Data-based mechanistic modelling of indoor temperature distributions based on energy input

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ABSTRACT

Energy used for building heating, ventilating and air conditioning contributes to a great share in the total energy consumption worldwide. Better understanding and management of energy distribution in those processes is essential for the improvement of process quality and efficiency of energy use. This paper presents a data-based mechanistic modelling approach to model the dynamic indoor temperature distribution in an imperfectly mixed ventilated airspace based on energy input to the system. The combination of classical heat balance differential equations and the data-based modelling techniques for continuous-time system has brought a robust dynamic model suitable for model-based controlling and yet providing a profound insight the energy and temperature distribution in ventilated systems. The effect of changing heat input on the temperature distribution inside a ventilated structure was studied. Dynamic response of indoor temperature to varying energy input could be explained by a second order transfer function model with a high coefficient of determination ($R^2 > 0.99$), a low Young Identification Criterion (YIC < –2.3) and a low model standard error (SE < 0.028 °C). The physically meaningful model parameters as local heat load fraction γ and local temperature change rate coefficient by heat load h (°C/J) were revealed. This modelling approach is very useful for future design of model-based predictive controller for zonal control of indoor temperature by the direct adjustment of heat load into ventilated structures. This approach will allow to energy in climate control.

KEYWORDS
Dynamic model, temperature distribution, energy distribution, zonal control.

INTRODUCTION

Temperature fluctuation and spatial heterogeneous temperature distribution in ventilated systems are indispensable results of imperfect mixing of the airflow, fluctuation of surrounding environmental conditions and variation of heat and moisture generation of occupants themselves. For many researchers concerned with heating, ventilating and air conditioning, improvement in controlling temperature distribution and heat transfer dynamics at local microenvironment by using model-based control theory is a huge challenge (Janssens *et al.*, 2004). Although there are different kinds of control strategy, the use of model-based controller is getting considerable attention in recent years due to its robustness to disturbances (Desta *et al.,* 2004). Therefore, first challenge in the development of such controllers is to have a good dynamic model for the concerned process. Data-based mechanistic (DBM) model, which is hybrid of a mechanistic dynamics model and data-based model, can describe the process in a dynamic simplified manner and provide physical under-standing about the process (Janssens *et al.*, 2004). Further advantage of it is fast

applicability to different systems (Berckmans and De Moor, 1996). In previous studies (Van Brecht, 2004, Janssens *et al.*, 2004, Thanh, 2008), DBM modelling approach has been developed for indirect control of air inside an empty ventilated airspace through adjustment of the inlet air temperature. However, the variation of heat load which also contributes considerably to the dynamic variation of indoor temperature has not been taken into account. This paper outlines the data-based mechanistic modelling approach to model the dynamic indoor temperature distribution in an imperfectly mixed ventilated airspace based on energy input. Profound insight at the distribution of heat load and temperature were obtained through the revelation of physical meaningful model parameters.

MATERIALS AND METHODS

Test installation

Experiments were carried out in a small-scale test chamber (*Figure 1*) at Division of Measure, Model and Manage Bioresponses (M3-BIORES), Catholic University Leuven, Belgium. The test chamber whose inner dimension is 0.8 × 0.6 × 0.4 m was equipped with two aluminium heater fans on the bottom which create forced air ventilation and supply maximum total heat of 200 W to the system. The 4 cm thick surrounding wall is made of propylene. A pipe system with a three-way valve adjusts the flow of fresh air (ventilation) and re-circulated air (recirculation). With 100% valve open for ventilation, maximum ventilation rate of 0.17 m^3 h^{-1} was achieved and no air recirculation. A circulation fan carried out the internal circulation within the chamber with capacity of 12.4 m^3 h^{-1}. Moisture was applied by the steam generator. Temperature and humidity distribution inside the test chamber were recorded by nine temperature-humidity sensors which were placed on a two-dimensional measurement grid. This grid was placed horizontally inside the test chamber.

