} X_n T_{os,\theta-n\delta} + \sum_{n=1}^{N_\phi} \phi_n q_{o,\theta-n\delta} \tag{3}$$

$$q_{i,\theta} = -\sum_{n=0}^{N_z} Z_n T_{is,\theta-n\delta} + \sum_{n=0}^{N_y} Y_n T_{os,\theta-n\delta} + \sum_{n=1}^{N_\phi} \phi_n q_{i,\theta-n\delta} \tag{4}$$

where q_o and q_i are heat flux at exterior and interior surface, respectively. X_n, Y_n and Z_n are surface-to-surface exterior, cross and interior CTFs, respectively. T_{is} and T_{os} are interior and exterior surface temperature, respectively. N_x, N_y and N_z are number of exterior, cross and interior CTFs terms, respectively. ϕ_n is flux coefficient. N_ϕ is the number of flux history terms.The subscript θ represents the current time, and δ is time step. The zero subscript represents a current value.

The linear relationship can greatly reduce computational effort and facilitate computer implementation of the method. Since CTFs are temperature independent, they are usually pre-determined. Pre-calculated CTFs of some typical constructions are available in the literature ASHRAE 1997 and 2001.

CALCULATION PRINCIPLE

While there are a number of numerical methods for solving equations (1) and (2), the Laplace transform (LP) method and the state-space (SS) method are the most widely used in cooling load and energy calculations. The frequency-domain regression (FDR) method is developed recently (Chen et al. 2003).

LP Method

Hittle (1979) introduced a procedure to solve the conduction heat transfer governing equations (1) and (2) by using the LP method. The system in the Laplace domain is shown in equation (5).

$$\begin{bmatrix} q_i(s) \\ q_o(s) \end{bmatrix} = \begin{bmatrix} \dfrac{D(s)}{B(s)} & \dfrac{-1}{B(s)} \\ \dfrac{1}{B(s)} & \dfrac{-A(s)}{B(s)} \end{bmatrix} \begin{bmatrix} T_i(s) \\ T_o(s) \end{bmatrix} \qquad (5)$$

where $A(s)$, $B(s)$ and $D(s)$ are overall transmission matrices that depend on material properties, and/or film coefficients.

Response factors are generated by applying a unit triangular temperature pulse to the inside and outside surface of the multilayered slab. Response factors are defined as the discretized heat fluxes on each surface due to both the outside and inside temperature pulse. Response factors are an infinite series. Hittle also described an algebraic operation to group response factors into CTFs, and to truncate the infinite series of response factors by the introduction of flux histories coefficients. A convergence criterion shown in equation (6) is used to determine whether the numbers of CTFs and flux history terms are sufficient such that the resulting CTFs accurately represent response factors. Therefore, through Laplace inverse transform of transfer functions $D(s)/B(s)$, $-1/B(s)$ and $-A(s)/B(s)$ with unit triangle temperature, response factors and CTFs can be worked out.

$$\sum_{n=0}^{N_x} X_n = \sum_{n=0}^{N_y} Y_n = \sum_{n=0}^{N_z} Z_n = U(1 - \sum_{n=1}^{N_\phi} \phi_n) \qquad (6)$$

where U is the overall heat transfer coefficient.

SS Method

The use of the SS method in solving the governing equations (1) and (2) was introduced by Jiang (1982), Seem (1987). The SS expression relates the interior and exterior boundary temperatures to the inside and outside surface heat fluxes at each node of a multi-layered slab as shown in equations (7) and (8).

$$\begin{bmatrix} dT_{si}/d\tau \\ \vdots \\ dT_{so}/d\tau \end{bmatrix} = \boldsymbol{a} \begin{bmatrix} T_{si} \\ \vdots \\ T_{so} \end{bmatrix} + \boldsymbol{b} \begin{bmatrix} T_i \\ T_o \end{bmatrix} \qquad (7)$$

$$\begin{bmatrix} q_i \\ q_o \end{bmatrix} = \boldsymbol{c} \begin{bmatrix} T_{si} \\ \vdots \\ T_{so} \end{bmatrix} + \boldsymbol{d} \begin{bmatrix} T_i \\ T_o \end{bmatrix} \qquad (8)$$

where T_s is the temperature of each node. ***a*, *b*, *c*, *d*** are coefficient matrices that depend on material properties, and/or film coefficients.

Coefficients matrices ***a*, *b*, *c*, *d*** can be worked out through numerical computation. And then CTFs can be obtained by Leverrier's algorithm (Seem 1987) from coefficients matrices ***a*, *b*, *c*, *d***.

