# ECONOMICALLY OPTIMAL DIMENSIONING OF A COUNTERFLOW AND CROSSFLOW HEAT EXCHANGER FOR FREE COOLING

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## ABSTRACT

During the cold seasons, it is possible to use ambient air to cool down the coolants in air-conditioning or other refrigeration applications. Applying this free cooling reduces the energy costs of a refrigeration machine. This paper presents a general method for the economically optimal dimensioning of a free cooling system in which a heat exchanger of the counterflow or crossflow type is applied to the heat transfer between the ambient air and the coolant.

The following costs are taken into account in deriving the method: 1) the investment and annual maintenance costs of the free cooling system, 2) the energy costs in order for the pumps and fans to overcome the pressure losses in the heat exchanger for the free cooling, and 3) the avoided energy costs of the refrigeration machine. The optimization is based on maximizing the net present value of the economical gain obtained from using free cooling.

As a result of the optimization using the presented method, economically optimal values are obtained, for instance, for the recuperation ratio and dimensionless conductance of the heat exchanger, as well as for the design temperatures for the dimensioning of the free cooling system.

The method proposed here for the economically optimal dimensioning of the free cooling system improves the quality of the design and increases the profitability of the investments made in it while saving finite resources. One advantage of the proposed method is that a customer-oriented investment strategy and investment criteria, such as the interest rate, operating time and local price of electricity, can easily be taken into account in the dimensioning of the free cooling system.

## KEYWORDS

free cooling, heat exchanger, dimensioning, optimization

## INTRODUCTION

In the Northern and Southern regions of the globe, ambient air can be utilized during the cold seasons for cooling down the coolants in air-conditioning or other refrigeration applications. The use of free cooling permits the reduction of the consumption of electricity and, thus, the energy costs of a refrigeration machine. Applying free cooling requires an investment, and the pumps and fans in the system also incur energy costs as well as annual maintenance costs. Consequently, an economic optimum exists for the dimensioning of a free cooling system.

This paper considers a free cooling system (Figure 1) in which a heat exchanger of the counterflow or crossflow type is applied in cooling down the returning coolant from a cooling application. When the free cooling power is smaller than the cooling load, the remaining cooling is generated by the refrigeration machine.

The objective of this study is to develop a general method for the economically optimal dimensioning of a free cooling system.

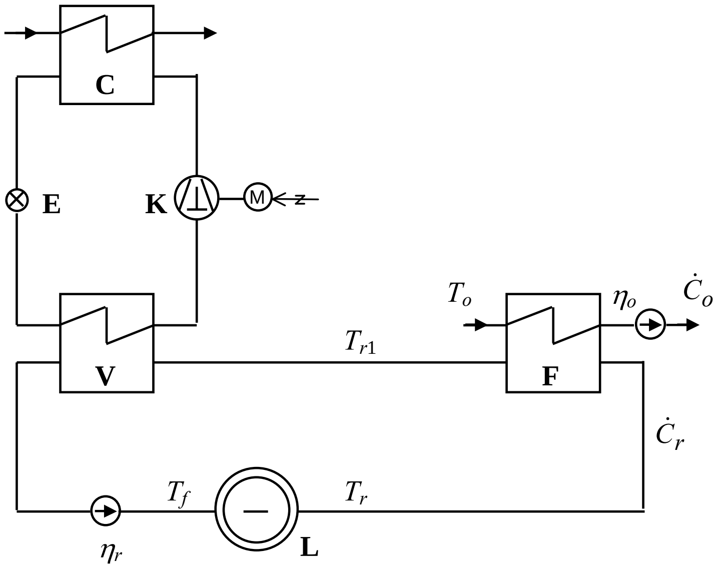


Figure 1: The schematic of a cooling system. F is the heat exchanger for free cooling, L the cooling load, V the evaporator, K the compressor, C the condenser and E the expansion valve of the refrigeration machine.

In deriving the method, the following costs are taken into account:

- the investment and annual maintenance costs of the free cooling system
- the energy costs in order for the pumps and fans to overcome the pressure losses in the heat exchanger for free cooling
- the avoided energy costs of the refrigeration machine

The method is based on maximizing the net present value of the economical gain obtained from the application of the free cooling system.

Parallel to the method presented in this paper, Sarkomaa et al. (2003) have presented a general method for the economically optimal dimensioning of a refrigeration machine when applying free cooling.

## DIMENSIONING EQUATIONS FOR THE FREE COOLING HEAT EXCHANGER

The ratio of the smaller heat capacity flow to the higher heat capacity flow in the heat exchanger is denoted with $R$. The heat capacity flow of the cooling ambient air is selected to be higher than that of the coolant flow so that, when following the notation in Figure 1, we obtain

$$R = \frac{\dot{C}_r}{\dot{C}_o} = \frac{q_{mr}\ c_{pr}}{q_{mo}\ c_{po}} = 0.2 \ldots 0.7 , \qquad (1)$$

where $q_m$ is the mass flow and $c_p$ the specific heat capacity at constant pressure.