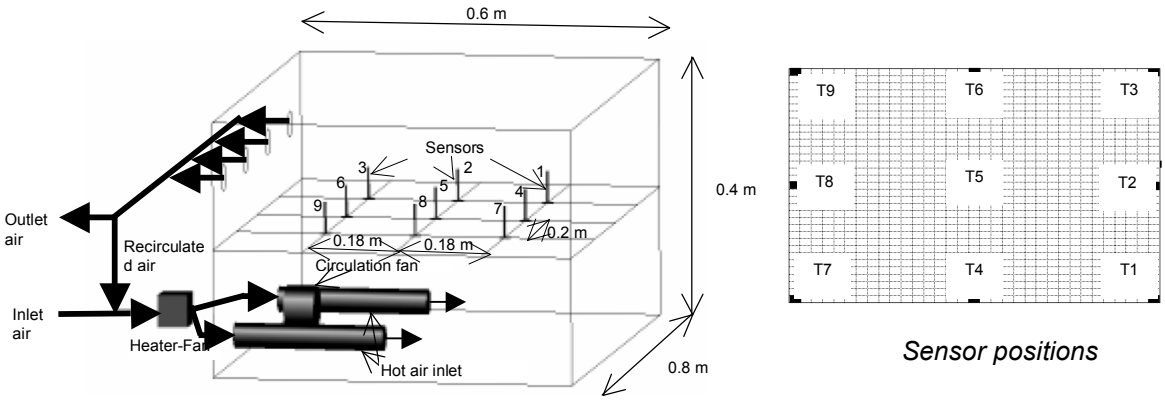

Figure 1. Schema of the small-scale test installation

DBM modelling of indoor temperature response to the variation of energy input

According to Randall (1979, 1981), Timmons (1980), and Barber (1982) total volume of air inside a ventilated space is never perfectly mixed. There is a three-dimensional airflow pattern in the concerned volume with spatial gradients in the local

environment variables (e.g. temperature, humidity…). Assumption was made, although the entire space is not perfectly mixed, it is always possible to exist a *control volume* or a *well-mixed zone* (WMZ) in which there is a better mixing and consequently an acceptable gradient of the variable to be controlled (Berckmans, De Moor, 1996). Assume that there was a homogeneous imaginary buffer zone closed to the boundary of the test chamber. Fresh air from buffer zone (*Figure 2*) which has temperature of $T_{o(t)}$ at time t, constant volumetric flow rate of V ($m^3\ s^{-1}$) passes the heater fan and enters the test chamber. Only a part of this fresh air (V_{ci}) enters the i^{th} WMZ whose volume is vol_i (m^3), the rest passes through other WMZs before going out though the outlet. External heat load to the test chamber at time t is $\dot{Q}_{(t)}$. A fraction of this heat load ($\dot{Q}_{ci(t)} = \gamma_i . \dot{Q}_{(t)}$) is transferred to the i^{th} WMZ by convection, conduction and radiation.

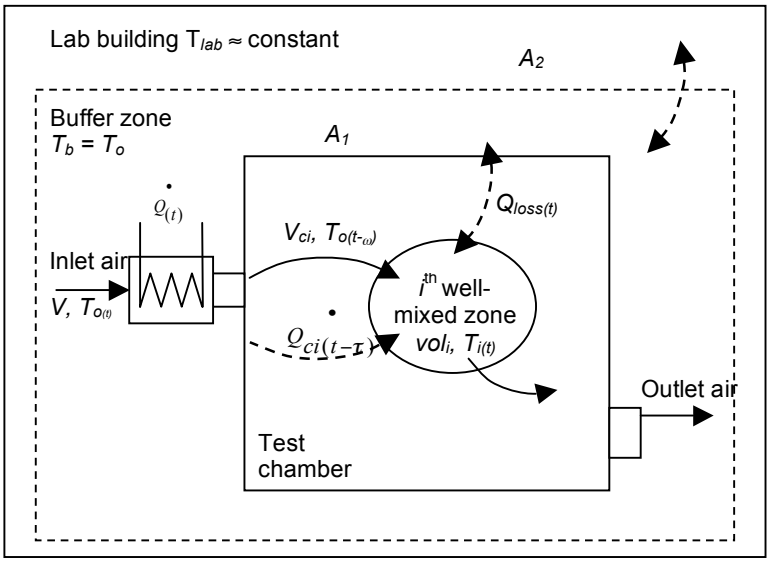

Figure 2 Schematic description of heat and mass transfer in the test installation

