Prediction and Control of Temperature in Air-Conditioned Indoor Spaces Using Proper Orthogonal Decomposition

A. Sempey, C. Allery and C. Inard

LEPTAB, University of La Rochelle, av. Michel Crépeau, 17042 La Rochelle Cedex 1, France

ABSTRACT

A method for controlling the temperature of the occupancy zone in a room equipped with a fan coil is presented. The heterogeneity of the air velocity field and the temperature distribution is considered. As it leads to a system with a great number of differential equations, the Proper Orthogonal Decomposition is applied to build a low order model. Moreover, the value given by a sensor of temperature enables the estimation of the temperature in the occupancy zone with a state estimator. The performances of the model are shown through the first results of simulation. Not only the method significantly increases the calculation speed, but the accuracy turns out to be good for controlling the temperature.

KEYWORDS

Proper Orthogonal Decomposition, State estimator, Control, CFD, Thermal comfort

INTRODUCTION

In air conditioning spaces, the air velocity field and the temperature distribution are not uniform with significant implications on the thermal comfort. Nowadays, this characteristic is generally not taken into account by the control system which also has consequences on energy consumption. Even though computational fluid dynamics (CFD) may predict the temperature distribution and the velocity field, computing time is prohibitive for real time control. For the problem of indoor air flow, Peng (1996) proposed a method to calculate the dynamic temperature distribution in a fixed flow field, provided that it is correctly calculated by the CFD code. In this condition, only the energy balance equation is discretized and then reduced using Proper Orthogonal Decomposition. Lastly, a sensor of temperature allows estimating and controlling the temperature in the occupancy zone.

THEORY

The energy balance is described by a differential equation:

$$\frac{\partial \theta}{\partial t} + \vec{V} \bullet \overrightarrow{grad\theta} = div\left(a_{eff}\, \overrightarrow{grad\theta}\right) + \frac{S_p}{\rho C_p} \qquad \textbf{(1)}$$

where ρ is the density, C_p is the constant-pressure specific heat, θ is the temperature and S_p is the source term. Normally, the airflow in room environments can be fully turbulent. To treat turbulence, the Reynolds averaging is used. Consequently, in equation (1), a_{eff} is the effective diffusivity, and can be split into a laminar term and a turbulent term:

$$a_{eff} = \frac{\lambda}{\rho C_p} + \frac{\nu_t}{\sigma_t} \tag{2}$$

where λ is the thermal conductivity, ν_t is the turbulent viscosity and σ_t is the turbulent Prandt number here equal to 0.9. The velocity field $\vec{V}$ and the turbulent viscosity field are calculated with the CFD software StarCD. The temperature distribution obtained is considered as the reference and the initial condition. The high Reynolds number k-ε model, Lu et al. (1997), is chosen for turbulence modelling, and the robust scheme MARS, STARCD (1999), for space discretization of the convective term.

To discretize equation (1), the same mesh and method of space discretization used in StarCD are adopted. The control volume method is the leading method in field of computational fluid dynamics, Murakami (1990). Furthermore the concept of the conservation of physical quantity is very clear in. For the discretization of the convective term, the widely applied QUICK scheme is chosen. It has a good stability a relatively small numerical viscosity. Near the wall, convective heat transfer coefficients are calculated by common formula given by Inard et al. (1998). At this stage, the order of the system corresponds to the number of discretization cells, and the form of the discretized equation is:

$$\begin{aligned}\frac{\partial \theta}{\partial t} &= a\theta + a_E\theta_E + a_W\theta_W + a_H\theta_H + a_L\theta_L + a_N\theta_N + a_S\theta_S \\ &+ a_{EE}\theta_{EE} + a_{WW}\theta_{WW} + a_{HH}\theta_{HH} + a_{LL}\theta_{LL} + a_{NN}\theta_{NN} + a_{SS}\theta_{SS} + \frac{S_\theta}{\rho C_p}\end{aligned} \tag{3}$$

where E, W, H, L, N, S are for east, west, high, low, north and south neighbouring cells and EE, WW, HH, LL, NN, SS are for east, west, high, low, north and south neighbouring cells of respectively E, W, H, L, N, S cells. A more advantageous representation is the state space form in which the temperatures are written in a vector, Ghiaus et al. (1995).

$$\begin{cases}\dot{\theta} = A\theta + Bu \\ y = C\theta + Du\end{cases} \tag{4}$$

where $\dot{\theta} = \frac{\partial \theta}{\partial t}$, θ is the state vector, u is the input vector and y is the output vector.