FDR Method

The use of the FDR method in solving the governing equations (1) and (2) was introduced by Chen et al. (2003). Base on Laplace transform, transmission matrix of a single layer slab can be obtained. The interior, cross and exterior heat transfer frequency characteristics of a multilayer slab are calculated through transmission matrix multiplication by substituting s with $j\omega$ (where, $j = \sqrt{-1}$). It is very easy to calculate the frequency characteristics of a multilayer slab. The FDR method is used to estimate some simple s-transfer functions in the form of equation (9) from the interior, cross and exterior frequency characteristics.

$$G(s) = \frac{\beta_0 + \beta_1 s + \beta_2 s^2 + \cdots + \beta_r s^r}{1 + \alpha_1 s + \alpha_2 s^2 + \cdots + \alpha_m s^m} \qquad (9)$$

where α_i and β_i are real coefficients. r and m are the orders of the numerator and denominator.

The simple s-transfer functions are in the forms of the polynomial ratio of variable s. For short, they are called polynomial s-transfer functions. In the FDR method, by minimizing the sum of the square error between the frequency characteristics of the multi-layer slab and the polynomial s-transfer function at all frequency points, the coefficients of the polynomial s-transfer function are easily obtained by solving a set of linear equations. The frequency characteristic of the polynomial s-transfer functions is evaluated by substituting s with $j\omega$. Through Laplace inverse transform of interior, cross and exterior simple polynomial s-transfer functions $G(s)$ with unit triangle temperature, response factors and CTFs can be easily and accurately worked out.

VERIFICATION OF CTFS ACCURACY AND THE TEST PROCEDURE

The heat flux comparison method is employed here. This method verifies CTFs accuracy by checking difference between heat flux calculated by CTFs and an analytical solution. The analytical method is explained as follows.

The conduction governing equations (1) and (2) are usually not solved analytically in building thermal load and energy calculations due primarily to the computational intensity of the implementation. However, with a periodic temperature boundary condition on one side of the slab and a constant temperature boundary condition on the other side, the analytical solution is tractable. Spitler et al. (2001) presented an analytical solution for a multi-layered slab subject to a sinusoidal outside temperature and a constant inside temperature. For single-layered slabs of thickness L, the inside temperature and heat flux are related to the outside temperature and heat flux by the following set of equations:

$$\begin{bmatrix} T_i \\ q_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_1 \end{bmatrix} \begin{bmatrix} T_o \\ q_o \end{bmatrix} \quad (10)$$

where

$$m_1 = \cosh(p + jp) \quad (11)$$

$$m_2 = \frac{L \sinh(p + jp)}{k(p + jp)} \quad (12)$$

$$m_3 = \frac{k(p + jp)\sinh(p + jp)}{L} \quad (13)$$

for a 24-hour cycle… $$p = \left(\frac{\pi L^2 \rho c_p}{86400k} \right)^{0.5} \quad (14)$$

where ρ and c_p are density and specific heat, respectively.

For a resistive layer, equation (11) to (13) can be simplified as follows.

$$m_1 = 1 \quad (15)$$

$$m_2 = R \quad (16)$$

$$m_3 = 0 \quad (17)$$

The matrix formulation shown in equation (10) can be extended for multi-layered slabs with appropriate changes on the m matrix, equation (18).

$$\begin{bmatrix} T_i \\ q_i \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_1 \end{bmatrix} \begin{bmatrix} T_o \\ q_o \end{bmatrix} \quad (18)$$

where

$$\begin{bmatrix} M_1 & M_2 \\ M_3 & M_1 \end{bmatrix} = \begin{bmatrix} 1 & R_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} m_1 & m_2 \\ m_3 & m_1 \end{bmatrix}_{layer,1} \cdots \begin{bmatrix} m_1 & m_2 \\ m_3 & m_1 \end{bmatrix}_{layer,n} \begin{bmatrix} 1 & R_o \\ 0 & 1 \end{bmatrix} \quad (19)$$

where n is the number of layers. R_i and R_o are thermal resistance from inside and outside film coefficient, respectively.

