Derivation of simple ventilation and thermal models for a naturally ventilated auditorium with high internal heat gains.

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Abstract

Measurements of ventilation rates and internal temperatures have been recorded in a naturally ventilated auditorium with high intermittent heat gains for a wide range of weather conditions at a UK site. Satisfactory internal temperatures and high ventilation rates have been found for "winter", "mid-season" and "summer" external conditions.

Simple ventilation and thermal models have been derived from experimental data which allow the prediction of ventilation rates and internal temperatures within the auditorium despite the complex natures of the flow regimes and heat transfer mechanisms present. The ventilation model assumes that flowrates are driven by the buoyancy effect. The thermal models have been derived assuming there is one dominating effect driving internal temperatures in the space (i.e. the heating system in winter and external temperatures in summer).

Good correlations have been found between predicted flowrates and internal temperatures and those actually measured in the auditorium. The models could be used to estimate flowrates and internal temperatures in similar naturally ventilated enclosures.

Notation

A_E	Effective area (m^2)
C_d	discharge coefficient
$G(s)$	Transfer function (Laplace domain)
h_1	height above datum of "inlet" (m)
h_2	height above datum of "outlet" (m)
$I(s)$	Input (Laplace domain)
k_{os}	flow coefficient ($m^3\ s^{-1}$)
$O(s)$	Output (Laplace domain)
ΔP	Pressure difference (Pa)
q_v	volumetric flowrate ($m^3\ s^{-1}$)
ρ_c	density of outside air ($kg\ m^{-3}$)
ρ_h	density of internal air ($kg\ m^{-3}$)
T_{ai}	internal air temperature (^oC)
T_{ao}	external air temperature (^oC)
T_c	ceiling slab surface temperature (^oC)
τ_c	time constant (h; cooling)
τ_h	time constant (h; heating)

1. Introduction

General description of the building and installed systems

The Queens Building, De Montfort University, Leicester, contains two naturally ventilated lecture theatres, each of which have maximum occupancy levels of 150. Prime features of these spaces are the use of tall ventilation stacks to promote natural buoyancy driven ventilation as a means of avoiding the use of mechanical ventilation and/or full air conditioning systems. A high percentage of openable facade allow effective use to be made of natural ventilation, while large areas of exposed walls and floors increase the effects of radiant cooling. Night venting is used in summer to pre-cool these enclosures.

The two lecture theatres have self contained natural ventilation systems. In the theatre under investigation, outside air enters adjacent to a lightly used road into three plenums via modulated control dampers (see figure 1), disperses within voids underneath the seating and enters the room via vertical grilles positioned at ankle height. Air leaves the space via two large openings near the base of two stacks. Outlets are positioned at the top of each stack in the form of eight windows. Inlet and outlet positions can be varied by the operation of motors controlled by the building management system (BMS). A "punka" type or "ceiling" fan is installed in one of the stacks to provide back-up ventilation in the event of inadequate stack flow rates. Heating in winter is provided by finned tubes placed within the seating void adjacent to the vertical grilles.

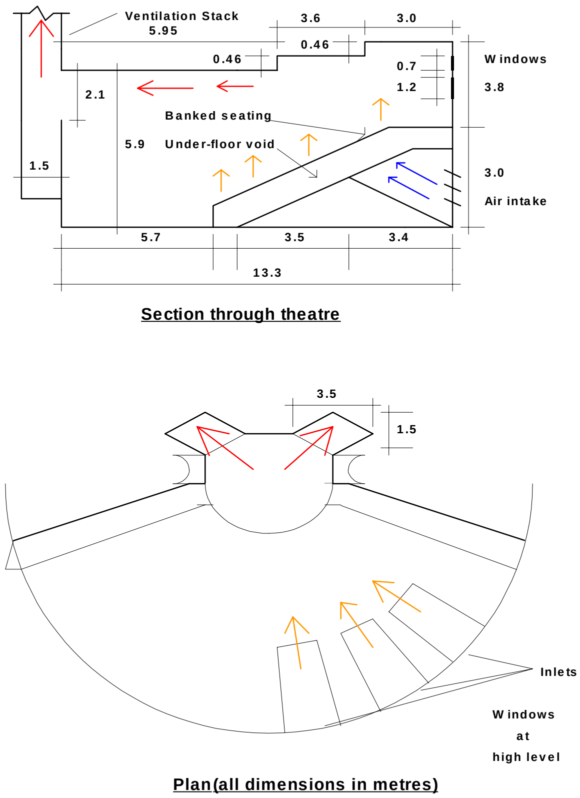

Figure 1 Section and plan of auditorium 1.10 showing the general directions of air movement. (Figures shown are the room dimensions).