Because of the high order of this model, its reduction is considered in studying the way to produce an optimal basis with the Proper Orthogonal Decomposition (POD), Allery et al. (2005) and Gunes (2002). This method needs snapshots extracted from a transient simulation made with StarCD. Then the temperature of this simulation is approximated in a series in terms of orthonormal set of basis functions φ_i:

$$\theta(x,t) = \theta_{moy}(x) + \sum_{i=1}^{m} a_i(t)\varphi_i(x) \qquad \textbf{(5)}$$

where θ_{moy} is the mean temperature, m is the order of the reduction and a_i are the temporal coefficients. Only a very small number m of functions are sufficient to rebuild the temperature. In order to obtain a low dimensional model, equation (5) is substituted in equation (4) and after a change of variable leads to a system of order m:

$$\begin{cases} \dot{a} = A_r a + B_r u \\ \theta = C_r a + D_r u \end{cases} \qquad \textbf{(6)}$$

The low order model (6) gives the behaviour of coefficients a_i in reply to the inlet vector. Knowing the basis functions φ_i, the temperature is then rebuilt with use of the expression (5).
This model is used to construct a state-estimator able to control the temperature of the occupancy zone:

$$\begin{cases} \dot{\tilde{a}} = A_r \tilde{a} + B_r u + L\left(\theta_{mes} - \tilde{\theta}_{mes}\right) \\ \tilde{\theta} = C_r \tilde{a} + D_r u \end{cases} \qquad \textbf{(7)}$$

where ~ stands for the estimated quantities, and θmes is the measured temperature by a sensor. The difference with equation (6) is the introduction of an error term between the estimated temperature and the measured temperature at the sensor location. In order to design the estimator L, optimal control theory is applied to minimize the difference between the low order model (6) and the state estimator (7).

STUDIED CASE

The aim of this study is to access the control of a 4.90 m × 2.82 m × 2.76 m room equipped with a fan coil. Several configurations were considered, but only one is developed in this paper. The 2D temperature distribution and velocity field are calculated for outlet temperature ranging from 16 °C to 21 °C with a step of 1 °C, and the outlet air velocity equal to 1.5 m/s. All walls temperatures are fixed to homogeneous and constant values. The mesh contains 2992 cells. The sensor is placed at the inlet of the fan coil.

The POD-based basis function are extracted from the simulation of two successive 5 degrees steps for outlet temperature fields 21 °C to 16 °C and 16 °C to 21 °C, that is to say all the range on which the velocity fields is supposed to be fixed.

In reference to the European standard ISO 7730, the control of temperature is surveyed at three different points in the occupancy zone, respectively at 0.10 m (T1), 1.10 m (T2) and 1.80 m (T3) high. In addition temperature at the inlet of the fan coil (T4) is checked. The location of these points is illustrated on the Figure 2. At first, the system is stabilised for a setpoint of 20 °C, and then submitted to a step from 20 °C to 21 °C. All simulations are done with Simulink, a toolbox of Matlab. The Figure 1 shows the implementation of the system in Simulink.