As a result, the so-called decrement factor f and time lag ψ can be calculated as follows.

$$f = \left| \frac{1}{UM_2} \right| \quad (20)$$

$$\psi = -\frac{1}{\omega} \tan^{-1} \left[\frac{\mathrm{Im}(1/UM_2)}{\mathrm{Re}(1/UM_2)} \right] \quad (21)$$

where ω is the frequency of temperature boundary condition. The arctangent should be evaluated in the range of –π to 0 radians. For a sinusoidal outside temperature and a constant inside temperature, the inside heat flux can be formulated as:

$$q_i(\tau) = UfT_A \sin[\omega(\tau - \psi)] \quad (22)$$

where T_A is the amplitude of the sinusoidal temperature variation.

The heat flux calculated from equation (22) is thus the exact solution of equation (1) and (2). If conduction is the only heat transfer in a control volume, and with constant inside and outside film coefficients, equation (22) can be used to calculate the exact cooling load values. The test procedure uses the analytical solution to benchmark the accuracy of conduction calculations by the SS, LP and FDR CTFs.

The error is calculated as the percent deviation of the numerical, CTF calculated heat flux from the analytical solution as follows:

$$error = \frac{q_{num} - q_{exact}}{q_{exact}} \times 100\% \quad (23)$$

where q_{exact} is the peak value of the 24-hour conduction heat fluxes calculated from the analytical solution, q_{num} is the conduction heat flux calculated from the CTFs solution at the same time that q_{exact} is calculated.

In the test, the ASHRAE Loads Toolkit (2001) algorithm was used to calculate the SS and LP CTF solutions. The procedure adopts the default Toolkit CTF algorithm settings in the Table 1. The boundary conditions are assumed as follows: the inside and outside film coefficients are treated as resistive layers and the resistances R_i and R_o are respectively equal to 0.120 and 0.044 (m^2-K)/W. The sinusoidal outside air temperature profile is approximated for 1-hour time step and 24-hour period as shown below. The inside air temperature is the mean air temperature, T_m.

$$T_o = T_m + T_A \sin(\frac{\pi}{12}\tau) \qquad (24)$$

The mean air temperature is 20℃, and the amplitude temperature is also 20℃.

COMPARISONS AND DISCUSSIONS

ASHRAE (2001) single-layer wall 24 and 6-layer wall 19 described in the Table 2 and Table 3 are investigated in the test procedure. Both brick layer and heavyweight concrete layer of the wall 19 are simultaneously subject to changes of layer thickness (*L*), thermal conductivity (*k*), density (*ρ*) and specific heat (c_p) respectively until the CTF solutions fail to converge and so is the wall 24. When a property of the slab changes proportionally, the others properties are kept unchanged. Table 4 gives an example of thickness changing based on wall 24.

Figure 1 shows the relationship between CTFs accuracy and thickness. The errors of SS and LP methodes remain within 5% when the thickness changing scale is less than 1.5 for wall 24 and less than -0.2 for wall 19. The error range can be accepted. When the thickness changing scale is more than 1.5 for wall 24 and more than -0.2 for wall 19, the errors of SS and LP methodes increase and even reach 100%. This error is unacceptable. However, for FDR case, no matter how the scale is varied, the errors of the FDR method always remain within 1%.

Figure 2 shows the relationship between CTFs accuracy and thermal conductivity. The errors of SS and LP methodes remain within 5% when the thermal conductivity changing scale is more than -0.8 for wall 24 and more than 0.4 for wall 19. The error range can be accepted. When the thermal conductivity changing scale is less than -0.8 for wall 24 and less than 0.4 for wall 19, the errors of SS and LP methodes increase and even reach 100%. This error is unacceptable. However, for FDR case, no matter how the scale is varied, the errors of the FDR method always remain within 1%.

Figure 3 shows the relationship between CTFs accuracy and density. The errors of SS and LP methodes remain within 5% when the density changing scale is less than 6 for wall 24 and less than -0.16 for wall 19. The error range can be accepted. When the thickness changing scale is more than 6 for wall 24 and more than -0.16 for wall 19, the errors of SS and LP methodes increase and even reach 100%. This error is unacceptable. However, for FDR case, no matter how the scale is varied, the errors of the FDR method always remain within 1%.

Figure 4 shows the relationship between CTFs accuracy and specific heat. The errors of SS and LP methodes remain within 5% when the specific heat changing scale is less than 6 for wall 24 and less than -0.26 for wall 19. The error range can be accepted. When the specific heat changing scale is more than 6 for wall 24 and more than -0.26 for wall 19, the errors of SS and LP methodes increase and even reach 100%. However, for FDR case, no matter how the scale is varied, the errors of the FDR method always remain within 1%.