2. Experimental results

The measurement results [1] have indicated that thermally comfortable conditions and adequate ventilation rates can be achieved throughout the year for a thermally heavyweight, naturally ventilated auditorium. This has never been demonstrated before for this type of enclosure and ventilation system, due to the uniqueness of the space construction, and the type of ventilation system installed. It has also proved to be difficult to measure ventilation rates in the building due the variability of flowrates, and external conditions (i.e. wind speeds, and directions).

The results have shown the enclosure can provide comfortable internal conditions which show little variability despite large variations in external temperatures and high maximum temperatures (above 30 ^{o}C) in summer. Comfortable internal environments have also been measured for winter external conditions.

3 Prediction of ventilation rates and air speeds

3.1 Buoyancy driven flow only (winter and mid-season conditions)

The CIBSE stack equation [2] was used at the design stage to determine whether a set of given opening areas would be big enough to provide enough fresh air. The results showed that sufficient air could be provided to the space when the vents were open and that good agreement was reached between the flow rate calculated by the CIBSE formula and the measured flow rate. However, this assumes that the stack effect is the predominant driving force. It appears that for a temperature difference of less than about 2 ^{o}C wind induced pressure differences are approximately the same as those generated by the stack effect, assuming the wind speed (adjacent to the outlets) is about 3 m s^{-1}. It has been found that the formula gives satisfactory flow rate predictions in "winter" and mid-season" but less accurate predictions in "summer".

It is possible to derive a formula for deriving stack induced pressures. This can be incorporated into Bernouillis equation to find flowrates, i.e.;

$$q_v(m^3/s) = C_d A_E \sqrt{\frac{2\Delta P}{\rho_h}} \qquad (1)$$

Equation 1 can be re-written as (see appendix for derivation):

$$q_v(m^3/s) = C_d A_E \sqrt{2g(h_2 - h_1)\left[\frac{T_{ai} - T_{ao}}{T_{ao}}\right]} \qquad (2)$$

Assuming that very little variation occurs in h_2 and h_1 for different opening areas, then for a given opening setting, the above equation can be simplified to:

$$q_v(m^3/s) = k_{os}(i)\left[\frac{T_{ai} - T_{ao}}{T_{ao}}\right]^{0.5} \tag{3}$$

where $k_{os}(i)$ = stack ventilation constant for a particular os (outlet setting) (i = 1, 2, 3 for settings of 50 %; 75 % and 100 % respectively).

The outlet setting values can be found by plotting $((T_i\text{-}T_o)/T_o)^{0.5}$ versus measured volumetric flowrates (see figure 2). A "best fit" line passing through the origin has been drawn through the points and the slope of this line has been calculated.

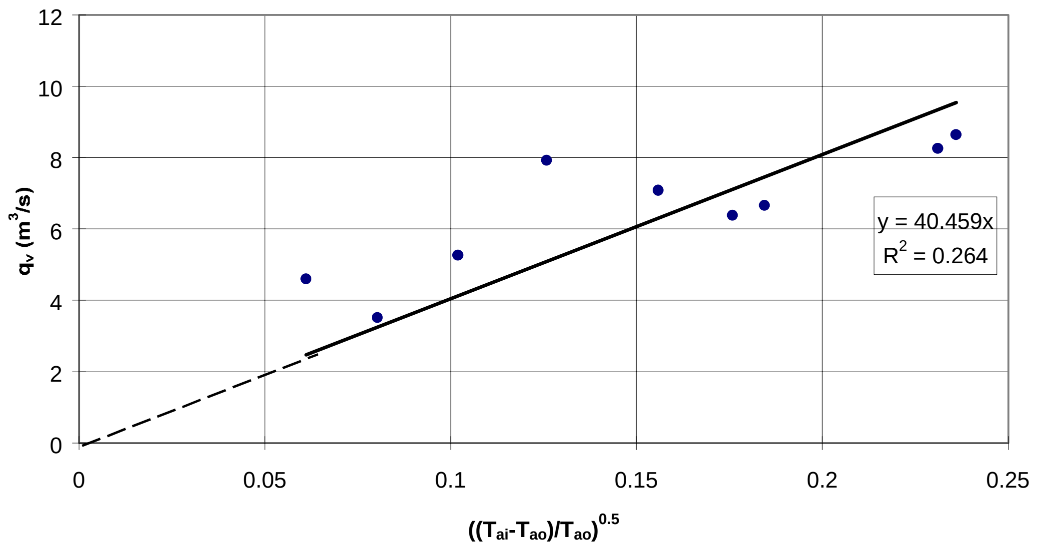

Figure 2 Measured flowrate versus $((T_{ai}\text{-}T_{ao})/T_{ao})^{0.5}$. Effective area is 100 % of maximum area.