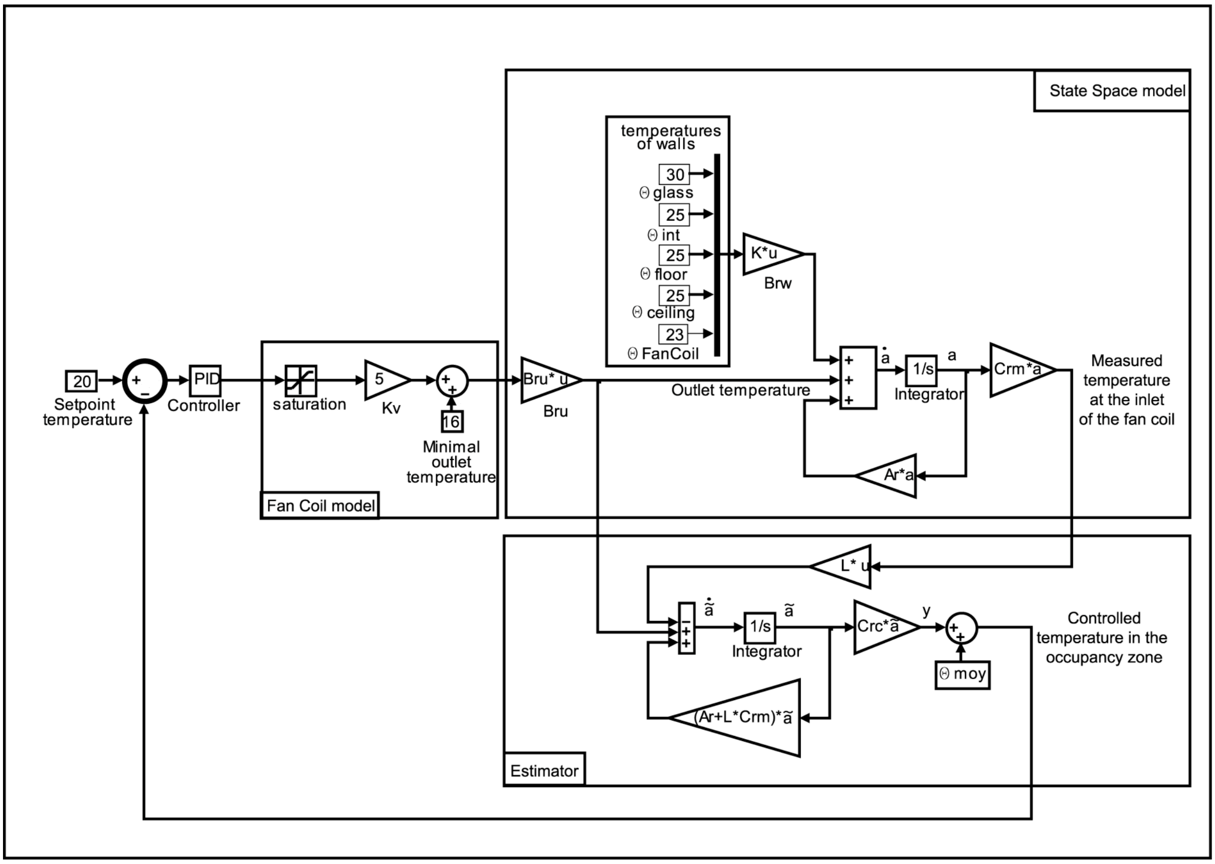

Figure 1: Control system in Simulink

RESULTS AND DISCUSSION

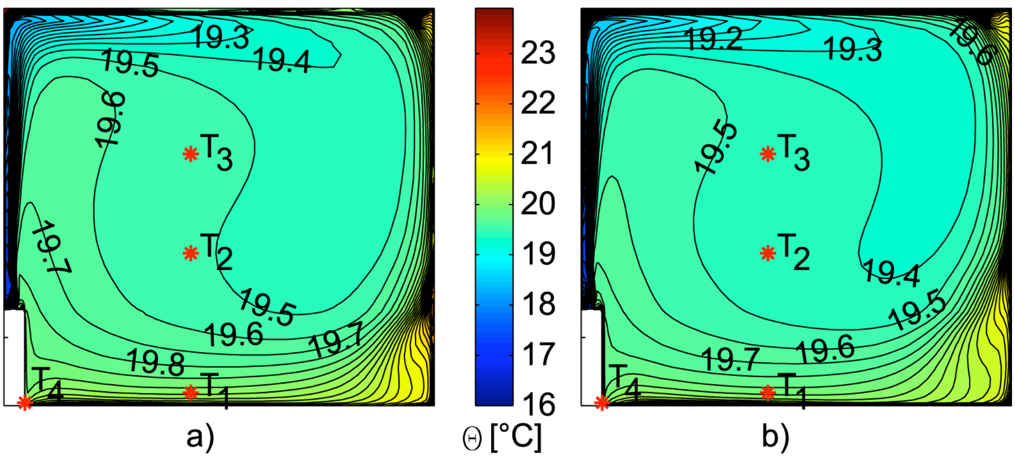

Figure 2: Temperature distributions for an outlet temperature of 17 °C extracted from: (a) the state space model (4), (b) the simulation with StarCD