The preceding figures show that CTFs accuracy is more sensitive to thickness than density or specific heat and that CTFs accuracy is inversely proportional to thermal conductivity. CTFs accuracy is more sensitive to multi-layer than single-layer walls. When material properties are varied greatly, errors in SS and LP methods increase substantially. However, errors in the FDR method remain within 1%. Therefore, the FDR method appears to be more robust and accurate than SS or LP methods.

SOURCES OF CTFS ERRORS

Since CTFs are the products of the numerical solution, numerical errors exist in the CTF solution. As the numerical methods: LP, SS and FDR methods are concerned, the error sources are categorized into followings:

- **Root finding tolerance:** This error applies only to the LP method. In order to calculate response factors, it is necessary to find the root of *B*(*s*) = 0 in equation (5) (Hittle 1979). Since the expression for *B*(*s*) becomes complicated for slabs with more than one layer, the root finding procedures rely on numerical methods. The procedures iteratively continue until the root is found within a root finding tolerance or the maximum number of iterations is reached. The tolerance value and number of iterations can cause error in the CTF calculation.
- **Number of nodes:** This error applies only to the SS method that uses state-space nodes to discretize the transient conduction equations. Seem (1987) demonstrated that CTFs accuracy is dependent on the number of nodes specified. CTFs accuracy is proportional to the number of nodes used in each material layer in the calculation. So the number of nodes can cause error in the CTF calculation.
- **Number of CTFs terms:** In the LP method, CTFs are derived from response factors and it is necessary to determine the number of CTFs terms so that the resulting CTFs can equivalently represent response factors. Equation (6) is used to check the equivalence of response factors and CTFs. While in the SS method, the number of CTFs terms is determined by tracking the ratio of the last CTFs flux term to the first term until the value is negligible. The number of CTFs terms is determined with an iterative process until the

conditions are satisfied within a tolerance limit or until the maximum number of iterations is reached. The tolerance value and number of iterations can introduce errors in the CTF calculation.

- **Frequency points:** This error applies only to the FDR method. The FDR method estimates a simple polynomial *s*-transfer function of transient heat conduction in a building construction on the basis of its theoretical frequency characteristics. The polynomial *s*-transfer function is approximately equivalent to the hyperbolic *s*-transfer function of the total transmission matrix in terms of frequency response characteristics. The concerned frequency range of different slabs is generally different. If the concerned frequency range does not agree with the actual frequency range of the slab, a calculation error can be introduced.

CONCLUSIONS

There are a number of methods to solve transient heat transfer through a building construction, such as LP, SS and FDR methods, etc.. In the paper, the calculation principle of these three methods is presented in detail. The heat flux comparison method is presented to verify CTFs accuracy. The test procedure is applied to investigate the relationship between CTFs accuracy and material properties of building slabs. The sources of error are briefly analyzed. Results show that CTFs accuracy is more sensitive to thickness than density or specific heat, and that CTFs accuracy is inversely proportional to thermal conductivity. CTFs accuracy is more sensitive to multi-layer than single-layer walls. The magnitude of error for LP and SS solutions increase substantially as the scale of change increases. The maximum errors for SS and LP solutions even reach 100%, which is unacceptable. However, regardless of the change in scale, the FDR method determined CTFs with very high accuracy. Therefore, the FDR method might be a more accurate, robust and reliable than the other two methods in common use.

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NOMENCLATURE

English letter symbols

$\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$	coefficient matrices that depend on material properties and/or film coefficients in the SS domain
$A(s)$, $B(s)$, $D(s)$	overall transmission matrix elements that depend on material properties and/or film coefficients in the LP domain
α	thermal diffusivity, m^2/s or ft^2/s
c_p	specific heat, J/(kg-℃) or Btu/(lb-℉)
f	decrement factor, dimensionless
$G(s)$	polynomial *s*-transfer functions in the FDR domain
j	imaginary number unit
k	thermal conductivity, W/(m-℃) or Btu/(hr-ft-℉)
L	thickness, m or ft
r	orders of the numerator in the FDR domain
m	orders of the denominator in the FDR domain
n	number of layers
N_x,N_y, N_z	number of exterior, cross and interior CTFs terms
N_ϕ	number of flux history terms
q	heat flux, W/m^2 or Btu/(hr-ft^2)
R	thermal resistance of film coefficient, (m^2-℃)/W or (hr-ft^2-℉)/Btu
s	Laplace transform variable
T	temperature, ℃ or ℉
T_A	amplitude of the sinusoidal temperature variation, ℃ or ℉
T_s	temperature of each node in the SS domain, ℃ or ℉
U	overall heat transfer coefficient, W/(m^2-K) or Btu/(hr-ft^2-℉)
x	heat flow direction, m or ft
X_n, Y_n, Z_n	exterior, cross and interior CTFs, W/(m^2-K) or Btu/(hr-ft^2-℉)