To describe the dynamic temperature response in the i^{th} WMZ to step up of heat load, general heat balance equation at time t was written for this WMZ. Assume that there was no significant difference in properties (density, specific heat) of the air in the WMZ, buffer zone and lab zone. Then the heat balance equation can be written as:

$$\underbrace{vol_i.\rho.C_p.\frac{dT_{i(t)}}{dt}}_{Q_{acc(t)}} = \underbrace{V_{ci}.\rho.C_p.T_{o(t-\omega)} - V_{ci}.\rho.C_p.T_{i(t)}}_{Q_{ventilation}} + \underbrace{\gamma_i.\dot{Q}_{(t-\tau)}}_{Q_{load(t)}} - \underbrace{k_{1i}.A_1.\left(T_{i(t)} - T_{o(t)}\right)}_{Q_{loss(t)}} \quad \text{(Eqn. 1)}$$

where C_{pa} and C_{pw} is the specific heat of dry air and moisture respectively; ρ_a is the density of dry air; $Q_{acc(t)}$ is the heat accumulated in the WMZ at time t; $Q_{ventilation}$ is heat of ventilation at time t respectively; $Q_{load(t)}$ and $Q_{loss(t)}$ are external heat load to the WMZ and heat loss through exchanging with buffer zone at time t respectively.

Assume that $T_{o(t-\omega)} \approx T_{o(t)}$ due to slow change in buffer zone temperature then the parameters of (Eqn. 1) can be simplified into (Eqn. 2).

$$\frac{dT_{i(t)}}{dt} = \alpha_i.T_{o(t)}. - \alpha_i.T_{i(t)} + h_i.\dot{Q}_{(t-\tau)} \qquad \text{(Eqn. 2)}$$

where $\alpha_i = \beta_i + K_{1i}$ $\beta_i = \frac{V_{ci}}{vol_i}$ $K_{1i} = \frac{k_{1i}.A_1}{vol_i.\rho.C_p}$ $h_i = \frac{\gamma_i}{vol_i.\rho.C_p}$

Temperature of buffer zone was changed due to heat exchange with all the WMZs (n WMZs) in the test chamber, heat load fraction from heat supply source to the buffer zone ($\dot{Q}_{b(t)} = \gamma_b.\dot{Q}_{(t)}$), and the heat of ventilation. This buffer zone then exchanged heat with the air space in the lab building which acted as such a very large heat sink that its temperature (T_{lab}) was approximately temporally constant. General heat balance equation was written for the imaginary buffer zone with volume of vol_b (m^3) as following:

$$\underbrace{vol_b.\rho.C_p.\frac{dT_{o(t)}}{dt}}_{Q_{accb(t)}} = \underbrace{\sum_{i=1}^{n}\left(k_{1i}.A_1.\left(T_{i(t)} - T_{o(t)}\right)\right)}_{\sum_{i=1}^{n} Q_{loss(t)}} + \underbrace{\gamma_b.\dot{Q}_{(t-\tau)}}_{Q_{loadb(t)}} + \underbrace{V.\rho.C_p.T_{i(t-\varsigma)} - V.\rho.C_p.T_{o(t)}}_{Q_{ventilationb}} - \underbrace{k_2.A_2.\left(T_{o(t)} - T_{lab(t)}\right)}_{Q_{lossb(t)}}$$

(Eqn. 3)

where $Q_{acc\ b(t)}$ is heat accumulation in the buffer zone at time *t*; $Q_{exi(t)}$ is heat exchange from the i^{th} WMZ to buffer zone at time *t*; $Q_{ventilation\ b}$ is heat of ventilation of the buffer zone at time *t* respectively; $Q_{load\ b(t)}$ and $Q_{loss\ b(t)}$ are external heat load to the buffer zone and heat loss through exchanging with heat sink at time *t,* respectively.