At first, the accuracy of the high order model (4) has to be check. Figure 2 shows the temperature distribution for an outlet temperature of 17 °C. The difference between the results of StarCD and the high order state space model is evaluated by:

$$Eqm = \sqrt{\frac{\sum (x_{ref} - x)^2}{n_x}} \quad \textbf{(8)}$$

where the index ref is in reference to StarCD, and n_x is the number of discretization cells for steady state simulations or the number of time steps for one location for transient simulations. Thus, the difference is 0.296 °C in the entire room, 0.095 °C at the outlet and 0.13 °C in the occupancy zone. The highest value for the whole room is in part due to the difficulty to predict accurately the driving flow. Simulink needs only few minutes to give the results. Nevertheless, the size of the model is too important to use for designing a controller. So, it is obviously necessary to reduce the model.

Only two basis functions are necessary to approximate the temperature of the two successive 5 degrees step with a very good accuracy. The dimension of the reduced order model (6) is then only two. Figure 3 shows the results of this reduced order model for a step from 20 °C to 21 °C. Comparison with the results of StarCD shows acceptable agreement. Above all, the system needs only few seconds to be solved.

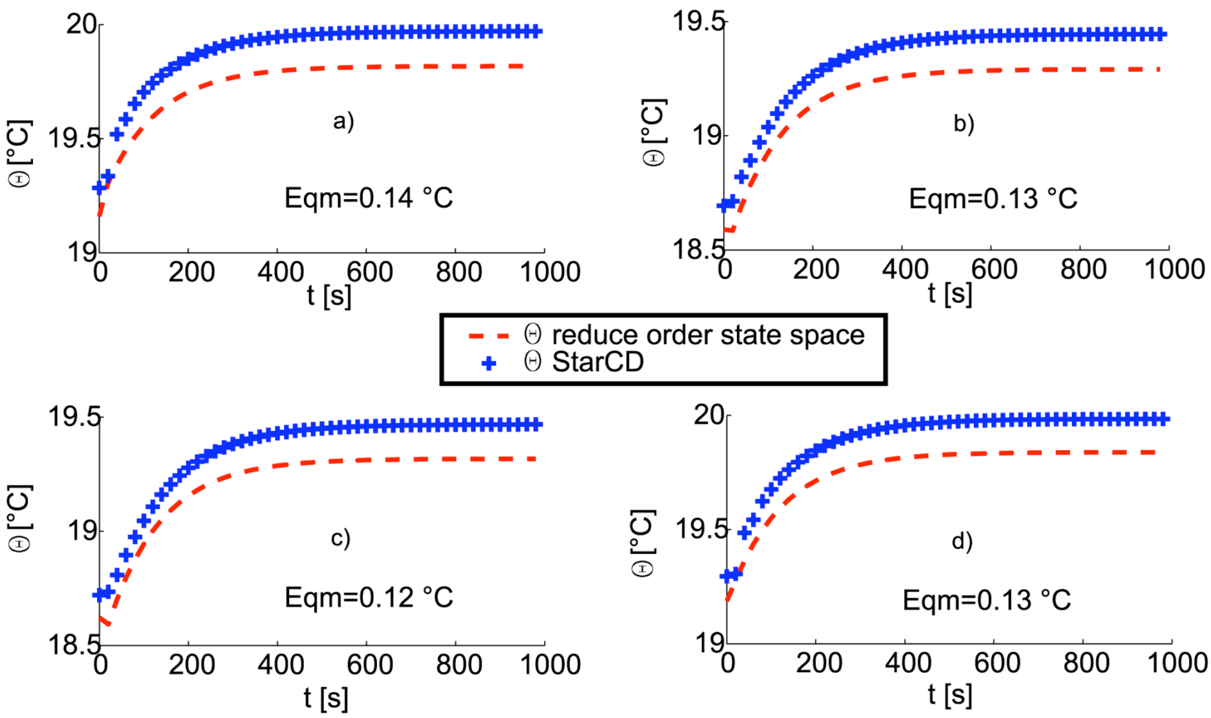

Figure 3: Computed temperatures: (a) T1, (b) T2, (c) T3, (d) T4.