The recuperation ratio, $\varepsilon$, for the heat exchanger is defined as

$$\varepsilon = \frac{T_r - T_{r1}}{T_r - T_o} \tag{2}$$

The dimensionless conductance, $z$, is defined using the smaller heat capacity flow

$$z = \frac{k\ A}{\dot{C}_r} = z(\varepsilon, R), \tag{3}$$

where $A$ is the heat transfer area and $k$ the overall heat transfer coefficient of the heat exchanger.

For a counterflow heat exchanger with $0 \le R < 1$, the dimensionless conductance is

$$z = \frac{1}{1-R} \ln \frac{1-R\varepsilon}{1-\varepsilon} \tag{4}$$

For a crossflow heat exchanger (single pass) with both fluids unmixed and with $0 < R \le 1$, the recuperation ratio can be calculated, using the Bessel functions $I_1(x)$, from the equation derived by Sahlberg (1969)

$$\varepsilon = \frac{1}{\sqrt{R}} \int_0^{z\sqrt{R}} e^{-\left(\frac{1}{\sqrt{R}} + \sqrt{R}\right)t} I_1(2t) \frac{dt}{t} \tag{5}$$

## THE COSTS AND ECONOMICAL GAIN OF THE SYSTEM

The ambient air cools down the returning coolant when the ambient temperature $T_o < T_r$. At higher ambient temperatures, the heat exchanger for free cooling is bypassed. Free cooling fulfils only a fraction of the need for cooling, and the free cooling system operates at full load when

$$T_\varepsilon = T_r - \frac{T_r - T_f}{\varepsilon} \le T_o < T_r \tag{6}$$

In the equation, the lower temperature limit is denoted with $T_\varepsilon$. The possible power of free cooling is higher than the cooling load and the free cooling system must be regulated when $T_o < T_\varepsilon$. The times that correspond with the temperature limits $T_\varepsilon$ and $T_r$ at a duration curve for ambient temperature (Figure 2) are denoted with $t_\varepsilon$ and $t_r$.

In the optimization domain, the investment, maintenance and pumping costs of the free cooling system are presented as being linearly proportional to the heat transfer surface area in the free cooling exchanger. Denoting the ratio of the annual maintenance costs to the investment costs with $r$, the net present value, $K_I$, of the investment, maintenance and pumping costs becomes

$$K_I = K_{I0} + A h (1 + r a) + \left[ \dot{C}_r A \left( \frac{\Delta p'_r}{\rho_r c_{pr} \eta_r} + \frac{\Delta p'_o}{R \rho_o c_{po} \eta_o} \right) \right] t_r e a \,, \qquad (7)$$

where $K_{I0}$ refers to the constant part of the costs. The marginal costs of the heat transfer surface (the derivative of the costs of the heat transfer surface with respect to the surface area at the optimum) are denoted with $h$ [€/m$^2$] and the factor of the present value of the periodic payment with $a$. $e$ [€/kWh] is the unit price of electricity. $\Delta p'$ refers to the derivative of the pressure loss with respect to the surface area at the optimum and $\eta$ to the product of the efficiencies of the electric motor, transmission and pump or fan.

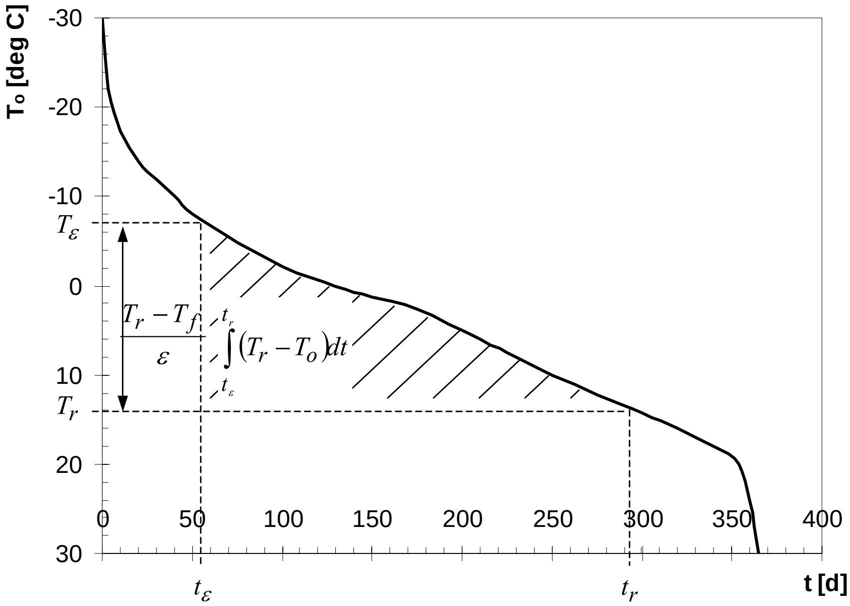


Figure 2: An example of a duration curve for the ambient temperature, $T_o$, during the year. Location: Lappeenranta, Finland.