The above plot has allowed the slopes of best fit lines (or k_{os} co-efficients) to be obtained. Hence the co-efficient is as follows: k_{100} = 40.46 m^3/s (co-efficients for the other opening settings have also been obtained, i.e. k_{50} = 26.56 m^3/s; k_{75} = 35.22 m^3/s)

One co-efficient can be derived from the above constants by plotting the above co-efficients against area ratios. Hence a more general formula can be produced:

$$q_v(m^3/s) = 39.8\frac{A_E}{A_{E(Max)}}\left[\frac{T_{ai} - T_{ao}}{T_{ao}}\right]^{0.5} \tag{4}$$

where A_E is the actual effective area and $A_{E(Max)}$ is the maximum effective opening area (3.57 m^2) used.

The following table provides measured and calculated flow rates for selected winter and mid-season runs and gives the percentage difference between the two sets of figures. Calculated flowrates have been found using the experimentally derived flow co-efficients (k_{50}; k_{75} and k_{100}; see above) and equation 3. Good agreement is evident for most runs.

Season	BMS Settings (%)	Opening Area (m^2)	Avg. Temp. Internal (^oC)	External (^oC)	Measured Flowrate (m^3/s)	Calculated Flowrate (m^3/s)	Difference (%)
Winter	50	2.43	21.2	3.3	6.53	6.76	-3.5
	75	3.18	20.5	3.2	8.26	8.82	-6.8
	100	3.57	19.8	4.4	8.64	9.55	-10.5
	50	2.43	19.5	2.2	6.43	6.67	-3.7
	75	3.18	19.7	4.3	8.50	8.32	2.1
	100	3.57	19.5	4.7	8.26	9.35	-13.3
	50	2.43	17.3	4.9	6.00	5.62	6.4
	75	3.18	16.9	5.0	7.25	7.29	-0.6
Mid-	50	2.43	19.9	10.0	5.24	4.97	5.1
season	75	3.18	19.2	9.2	6.05	6.63	-9.5
	100	3.57	18.8	9.2	6.66	7.46	-12.1
	50	2.43	20.4	12.0	4.48	4.56	-1.8
	75	3.18	20.8	11.6	6.47	6.33	2.1
	100	3.57	20.4	11.6	6.38	7.11	-11.5
	50	2.43	19.0	11.3	5.51	4.37	20.7
	75	3.18	18.8	10.9	6.73	5.88	12.7
	100	3.57	18.0	11.1	7.08	6.31	10.9
	50	2.43	20.2	14.5	7.95	3.74	52.9
	75	3.18	20.1	15.3	4.91	4.53	7.8
	100	3.57	20.0	15.4	7.92	5.09	35.8
Average							11.5

Table 2 Comparison of measured and calculated flow rates for selected mid-season runs.

Based on the data shown in Table 2 it would be reasonable to conclude that accurate predictions (to 12 %) can be generated using the CIBSE stack formula provided the average inside/outside temperature difference is of the order of 10 ^oC and wind speeds are less than about 3 m/s. An average error of 9.1 % was obtained for winter and mid-season conditions with inside minus outside temperature differences from approximately 8 ^oC to 15 ^oC.

4 Prediction of internal temperatures

4.1 Calculation of time constants (Winter conditions)

It is desirable to be able to predict internal air and fabric temperatures for a range of external conditions. As can be seen from figure 3 the air temperature rises asymptotically to the room set point when the heating system is on, then drops once the system has been switched off. It is evident that the start-up internal temperature is dependent on the day of the week (an extended plant shut-down period allows the building to lose more heat and reach a lower internal temperature) and external temperature. In figure 3 (winter conditions), the internal temperature drops to about 13 °C (Monday morning) and a pre-heat period of about 4 hours is needed to bring the room temperature up to 19 °C. Temperatures for milder (mid-season) conditions are shown in figure 4 and the corresponding temperature drops to only 18 °C. In this case, the pre-heat time has dropped to 2 hours. Thus, as the external temperature increases, the pre-heat time reduces. This pre-heat period is a function of external temperature and the temperature at start-up (of the heating plant).