Finally, Figure 4 shows the comportment of the controlled temperature. It‘s worth noticing that the stabilisation of the temperature for a setpoint of 20 °C is coherent with the results of StarCD. Moreover, a difference between the outlet temperature and the temperature in the occupancy zone clearly appears. Even though this difference is small in this case, it highlights the efficiency of the state estimator to control the occupancy zone temperature owing to the measure of the inlet temperature.

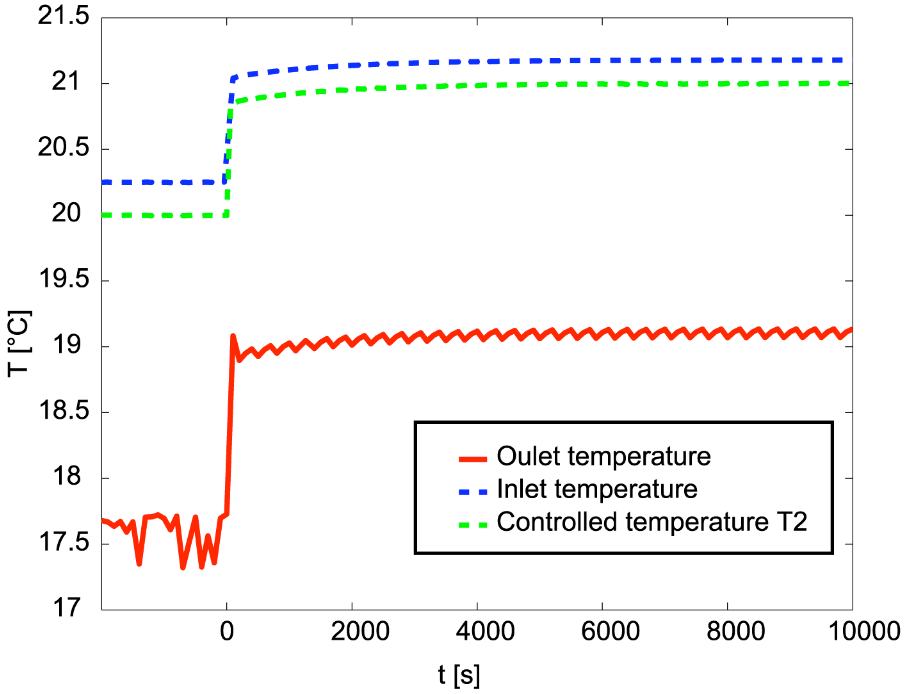

Figure 4: Outlet, Inlet and controlled (T2) temperatures

CONCLUSION

These first results show a good capacity of POD to control the temperature in an air-conditioned room. The dramatic power of reduction of POD allows the same procedure to be applied to three-dimensional cases. Nevertheless, the effectiveness of this method has to be studied with a higher thermal stratification of the indoor air. Indeed, the case studied here has a weak inhomogeneity. Besides, further developments will consist in analysing the validity domain of the reduced model as a function of the inlet temperature.

REFERENCES

Allery, C., Beghein, C. and Hamdouni, A. (2005). Applying proper orthogonal decomposition to the computation of particle dispersion in a two-dimensional ventilated cavity. *Communications in Nonlinear Science and Numerical Simulation,* **10:8,** 907-920.

Ghiaus, C. M., Peng, X. and Van Passen, A. (1995). A state-space representation of the dynamic model of air temperature distribution. *In 19th International Congress of refrigeration, The Hague.*

Gunes, H. (2002). Low-order dynamical models of thermal convection in high-aspect ratio enclosures. *Fluid Dynamics Research,* **30:1,** 1-30.

Inard, C., Meslem, A. and Depecker, P. (1998). Energy consumption and thermal comfort in dwelling-cells: A zonal-model approach. *Building and Environment,* **33:5,** 279-291.

Lu, W., Howarth, A. T. and Jeary, A. P. (1997). Prediction of airflow and temperature field in a room with convective heat source. *Building and Environment,* **32:6,** 541-550.

Murakami, S. (1990). Computational wind engineering. *Journal of Wind Engineering and Industrial Aerodynamics,* **36:1-3,** 517-538.

Peng, X. (1996) In *Faculty of mechanical engineering and marine technology* Delft university of technology, Delft, pp. 158.

STARCD (1999). *Methodology*(Ed, adapco) Melville,USA.