

# Identification of the Physical Parameters of an Experimental Solar House in Tunisia

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## ABSTRACT

This paper presents the application of multi-inputs single-output (MISO) models to estimate the thermophysical parameters of a building. ARARMAX, Box-Jenkins and the general MISO models are used to identify the U value, the time constant and the equivalent solar surface of the building. Optimization-based prediction error method (PEM) algorithm is used to estimate model parameters. This approach has been tested to analyze a passive solar house in Tunis. The identified parameters were compared to theoretical values; good results have been obtained for the tested building.

## KEYWORDS

Building physical parameters, System identification, Parameter estimation, Prediction error method..

## INTRODUCTION

In practice, several factors can involve distortions of the performances of a building, such as: defects in the construction of the envelope, system effectiveness of conditioning, behavior of the occupants. To be able to judge the energetic quality of a building, it is essential to determine its real physical parameters, as well as uncertainties on the given values. The estimation of the physical parameters of the building such as heat transmission coefficient or time-constant requires the knowledge of other parameters characterizing the building, which are rather difficult to determine. Identification theory is very well developed and there exist many results which can be applied to linear models (Ljung & Söderström, 1983; Söderström & Stoica, 1989).

The National School for Engineers of Tunis, ENIT, has built a passive solar pavilion, equipped with a Trombe wall and large south-facing glazed areas, which has been carefully instrumented. The purpose of the present work is to develop a methodology for the identification of the main physical parameters that govern the building thermal behavior, both in static and dynamic states and to test it on the solar pavilion of ENIT. Such a methodology will be very useful for the energy diagnosis of buildings with an improved envelope, oriented toward capture of high solar gains.

## THEORETICAL CONSIDERATIONS

Let us consider the problem of estimating a model for a multi-input single-output (MISO) system based on the observation of an ( $N$ )-set input-output data sequence  $Z^N = \{U(t), y(t)\}$ , where  $U(t) = [u_1(t), \dots, u_{nu}(t)]$  is the input vector and  $y(t)$  the output at time  $t$  (Walter E., Pronzato L. (1994)).

When identifying the system, we use the following general MISO structure with known order given by Eqn. 1:

$$A(z^{-1}, \theta)y(t) = \sum_{i=0}^{nu} \left[ \frac{B_i(z^{-1}, \theta)}{F_i(z^{-1}, \theta)} u_i(t) \right] + \frac{C(z^{-1}, \theta)}{D(z^{-1}, \theta)} e(t)$$

where the vectors  $A(z^{-1})$ ,  $B_i(z^{-1})$ ,  $F_i(z^{-1})$ ,  $C(z^{-1})$  and  $D(z^{-1})$  are given by Eqn. 2:

$$A(z^{-1}, \theta) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_{na} z^{-na}$$

$$B_i(z^{-1}, \theta) = b_{0i} + b_{1i} z^{-1} + \dots + b_{nbi} z^{-nbi}$$

$$C(z^{-1}, \theta) = 1 + c_1 z^{-1} + \dots + c_{nc} z^{-nc}$$

$$D(z^{-1}, \theta) = 1 + d_1 z^{-1} + \dots + d_{nd} z^{-nd}$$

$$F_i(z^{-1}, \theta) = 1 + f_{1i} z^{-1} + \dots + f_{nfi} z^{-nfi}$$

and  $z^{-1}$  is the backward shift operator:  $z^{-1} u(t) = u(t-1)$

In this work, many different linear submodel structures have been considered. Eqn. 1 contains several special model structures, namely ARX, ARMAX, ARARMAX and Box Jenkins.