If the "heating" and "cooling" time constants are known, then it is likely that a first order model of the system can be produced. This, when validated, could be used to make temperature predictions and estimations of heating energy consumption.

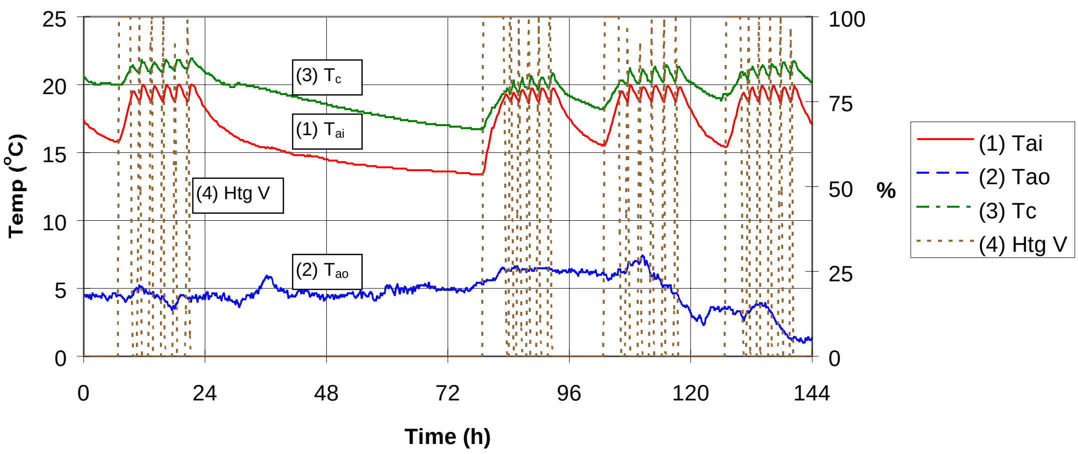

Figure 3 Internal and ceiling slab temperatures for winter external conditions. The position of a heating valve (Htg V) is also shown.

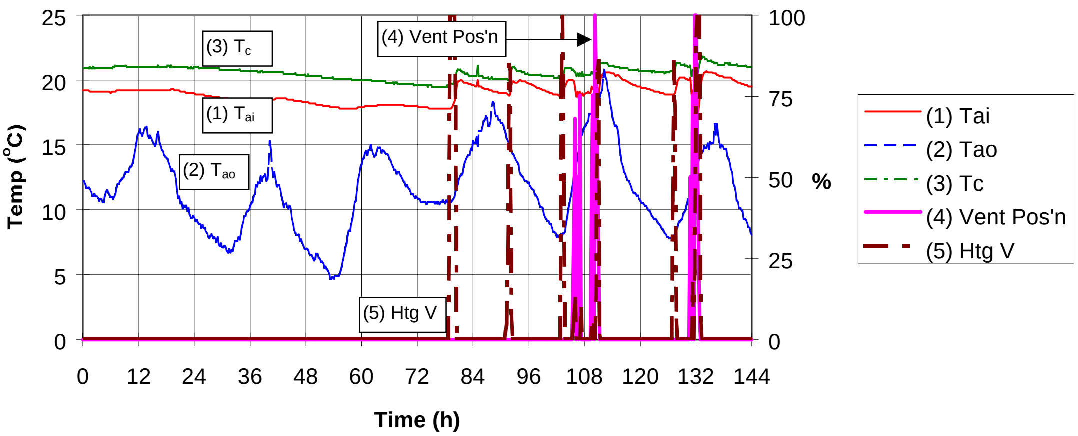

Figure 4 Internal and ceiling slab temperatures for mid-season external conditions; 72 hours is equivalent to a Monday (after a weekend heating plant shut-down). The position of a heating valve is also shown.

From the above plots, it is evident that the space cooling and heating curves can be approximated by outputs of first order differential equations (systems). If the space is assumed to have the same spatial distribution of air and fabric temperatures throughout, then solutions to first order differential equations can be readily derived.