Greek symbols

α_i , β_i	real coefficients in the FDR domain
$\Delta\tau$	time interval, s or h
ρ	density, kg/m^3 or lb/ft^3
τ	time, s
ϕ_n	flux coefficient, dimensionless
ψ	time lag, hour
ω	frequency, s^{-1}

Subscripts

i	inside or interior
o	outside or exterior
s	surface or each node
θ	current time
δ	time step

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Table 1 Default Toolkit settings for CTF solution

Error types	Error sources	Parameter settings	
		State-space CTF	Laplace CTF
CTF numerical error	(a) No. of state-space nodes	10 ~ 19	NA
	(b) Root finding tolerance	NA	1E-10
	(c) No. of CTF terms	TL = 1E-13 NI = Total no. of nodes≤19	TL = 1E-4 NI = No. of roots found ≤ 5
	(d) Solution time step	1 hour	1 hour
Application error	(e) Solution convergence*	TL = 1E-6 NI = 100	TL = 1E-6 NI = 100
	(f) No. of flux history terms	24	24

Note: NA = Not applicable TL = Tolerance NI = Number of iterations * = User defined parameters

Table 2 Details of ASHRAE wall 24

Description	Thickness and thermal properties				
	L (m)	k (Wm^{-1}K^{-1})	ρ (kg m^{-3})	c_p (J kg^{-1}K^{-1})	R (m^2 K W^{-1})
Outside surface film	-	-	-	-	0.044
LW concrete block (filled)	0.2032	0.26	465	879	0.783
Inside vertical surface film	-	-	-	-	0.120

Table 3 Details of ASHRAE wall 19

Description	Thickness and thermal properties				
	L (m)	k (Wm^{-1}K^{-1})	ρ (kg m^{-3})	c_p (J kg^{-1}K^{-1})	R (m^2 K W^{-1})
Outside surface film	-	-	-	-	0.044
Brick	0.1016	0.894	1922	795	0.114
Wall air space	-	-	-	-	0.153
Insulation board	0.0254	0.029	43	1214	0.876
Heavyweight concrete	0.3048	1.947	2243	921	0.157
Wall air space	-	-	-	-	0.153
Gyp board	0.0159	0.160	801	1089	0.099
Inside vertical surface film	-	-	-	-	0.120

Table 4 CTF error and changing scale with the proportionally-changing thickness based on wall 24

Changing Scale		0.00	0.10	0.20	0.30
L (m)		0.2032	0.2235	0.2438	0.2642
Error	SS	-0.7683%	-0.7127%	-0.8983%	-1.1408%
	LP	-0.5725%	-0.5922%	-0.5742%	-0.5821%
	FDR	-0.5698%	-0.5704%	-0.5702%	-0.5698%

