Contribution to the Stochastic Simulation of Buildings Thermal Behaviour

M. Abida[1], G. Lefebvre[2], N. Ghrab-Morcos[1]

[1]Unité de Recherche Energétique des Bâtiments et Systèmes Solaires,
Ecole Nationale d'Ingénieurs de Tunis, B.P. 37, 1002 Tunis-Belvédère, Tunisia
Phone : + 216 71 874 700 # 532, abidamed2003@yahoo.fr, nadia.ghrab@enit.rnu.tn
[2]CERTES, Université Paris 12, 61 avenue du Général de Gaulle, 94010 Créteil, France
Phone : +33 1 45 17 18 41, lefebvre@univ-paris12.fr

ABSTRACT

The paper discusses a methodology for thermal analysis applied to buildings in which the stochastic nature of the external forces is concerned. The considered forces are ambient temperature and solar radiation. The stochastic approach presented in this paper, consists in modelling the climatic inputs as a Markov process which have been injected on a reduced modal model describing the thermal behaviour of the building. The weather records used for stochastic identification have been gathered over 5-years (1975-1979) in Tunis and the case study has been carried on a typical Tunisian building. As a result of numerical computation, the distribution of indoor temperature has been directly obtained, as well as that of the comfort index of the building.

KEYWORDS

Stochastic simulation, Markov chain, modal method, thermal comfort.

INTRODUCTION

Energy consumption grows rapidly all over the world, and the residential sector is responsible of an important part of it. We are interested in decreasing this consumption by designing economical buildings which necessitate low amounts of energy to produce a good quality of comfort.

Different simulation methods have been developed to calculate more or less accurately, long-term energy performance of buildings. The input of these codes is the historical hourly weather data. The programs have some limitations because they give information only on the particular selected period. Moreover, they need a big effort of calculation to analyze the results.

The approach adopted in this study gives directly statistical information about the system outputs, in relation with the statistic characteristics of the input variables. We have replaced the analysis of results by modelling the climatic data to simplify calculations taking benefit of the modal method properties (independence and reduction). This approach presents then two main advantages: thermal evaluation of the building through a stochastic comfort note, and simplification of calculations.

This paper presents a simulation of a type of housing built in great series in Tunisia in order to predict the thermal quality of the indoor environment which can be obtained in a natural way. Comparison to the deterministic simulation results were made to improve the performance of this approach for the evaluation of the thermal quality of buildings.

METHOD OF APPROACH

Statistical Modelling of the Meteorological Inputs

The data files of outdoor temperature and solar radiation, consisting in hourly measurements, are transformed into a probability transition matrix and a long-term vector of probabilities by the discretisation of their values into different classes.

Computation of probability transition matrix

Computation of the stochastic matrix M is a long and tiresome stage, but it has to be carried only once; then the parameters of the Markovian model will be filed in a library to be used for any later simulation. Each variable is discretized on the basis of a limited number of classes; then, we count the number of transitions from a possible state to another. The transition matrix is thus built; it characterizes completely the random behaviour of each variable, over the chosen period. The elements of the matrix are normalized to obtain probabilities instead of number of occurrence.

Long-term State

We indicate by long-term state, the state of the chain at an infinite time. It is defined by the long-term probabilities vector μ_s. The dimension of this vector is equal to the number of configurations which the Markov chain can theoretically take which is equal to n_{pr} (number of classes of solar radiation)* n_{pt} (number of classes of the outdoor temperature). Each component of the vector represents the probability of finding the chain at infinite time in the correspondent configuration. Several methods permit to calculate the vector μ_s. We have used in our case an iterative method based on Eqn. 1:

$$^t\mu_n = {}^t\mu_{n-1} * M$$

This is done until the variation of the successive values of μ_n becomes negligible. The transition matrix and long term vector are better representation of the true climate, when the number of distinct classes for temperature and radiation increases.

Modelisation of the Thermal Behaviour of Building

We have used the modal method [G. Lefebvre 1987] for modelling thermal building behaviour.

The main properties of a modal model can be summarized as follows:

- The evolution of each modal state is solution of an equation decoupled from the other ones

- Only a reduced number of modes are significant to explain the response of the system. Reduction of the model can then be applied preserving a very good accuracy of the outputs.

These two properties permit to simplify calculations and to reduce the time of simulation. In the initial system, the Markovian matrix representing the state of the system is a squared matrix with a large size, its dimension being (n_{pt}*n_{pr}*n_{m1}*…*n_{mn}), with n_{mi} the number of distinct classes for mode i. Remembering that the number of modes is the number of nodes of building discretization, it can be seen that this is a very big matrix. If the number of modes is reduced to R (generally 3), the independence of the modes shows that the initial large matrix can be replaced by R small squared matrices, the dimension of each one being (n_{pt}*n_{pr}*n_{mi}). The resulting reduction of memory and calculation thus obtained can exceed 99% for a complex thermal system like a building.