The space temperature will respond in the following manner:

$$T_{ai}(t) = T_{\infty}\left(1 - e^{-\frac{t}{\tau_H}}\right) \qquad \text{(heating)} \qquad (5)$$

$$T_{ai}(t) = T_{\infty}\left(e^{-\frac{t}{\tau_C}}\right) \qquad \text{(cooling)} \qquad (6)$$

where T_{∞} = steady state temperature;
τ_H = heating time constant;
τ_C = cooling time constant.

A number of data files are available that give internal and external temperatures for a range of climatic conditions. Heating and cooling time constants can be found from plots of internal air and surface temperatures versus time. Heating time constants (where the space or building is being heated up) can be obtained from plots of the natural log of (1.0-internal temperatures) against time. The slopes of these curves will be the heating time constants. For cooling time constants (where the building is losing heat), if the natural logs of temperatures are plotted against time, then the slopes of the plots will equal the system cooling time constants.

If air, and surface temperatures can be predicted, it should be possible to estimate average thermal comfort parameters within the space (it is assumed that mean room air speeds are at or below 0.1 m/s and that relative humidity values are about 50 %).

To generate a model of the behaviour of a building when heating is required over a 24 hour period (or longer), the following variables are needed; i) cooling time constant, ii) heating time constant, iii) external temperature, iv) set point temperature (night set-back), and v) set point temperature (normal operation).

4.2 Prediction of internal temperatures in winter

Using previously derived time constants and equations 5 and 6 and knowing the operating times of the auditorium and temperature set points, the internal temperatures can be predicted (see figure 5).

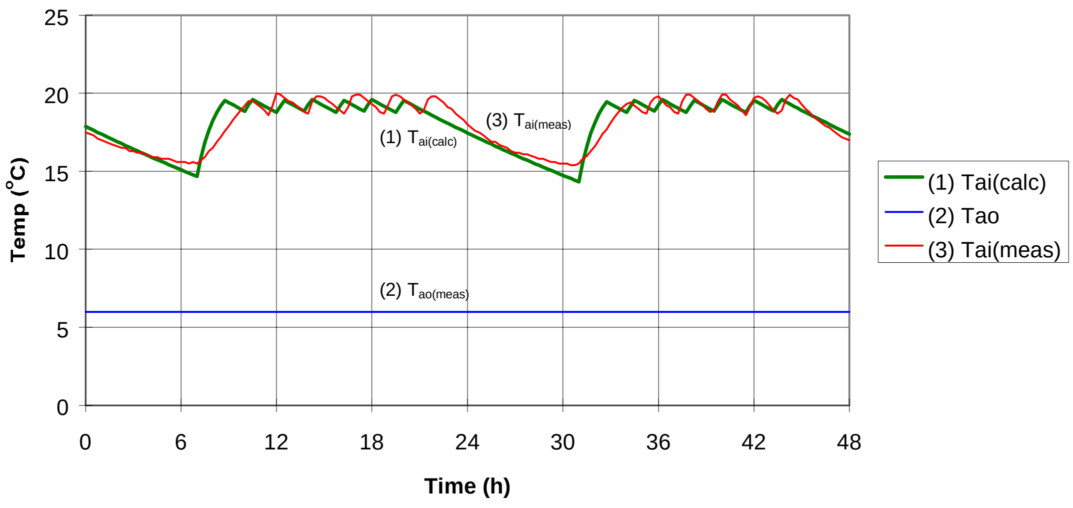

Figure 5 Comparison of measured ($T_{ai\ (meas)}$)and predicted internal air temperatures ($T_{ai\ (calc)}$) using a simple first order model.

4.3 Summer conditions

Internal temperatures in summer can be predicted using a simple model. The behaviour of the model can be calculated assuming it has first order characteristics (using the time constants already derived) and the output will be determined for a sinusoidal variation in external temperature. The variation in time constant with flowrate can be found using the formula supplied in the next section.

The output (variation in internal temperature) of the first order system for a sinusoidal variation in external temperature is found using Laplace transforms [3] (see appendix for the derivation of formulae used).

4.4 Summer conditions - comparison of predictions with experimental results.

Using the formulae above, predictions have been made for internal air temperatures and these are compared with measured values in figure 6.