ARARMAX model is one of the most popular linear models, given by Eqn. 3:

$$A(z^{-1})y(t) = B(z^{-1})u(t) + \frac{C(z^{-1})}{D(z^{-1})}e(t)$$

where  $\theta = (a_1 \dots a_{na} \ b_1 \dots b_{nb} \ c_1 \dots c_{nc} \ d_1 \dots d_{nd})^T$  is the parameter vector and  $e(t)$  the white noise with zero mean. Since  $e(t)$  is unknown, the parameters  $c_i$  should be identified based on the residual. By introducing a regression vector  $\varphi^T(t, \theta)$  the model (3) can be expressed in the linear regression form of Eqn. 4:

$$y(t) = \varphi^T(t, \theta)\theta + e(t)$$

where  $\varphi(t) = (-y(t-1) \dots -y(t-na) \ u(t-1) \dots u(t-nb))^T$

and  $\varepsilon(t, \theta) = y(t) - \varphi^T(t, \theta)\theta$  denotes the prediction error.

For this identification problem, most numerical schemes select  $\theta = \hat{\theta}$ , so that, for instance, a quadratic norm criterion function as given by Eqn. 5:

$$J_N(\theta) = \frac{1}{N} \sum_{t=1}^N |y(t) - \varphi^T(t, \theta)\theta|^2$$

is minimized, that is  $\hat{\theta}$  satisfies Eqn. 6:

$$\hat{\theta} = \arg \min \{J_N(\theta)\}$$

There is a large class of identification methods for solving (6). The optimization-based Prediction Error Method (PEM) described in Söderström and Stoica (1989) is a typical one which is very suitable for linear parametric model identification.

## METHODOLOGY

Dynamic analysis requires the use of a model to represent the system to be analyzed, including the main physical effects, a mathematical algorithm and a software tool to find the parameters required for the selected model.

## Data set

Instead of using measured values for the outdoor climate data and the indoor air temperature, a data set was constructed using outdoor climate data, and the simulated values for the indoor air temperature. These values were obtained by simulating the solar house behavior with TRNSYS (Transient System Simulation program) under real outdoor weather conditions, implying dynamic state.

## Specification of the model

The identification was done in two steps, firstly by applying an ARARMAX model with different model orders and secondly by using the general MISO structure. Although those models are able to describe very complex systems, data limitations and problems of inputs correlation makes it necessary to keep the models as small and simple as possible. The strategy for model selection will be to specify the model with as low order of the polynomials structure as possible and then increase the order of the model if necessary. We begin with a structure without noise model and we assume all polynomials  $A(z^{-1})$ ,  $B_i(z^{-1})$  and  $F(z^{-1})$  to be to the first order; then we increase their order one by one to obtain the best model (validation tests). Then, we specify the noise model, by choosing the order of the polynomials  $C(z^{-1})$  and  $D(z^{-1})$ . Optimization-based prediction error method (PEM) algorithm was used to estimate models parameters, by minimizing the criterion (5), function of the prediction error (Eykhoff P. (1974).).

The MATLAB identification toolbox was used to select the model structure and order and also to generate matrix containing information about the selected model structure, and the estimated parameters. This matrix was exported to the MATLAB workspace, where the required parameters and the criterion error were calculated.

## Model validation

Various statistical tests are applied in validating the model, such as residual analysis (auto-correlation and inter-correlation tests), test for model order and parameter significance. Various statistical tests were realized (Ljung L. (1999)):

**T1:**  $\varepsilon(t)$  is a white noise with zero average

**T2:**  $\varepsilon(t)$  is symmetrically distributed

**T3:**  $\varepsilon(t)$  is independent from past inputs ( $E\varepsilon(t)u(s) = 0, \forall t > s$ ).

**T4:**  $\varepsilon(t)$  is independent from all inputs ( $E\varepsilon(t)u(s) = 0, \forall t \text{ and } s$ )

## A GENERAL LINEAR MODEL

Although climate characteristics are continuous variables, they are measured and recorded at time steps which give them a discrete character. In our case, we shall consider the climate variables  $T_{ext}$  (outdoor temperature),  $P_{sol}$  (solar radiation) and a third input  $P_u$  (heating power). The system output is  $T_{int}$ , the indoor temperature. In this discrete domain, the dynamic system can be modeled in several ways, one of which is by means of linear auto regressive relations between the discrete output  $T_{int}(t)$  and the discrete inputs  $T_{ext}(t)$ ,  $P_u(t)$  and  $P_{sol}(t)$  such as the general MISO model, given by Eqn. 7:

$$A(z^{-1})T_{\text{int}}(k) = \frac{B_1(z^{-1})}{F_1(z^{-1})}T_{\text{ext}}(k) + \frac{B_2(z^{-1})}{F_2(z^{-1})}P_u(k) + \frac{B_3(z^{-1})}{F_3(z^{-1})}P_{\text{sol}}(k) + \frac{C(z^{-1})}{D(z^{-1})}e(k)$$

## Physical interpretation of the linear model

### Steady state thermal properties

Consider the systematic part of Eqn. 1. Assume that all variables are constant, i.e. there is no variation over time. The linear model must hold also in this case. Dropping the time argument of the variables and putting  $z^{-1}=1$  we obtain Eqn. 8:

$$A(1)T_{\text{int}} = \frac{B_1(1)}{F_1(1)}T_{\text{ext}} + \frac{B_2(1)}{F_2(1)}P_u + \frac{B_3(1)}{F_3(1)}P_{\text{sol}}$$

If there is no heating ( $P_u=0$ ) and no solar radiation ( $P_{\text{sol}}=0$ ), the indoor temperature must be equal to the outdoor one. This gives the following constraint:  $A(1) = \frac{B_1(1)}{F_1(1)}$

Replacing in Eqn 8 and comparing with the equation :  $U(T_{\text{int}} - T_{\text{ext}}) = P_u + S.P_{\text{sol}}$

We obtain:  $U = \frac{A(1)F_2(1)}{B_2(1)}$  and  $S = \frac{B_3(1)F_2(1)}{F_3(1)B_2(1)}$

where  $U$  and  $S$  are the steady state properties of the building. The parameter  $U$  [W/°C] may be defined as the U-value or heat loss coefficient describing the heat loss per unit indoor-outdoor temperature difference. The parameter  $S$  [m<sup>2</sup>] is an effective solar aperture equivalent with a south facing windows. Hence, we have obtained meaningful steady state properties as rational functions of the mathematical parameters in the linear model.

### Transient thermal properties

Consider the case when the relationship between the indoor and outdoor temperatures is:

$$(1 + \alpha z^{-1})T_{\text{int}}(t) = (1 + \alpha)T_{\text{ext}}(t)$$

The solution of this difference equation may be written as:

$$T_{\text{int}}(t) = h(-\alpha)^t + T^*(t)$$

where  $h$  is a constant and  $T^*(t)$  is a particular solution.

The constant  $h$  and the particular solution are determined by the initial conditions and the input signal. Assume that both temperatures are initially equal to zero and that the outdoor temperature is an impulse signal :  $T_{\text{ext}}=1$  if  $t=0$ , and  $T_{\text{ext}}=0$  if not; then the solution of the difference equation is:

$$T_{\text{int}}(t) = 1 - (-\alpha)^t = 1 - e^{-\frac{t}{\tau}}$$

where  $\tau = \frac{-1}{\log(-\alpha)}$  is the thermal time constant of this input-output process.

This reasoning may be extended for higher order polynomials in the lag operator and for interpretation of difference equations for all input-output relations described by Eqn. 7. We may describe such dynamic processes by parameters  $\tau_1, \tau_2 \dots \tau_n$ , which

are the time constants of the building; they can be calculated as the zeros of the polynomials  $A(z^{-1})$  and  $F(z^{-1})$ .

## RESULTS FOR VARIOUS PROTOCOLS

The different inputs-outputs were obtained by imposing to the building two different protocols, described below. Tables 1-3 summarize the results obtained with the validated models.

### Protocol 1

The solar house was excited with a constant 30000 kJ/hr heat flow; this protocol was used to obtain an approximation of the physical parameters of the building and to verify the adequacy of the used model to describe its behavior.