The Integration of a modal model excited by a Markovian Process gives the following system Eqn. 2:

$$X_i(n+1) = X_i(n) * e^{\lambda_i \Delta t} - \frac{1 - e^{\lambda_i \Delta t}}{\lambda_i \Delta t} \sum_p B_{ip}[U_p(n+1) - U_p(n)]$$

$$Y(n+1) = H * X(n+1) + S * U(n+1)$$

where [F], [B], [H] and [S] are the modal matrices, λ_i an eigenvalue which is the i^{th} element diagonal of the diagonal matrix [F], X_i the mode i, U_p a solicitation and Δt the time step. These equations allow to construct the transition matrices of the simulated system.

The transition matrix of each conserved process is calculated in the following way:

- We define the classes of the modes
- We calculate the future value of each discrete value of the mode considered a step of time later with Eqn. 2.
- We locate the classes to which this value corresponds
- The probability of transition of the triplet (mode, solicitations) is equal to the probability transition of the couple of solicitations, because the transition of this couple from an interval to another, forces the discrete value of mode X_i to transit to a well defined value (see Eqn. 2).

Calculation of the Indoor Temperature

We start by discretizing the indoor temperature into intervals. Its values are calculated for each possible combination of the modes. Then we locate it by determining the interval to which it belongs.

From the stochastic matrices, we calculate the long-term vectors of each state of the chain. These probabilities are used to calculate the probability of each combination and consequently the probability of the classes of the indoor temperature. The component of stochastic vector $P(C_k)$ describing the probability attached to each class k of the indoor temperature, is equal to the sum of the probabilities of the various combinations which lead to a temperature belonging to this class. Eqn. 3

$$P(C_k) = \sum_j P^j(C_k) = \sum_j \frac{\prod_{i=1}^{R} Pf_i^{\,j}}{(Pf^{\,j})^{R-1}}$$

where R is the number of conserved modes, P^j: probability of having the combination j, $Pf^{\,j}$: long term probability of couple of solicitations and $Pf_i^{\,j}$: the long term probability vector of the triplet (mode $X_{i,}$ couple of solicitations).

Quantification of Comfort

The index PMV [P.O. Fanger 1973] can be predicted, when the metabolic activity, the thermal resistance of clothing are estimated, and the following parameters of environment: temperature of the interior air, radiant average temperature, air velocity and relative humidity are known. After a linear regression, the PMV can be written in the following form: Eqn. 4

$$PMV = a + b * T_{air}$$

The coefficients a and b depend on the other variables which are supposed constant for one selected period. The calculation of the long term vector of PMV is then deduced from the long term vector of indoor temperature. Thus, we can determine the quality of comfort by calculating the number of satisfied persons deduced from the PMV index.

Stochastic Program

The stochastic program, developed in Scilab environment [B.Ycart 2003], comprises the following phases: - construction of the transition matrices, - deduction of the long-term probability vector of the states, - quantification of comfort. The modal matrices needed for the execution of the stochastic program are generated with the help of m2m [G. Lefebvre 1996], a package based on modal method. Figure 1 represents the various blocks of the program, as well as the inputs and the outputs of each block.