Assume that $T_{i(t-\varsigma)} \approx T_{i(t)}$ due to slow changing of WMZ temperature. And assume that there is no significant difference between heat exchanged from every WMZ to the buffer zone, which means:

$$\sum_{i=1}^{n}\left(k_{1i}.A_1.\left(T_{i(t)} - T_{o(t)}\right)\right) \approx \left[A_1.\left(T_{i(t)} - T_{o(t)}\right)\right]\sum_{i=1}^{n}\left(k_{1i}\right) \qquad \text{(Eqn. 4)}$$

Then, (Eqn. 3) can be simplified as following:

$$\frac{dT_{o(t)}}{dt} = \left(K_3 + \beta_b\right)T_{i(t)}. - \left(K_3 + \beta_b + K_2\right)T_{o(t)} + h_b.\dot{Q}_{(t-\tau)} + K_2.T_{lab} \qquad \text{(Eqn. 5)}$$

where $\beta_b = \frac{V}{vol_b}$ $K_2 = \frac{k_2.A_2}{vol_b.\rho.C_p}$ $K_3 = \frac{A_1.\sum_{i=1}^{n}\left(k_{1i}\right)}{vol_b.\rho.C_p}$ $h_b = \frac{\gamma_b}{vol_b.\rho.C_p}$

It is possible to split the process variables into steady and fluctuation parts, such as $T_{i(t)}$ can be split into steady part $T_{i,s(t)}$ and fluctuation part $t_{i(t)}$; and let *s* be the time derivative operator, which means $s = \frac{d}{dt}$, then (*Eqn. 2*) and (*Eqn. 5*) can be rewritten for the fluctuation parts as (*Eqn. 6*) and (*Eqn. 7*), respectively.

$$t_{i(t)} = \frac{\alpha_i}{s+\alpha_i}.t_{o(t)} + \frac{h_i}{s+\alpha_i}.\dot{q}_{(t-\tau)} \quad \text{(Eqn. 6)}$$

$$t_{o(t)} = \frac{K_3+\beta_b}{s+(K_2+K_3+\beta_b)}t_{i(t)} + \frac{h_b}{s+(K_2+K_3+\beta_b)}\dot{q}_{(t-\tau)} \quad \text{(Eqn. 7)}$$

(Eqn. 8) was obtained by substituting of (*Eqn. 7*) into (*Eqn. 6*). It is clear that $t_{i(t)}$ relates to $\dot{q}_{(t-\tau)}$ by a second order transfer function [*n m τ*] of *s*, where (*n*=2) is the order of denominator, (*m*=2) is the order of nominator, *τ* is time delay (s). a_{1i}, a_{2i}, b_{oi}, b_{1i} are the model parameters.

$$t_{i(t)} = \frac{b_{oi}.s + b_{1i}}{s^2 + a_{1i}.s + a_{2i}}.\dot{q}_{(t-\tau)} \quad \text{(Eqn. 8)}$$

where $a_{1i} = \alpha_i + K_2 + K_3 + \beta_b$ $\quad b_{oi} = h_i$

$a_{2i} = \alpha_i(K_2 + 2K_3 + 2\beta_b)$ $\quad b_{1i} = h_i(K_2 + K_3 + \beta_b) + \alpha_i.h_b$

Application of this modelling approach to each of the nine zones inside the test chamber where the temperature sensors were positioned, transfer function matrix (TFM) which describes the spatio-dynamic response of temperature inside the test chamber to the variation of energy input was obtained. This model concept works without prior information about the position of heat source, temperature and position of inlet and outlet air, in the condition of no significant change of surrounding environment temperature. To estimate the TFM model parameters, dynamic experiments were conducted (*Table 1*). Heat load was sharply increased from 0 W to 200 W. The ventilation rate was kept constant at 0.17 m^3 h^{-1}. Internal circulation was 12.4 m^3 h^{-1}. Temperature outside was 17.1°C. Total heat load and temperatures response inside the test chamber were recorded every ten seconds. TFM model parameters were estimated from the experimental time-series data using refined instrumental variable algorithm for contiuous-time system (RIVC) (Young and Jakeman, 1980, Young *et al.,* 2002).

TABLE 1. Overview of dynamic experiments

Energy input (W)	Surrounding temperature (°C)	Duration (s)
0	17.1	1200
200		

To reveal the physical meaning of the model parameters, component analysis of (Eqn. 2) was done. In which $\frac{dT_{i(t)}}{dt}$ is the temperature changing rate (°C s^{-1}) in the WMZ *i; h_i* is the local temperature change rate coefficient by heat load (°C J^{-1});

RESULTS AND DISCUSSIONS

Model identification

Estimated model parameters and model accuracy for the nine zones inside the test chamber are presented in *Table 2.* All of the models yield high coefficient of determination, low YIC and low models standard error (high accuracy). *Figure 3* is an example for comparison of measured temperature and predicted temperature by the DBM model at sensor position T2. The residuals distributed randomly around the 0°C value with small absolute values (less than 0.06°C). This graph strongly confirms the fitness of the [2 2 *τ*] model for predicting temperature response based on energy input.