The net present value, $K_e$, of the energy that has been substituted by the system is

$$K_e = \dot{C}_r a \frac{e}{C_P} \left[ \int_{t_\varepsilon}^{t_r} (T_r - T_o) \varepsilon \, dt + (T_r - T_f) t_\varepsilon \right], \qquad (8)$$

where the heat capacity flow, $\dot{C}_r$, is assumed to be constant. The coefficient of performance of the refrigeration machine that is to be replaced is denoted with $C_P$.

The net present value, $K$, of the economical gain obtained from investing in free cooling is calculated from

$$K = K_e - K_I \qquad (9)$$

## OPTIMIZATION OF THE SYSTEM

Introduce $J$ as the economical efficiency of the free cooling heat exchanger as

$$J = \frac{k}{h\left[1 + a\,(r + e^{*})\right]}, \tag{10}$$

where

$$e^{*} = \frac{\dot{C}_r}{h}\left(\frac{\Delta p_r'}{\rho_r c_{pr} \eta_r} + \frac{\Delta p_o'}{R\,\rho_o c_{po} \eta_o}\right) t\, e \tag{11}$$

By substituting Eqns. 3, 7, and 8 into Eqn. 9 and using the notations from Eqns. 10 and 11, the net present value, $K$, of the free cooling investment can be presented as

$$K = K_0 + \dot{C}_r a \frac{e}{C_P}\left[\int_{t_\varepsilon}^{t_r} (T_r - T_o)\varepsilon\, dt + \left(T_r - T_f\right) t_\varepsilon\right] - \dot{C}_r \frac{h}{k}\left[1 + a\left(r + e^{*}\right)\right] z\,, \tag{12}$$

where $K_0$ refers to the constant part of the costs.

By multiplying the above equation by $J/\dot{C}_r$, the varying part, $K^*$, of the dimensionless net present value of the free cooling investment becomes

$$K^{*} = J\,a \frac{e}{C_P}\left[\int_{t_\varepsilon(\varepsilon)}^{t_r} \left[T_r - T_o(t)\right]\varepsilon\, dt + \left(T_r - T_f\right)\, t_\varepsilon(\varepsilon)\right] - z(\varepsilon, R) \tag{13}$$

The ratio $R$ of the heat capacity flows is given as an input parameter, and thus, the recuperation ratio, $\varepsilon$, remains a free variable in the optimization. The optimum recuperation ratio is determined by

$$\frac{\partial K^{*}}{\partial \varepsilon} = J\,a \frac{e}{C_P}\int_{t_\varepsilon}^{t_r} (T_r - T_o)\, dt - \frac{dz}{d\varepsilon} = D - \frac{dz}{d\varepsilon} = 0\,, \tag{14}$$

where $D$ is a dimensionless economy number

$$D = J\,a \frac{e}{C_P}\int_{t_\varepsilon}^{t_r} (T_r - T_o)\, dt \tag{15}$$

For a counterflow heat exchanger, the optimum recuperation ratio is gained by differentiating the dimensionless conductance in Eqn. 4 and substituting it into Eqn. 14. The resulting optimum becomes

$$\varepsilon = \frac{1+R}{2R} - \sqrt{\left(\frac{1-R}{2R}\right)^2 + \frac{1}{DR}} \tag{16}$$

For a crossflow heat exchanger with both fluids unmixed, the optimum dimensionless conductance is given by the following, as presented by Sarkomaa (1977) and Sarkomaa (1979)

$$z = \frac{1}{1+R}\left[\ln 2D + \ln \frac{I_1\left(2z\sqrt{R}\right)}{2z\sqrt{R}}\right] \tag{17}$$

## DISCUSSION

In this paper, general equations have been derived for the economically optimized dimensioning of a free cooling system in which a heat exchanger of the counterflow or crossflow type is applied in the heat transfer between the ambient air and the coolant. As a result of optimization using the presented method, economically optimal values are obtained, for instance, for the recuperation ratio and dimensionless conductance of the heat exchanger, as well as for the design temperatures for the dimensioning of the free cooling system.

An economically optimized system may also be unprofitable. If the dimensionless economy number from the optimization remains below unity, additional investment in free cooling will be unprofitable and the system must not be implemented unless other needs arise. In this case, the optimization minimizes the losses incurred in the implementation of the system. Consequently, the investment decision must always be based on a feasibility study performed for the system.

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