In automatic control, it is recognized that the length of the sequence used for identification, should be at least three times the response time of the system to a step excitation; in our case, we took a sequence of 750 hours with a 30 mn time step.

TABLE 1  
Results obtained with ARARMAX models for protocol 1

| Model            | $U (W/^{\circ}C)$ | $J (^{\circ}C)^2$ | $S (m^2)$ | $\tau_1 (h)$ | $\tau_2 (h)$ |
|------------------|-------------------|-------------------|-----------|--------------|--------------|
| ARARMAX 2 212 00 | 128,83            | 49,85             | 12        | 73,28        | 0,37         |
| ARARMAX 2 212 10 | 136,06            | 43,1              | 15,04     | 67,31        | 0,77         |

ARARMAX 2 212 1 is an ARARMAX model such as  $n_a=2$   $n_{b1}=2$   $n_{b2}=1$   $n_{b3}=2$  and  $n_c=1$

### Protocol 2

The solar house was excited with a 15000 kJ/hr pseudo-random binary sequence PRBS heat flow with a 6 hours clock period, 6 register discrepancy stages and 380 hours heating sequence.

#### Protocol 2.1

Inputs: PRBS heat flow + solar flow on horizontal surfaces + outside air temperature

Output: inside air temperature.

TABLE 2  
Results of identification with protocol 2.1

| Model             | $U (W/^{\circ}C)$ | $J (^{\circ}C)^2$ | $S(m^2)$ | $\tau_1 (h)$ | $\tau_2 (h)$     |
|-------------------|-------------------|-------------------|----------|--------------|------------------|
| ARARMAX 2 232 00  | 133,24            | 4,90              | 5,76     | 118,8        | 0,17             |
| ARARMAX 2 232 10  | 169,4             | 3.35              | 10.59    | 79,11        | ( <sup>1</sup> ) |
| MISO 1 121 222 00 | 155,85            | 3,75              | 7,45     | 118,8        | 0,47             |
| MISO 1 121 222 11 | 181,67            | 3.25              | 10.86    | 101,8        | 0,32             |

#### Interpretation of results

The results of the identification with a pseudo-random binary sequence seem rather good and physically meaningful, except the appearance of a low instability on the identified parameters. This is probably due to a correlation between the model inputs. We note also the appearance of negative roots for some models, which is a source of

instability. The analysis of the identified parameters shows that the building U values are often consistent with the theoretical one. The values of equivalent solar surfaces are different from one model to another.

We used solar radiation on horizontal surface as input, but the windows are vertical, so the calculated surface is not the solar equivalent surface but the product of this one by a coefficient, which is function of the position of the sun. It would then be more reasonable to use solar radiation on vertical surface instead of that on horizontal one. This was done in Protocol 2.2.

#### Protocol 2.2

TABLE 3  
Results obtained for protocol 2.2

| Model             | $U (W/°C)$ | $J (°C)^2$ | $S(m^2)$ | $\tau (h)$ |
|-------------------|------------|------------|----------|------------|
| ARARMAX 1 221 11  | 134,96     | 4,72       | 11,85    | 131,3      |
| MISO 2 122 111 00 | 149.98     | 3.41       | 9.47     | 155.9      |

#### Observation

The use of solar radiation on south vertical surface for the identification of the physical parameters of the solar house enabled us to improve the values of identified equivalent solar surfaces and it was the goal of its use.

## CONCLUSION

In order to identify the thermal parameters of buildings, using performance data, we have tested several identification methods, based on the assumption of constant parameter linear system. The methods gave results which are consistent with each other and with straightforward calculations.

The developed approach will be very useful for the estimation of the real parameters for solar buildings. However we plan to introduce some improvements, for the identification of the parameters of each solar component alone to improve the method.

Also, some correlation has been noticed between residual and input, essentially due to the solar radiation. Moreover, the stability of the time constant should be improved.

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