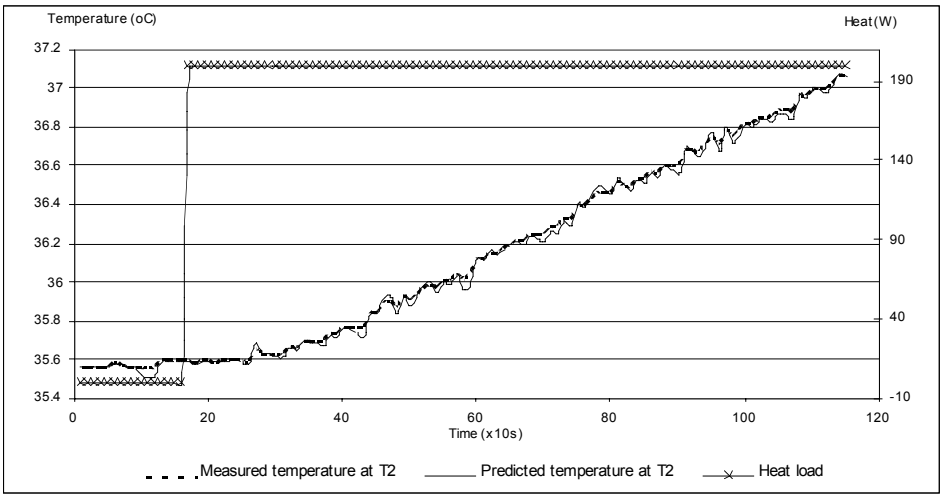

Figure 3. Predicted temperature by model [2 2 9] and measured temperature at the sensor position T2

TABLE 2. The model transfer function [2 2 τ] for nine sensor positions

Sensor position	Transfer function [*n m τ*]	Parameter estimates				Standard error of model (°C)	R_T^2	*YIC*
		a_1	a_2	b_0	b_1			
T1	[2 2 4]	0.001170	0.000038	0.000005	0.000000	0.021	0.999	-6.661
T2	[2 2 9]	0.006418	0.000023	0.000004	0.000001	0.019	0.999	-7.698
T3	[2 2 2]	0.000164	0.000037	0.000002	0.000000	0.018	0.998	-2.304
T4	[2 2 0]	0.001003	0.000032	0.000002	0.000000	0.023	0.998	-5.484
T5	[2 2 6]	0.002539	0.000027	0.000002	0.000000	0.021	0.998	-6.705
T6	[2 2 7]	0.000446	0.000036	0.000003	0.000000	0.023	0.998	-3.185
T7	[2 2 10]	0.002133	0.000035	0.000005	0.000000	0.019	0.999	-6.981
T8	[2 2 5]	0.004853	0.000037	0.000002	0.000001	0.019	0.999	-8.616
T9	[2 2 2]	0.006270	0.000037	0.000002	0.000001	0.028	0.998	-7.355

R_T^2: *coefficient of determination; YIC: Young Identification Criterion value; n, m and τ: denominator order, nominator order and time delay;* a_1, a_2, b_0, b_1 *: estimated parameters of denominator and nominator in the transfer function.*

Local temperature change rate coefficients by heat load

Local temperature change rate coefficient by heat load (h_i = b_{oi}) (°C J^{-1}), which illustrates how much temperature (°C) in a certain zone increase corresponding to 1J of energy input, were estimated for each of the nine zones. It is indicated in *Table 3* that the temperature change rates at position T1 and T7 are the most sensitive to heat load as these positions are close to the hot inlet air. This information is useful for

later improvement of the test installation to have better homogeneous temperature change rate as well as better temperature uniformity inside the ventilated structure.