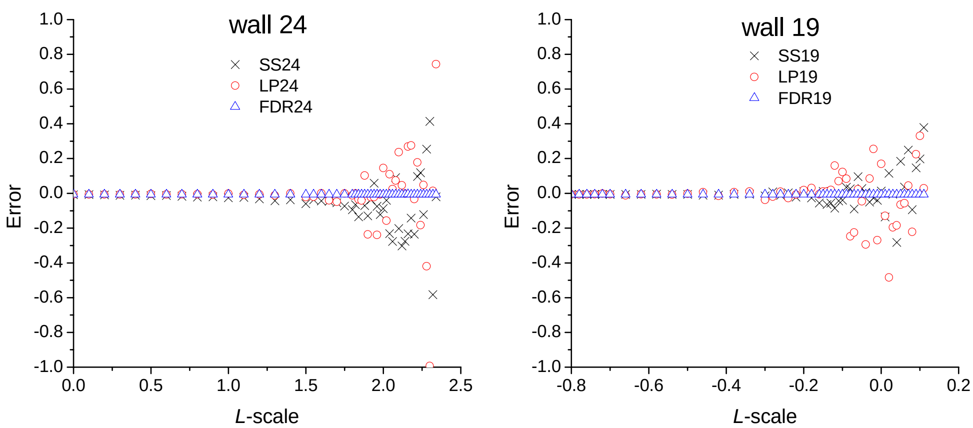

Figure 1 Relationship between CTFs accuracy and thickness

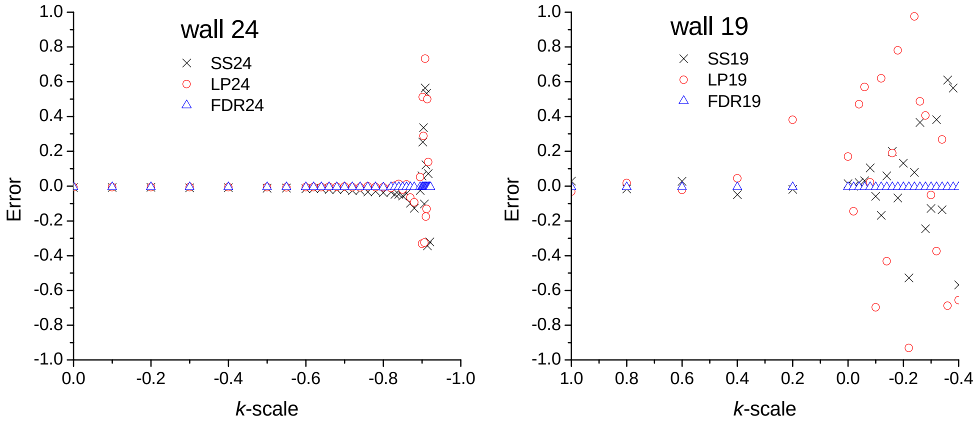

Figure 2 Relationship between CTFs accuracy and thermal conductivity

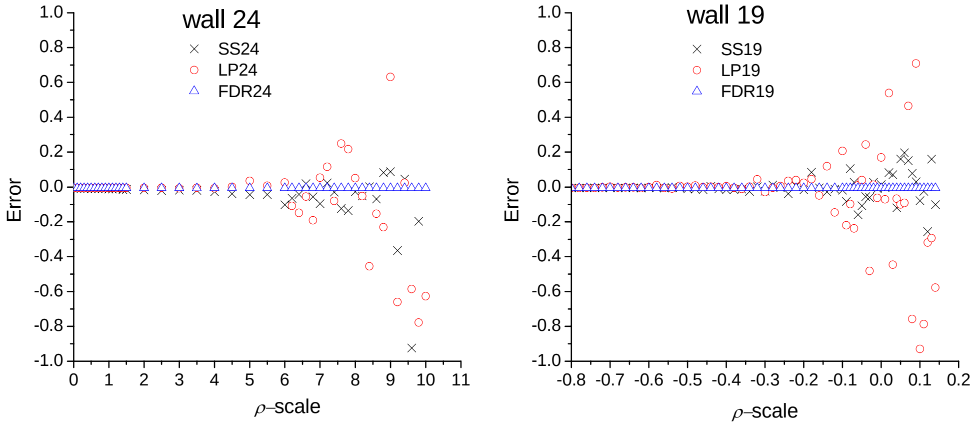

Figure 3 Relationship between CTFs accuracy and density

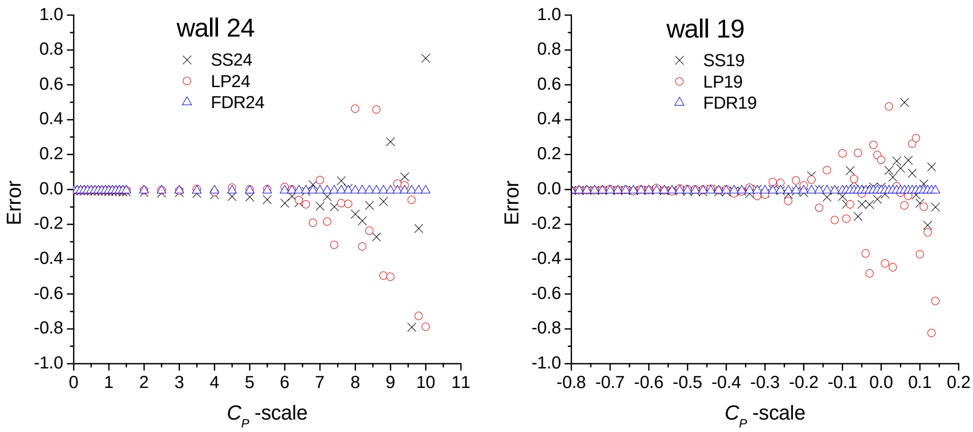

Figure 4 Relationship between CTFs accuracy and specific heat