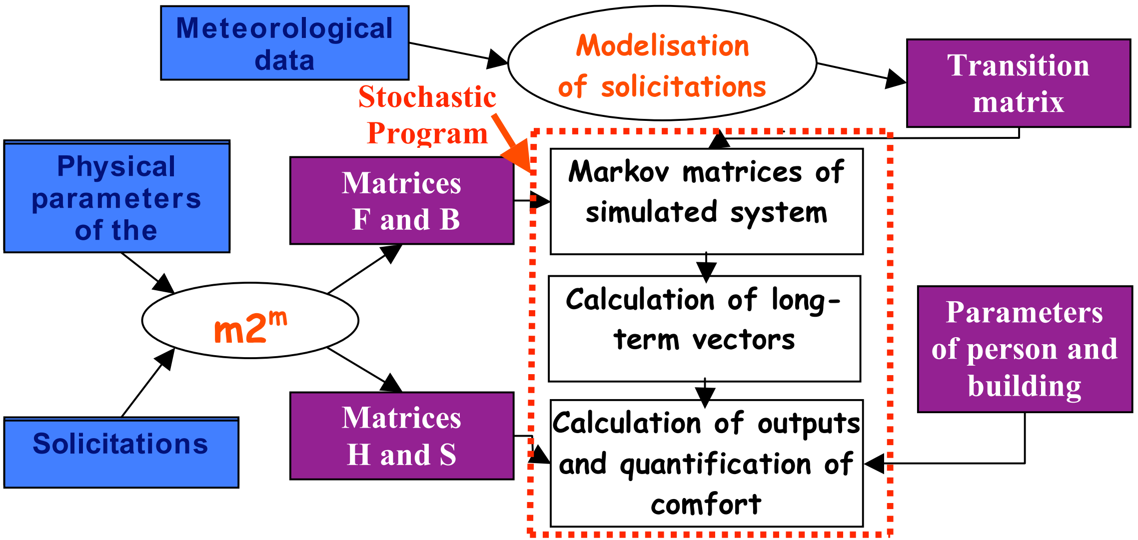


Figure 1: Blocks of stochastic program

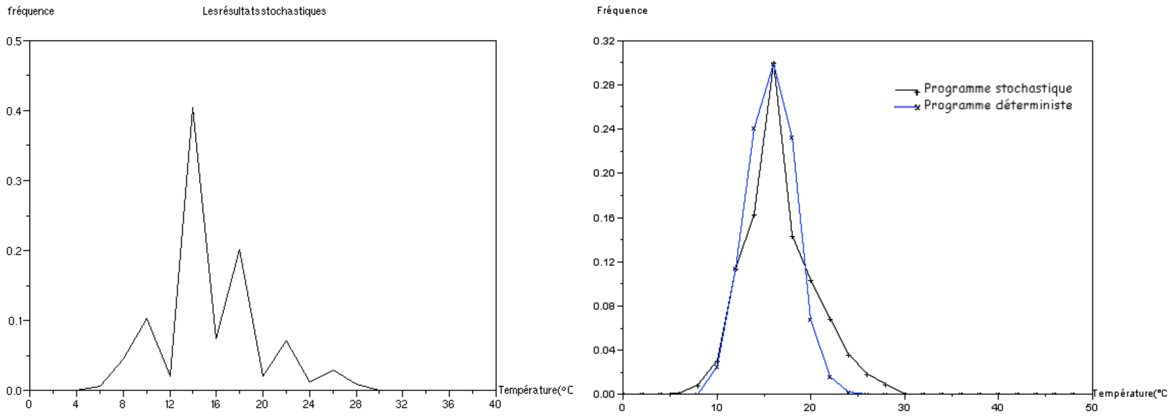
RESULTS AND ASSESSMENTS

Description of the building

The simulations are performed for months of January for the years 1975:1979, of Tunis on an example of building built in great series in Tunisia. The building is a four floor rise, with an area of 86 m² per floor, with a south glazed area of 9m², and a north one of 6m².
We wished to model the thermal behaviour of the building with all of these solicitations (southern solar radiation, northern ...); but as these solicitations are dependent, we cannot model them like a chain of Markov. We had to choose only one that is the southern radiation coupled with the outdoor temperature. The building was divided into four zones; one zone per floor.

Results

Only 2 modes are conserved from 231 modes. The choice of the number of classes for each introduced variable (solicitations, modes) influences the quality of the stochastic results. Indeed, if we take 4 classes for each variable (Figure 2a), we notice that the stochastic results are approximate and generate significant errors. In addition, the stochastic graph presents some limits because it generates jumps which indicate errors of representation. It is improbable to find a class which has a frequency lower than the frequencies of the two neighbouring classes.
Then, we carry out a more refined subdivision on the couple of solicitations and modes passing from 4 to 12 classes for each one (figure 2b), hoping to improve the quality of the stochastic results.



a) 4 classes b) 12 classes

Figure 2: Representation of the distributions of the outside temperature of one zone

With this refined subdivision, we notice that the shape of the curve becomes perfect. The deterministic curve and the stochastic one are comparable: they are well centred and have the same majority frequency for the same class of temperature. Thus, the stochastic results are reliable and can be used for the quantification of comfort.

Quantification of Comfort

The performances obtained are summarized in table1 where we find the average notes of comfort in January for each floor of the building.

TABLE 1
Indoor temperature and comfort notes

	stage_0	***stage_1***	***stage_2***	***stage_3***
Mean of indoor temperature	16.64	17.01	16.65	15.94
PMV	- 1.77	- 1.67	-1 .76	- 1.95
PPS	42.7	46.6	42.9	35.8

The simulation of a typical building of Tunisian construction, by the stochastic code gave probable results which are in good agreement with those of deterministic simulation. A subdivision into 12 classes for each dominant mode leaded to the best results.

The developed stochastic technique of simulation proved to be able to directly provide information of a probabilistic type on the quality of comfort carried out in an unconditioned building.

CONCLUSION

The object of this work is to develop a computer code allowing the quantification of the comfort in a building based on the stochastic approach. This type of simulation is a simple and fast tool for the thermal optimization of buildings, in order to obtain the best quality of comfort without using active systems. The results obtained have been compared to the deterministic results to study the reliability of this method.

On the other hand, this method has some limitations: indeed, knowledge of the hour is lost and we can’t model all the solicitations because some of them are dependent. Solutions to these limitations will be investigated in our subsequent research work.

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