TABLE 3. Estimated local temperature change rate coefficient, local heat load fraction, local heat load

Sensor position	Local temperature change rate coefficient by heat load h_i (°C J^{-1})	Local heat load fraction γ_i	Local heat load $\dot{Q}_{ci}$ (W)
T1	0.000005	0.000121	0.024203
T2	0.000004	0.000085	0.016936
T3	0.000002	0.000042	0.008303
T4	0.000002	0.000056	0.011166
T5	0.000002	0.000055	0.011007
T6	0.000003	0.000069	0.013760
T7	0.000005	0.000115	0.022969
T8	0.000002	0.000039	0.007769
T9	0.000002	0.000059	0.011749
Total		0.000639	0.127863

Energy distribution in the empty installation

Total volume of the test chamber was divided in to 3×3×1 zones with equal volume of vol_i = 0.021 m^3. Local heat load fraction γ_i (0 < γ_i < 1) for a i^{th} well zone was calculated as: $\gamma_i = h_i.vol_i.\rho.C_p$ (*Table 3*). Finally, local heat load was calculated as: $\dot{Q}_{ci} = \gamma_i . \dot{Q}$ (W), where $\dot{Q}$ = 200 W (*Table 3*).

Heat load distribution is a result of internal air circulation and hot air inlet position. Evidence of heterogeneous heat load distribution is visualized in *Figure 4.* Zones at positions T1, T7, T2 received larger amount of heat load than the others due to the effect of hot air inlet position. However, in comparison with the temperature distribution (*Figure 5*) they were not the positions which had the highest temperature. Large amount of heat load to these zones was compensated by sufficient heat loss through the wall and ventilation. Zones at the corners suffered more from heat loss through the wall, especially position T3 which is closed to the cold inlet air tube.

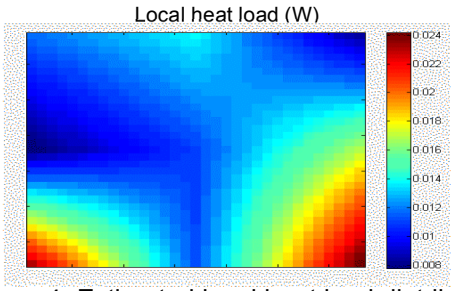

Figure 4. Estimated local heat load distribution inside the test chamber when heat supply is 200 W

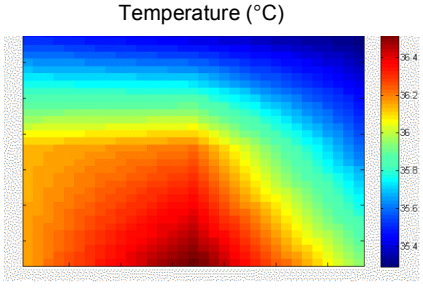

Figure 5. Visualization of 2D measured temperature distribution inside the test chamber

Considering the effectiveness of heat supply when ventilation is turned on, the total heat load which have effect on increasing temperature of air inside the test chamber was 0.128 W, which takes only 0.064% of the total heat load of 200 W. The rest of heat supply did not contribute to the increase of temperature inside the test chamber

but lost due to heating up the test installation and loss to the surrounding. Huge amount of heat may lose through the plexiglas upper cover of the test installation due to low heat insulation capacity compared with the side walls.

CONCLUSIONS

This study has shown that data-based mechanistic approach could be applied for modelling temperature responses inside the empty test installation based on energy input. The dynamic response of inside air temperature to the variation of energy input could be modelled by a second order transfer function with a high coefficient of determination (R_T^2 > 0.998), a low Young Identification Criterion (YIC < -2.3) and a low model standard errors (SE < 0.028°C) (*Table 2)*. Furthermore, physical meaningful parameters as local heat load fraction γ and local temperature change rate coefficient by heat load *h* (°C J^{-1}) were revealed from the model (*Table 3.*). Experimental determination of these parameters provided a profound insight in distribution of heat load and temperature distribution in a ventilated structure under controlled conditions.

DBM modelling of indoor temperature based on energy input is very promising for applications where indoor temperature distribution is controlled by direct adjustment of energy input (e.g. mechanical heating, cooling, air conditioning) and/or the variation of heat generation by occupants (e.g. food products, plants, animals) or by processes (e.g. fermentation, incubation) have considerable effect on the dynamic response of temperature inside the concerned ventilated systems.

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