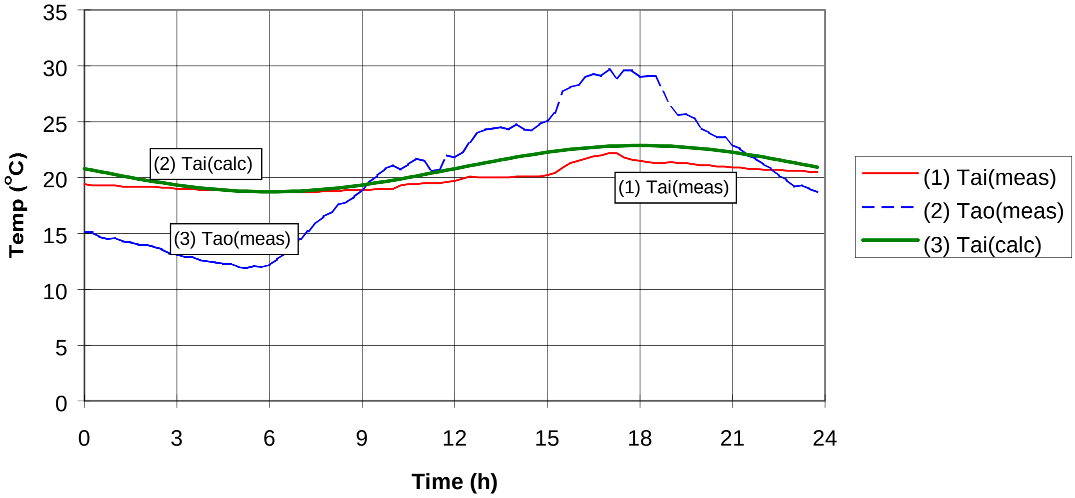

Figure 6 Summer conditions: comparison of predicted and measured internal air temperatures.

5. Applicability of models

Simple models to allow the prediction of ventilation rates and internal temperatures have been developed. However, these have been produced with limited sets of experimental data, i.e. limited ranges of external temperature, wind speeds and directions and internal heat gains. Ideally more data sets could have been generated, thus extending the applicability of the derived models.

To keep the models relatively simple, a limited number of inputs were used. In practice, ventilation rates and internal temperatures are affected by a range of inputs; these include external temperatures, wind velocities, opening areas, thermal mass of the enclosure, internal gains and other factors.

The applicability of each model will now be considered:

(i) *Ventilation model*:
This model assumes that buoyancy effects are dominant (i.e. temperature differences are greater than 3 ^{o}C and wind speeds are less than 3 m/s).

(ii) *"Winter" temperature model*:
The variations in internal temperature are assumed to be dominated by the effects of the heating system (i.e. the heat output of the system is much greater than the magnitude of steady-state heat losses or internal heat gains). The system behaves like a first order process.

(iii) *"Summer" temperature model*:
The changes in internal temperature are considered to be dominated by the effects of external temperature and heat gains are assumed to be low (i.e. at or below average values).

If the above conditions are not satisfied, then the models are not applicable.

6 Conclusions relating to the models used.

6.1 Ventilation models

Accurate predictions of stack driven ventilation rates can be made using the formula previously derived (equation 4). When temperature effects were dominant, the formula gave satisfactory predictions of average ventilation rates. However, inaccuracies increased when stack effects diminished relative to wind induced effects. The large differences between measured and calculated flowrates occurred when stack flowrates were similar in magnitude to wind driven flowrates.

To obtain relatively accurate predictions of ventilation rates driven by both stack and wind effects, it was important to have accurate values for pressure co-efficients; air speeds and directions. Predictions improved when more reliable air speeds were available (when comparing measured and calculated ventilation rates).

It is reasonable to expect an accuracy of +/- 10 % when predicting stack flowrates assuming an inside outside temperature difference of the order of 10 ^{o}C and wind speeds of less than about 3 m/s. For lower temperature differences, to maintain the same level of accuracy, use should be made of the best available values for surface pressure co-efficients and accurate wind profiles or one should use one of the zone ventilation models available in the public domain.

6.2 Thermal models

When viewing plots of internal air and surface temperature versus time for days in winter and mid-season, it appears that it should be possible to predict temperatures using solutions to a first order system. It has been shown that temperatures rise asymptotically and fall exponentially and there are good correlations between "least squares" plots and the actual measured data when the natural log of temperatures is plotted against time. It is also possible to use another first order model to predict internal temperatures in summer.

Average cooling time constants were based on figures for internal air and surface temperatures. Average heating time constants were estimated from the rate at which temperatures increased in the morning after an overnight cooling period.

Time constants should be reduced if the enclosure is ventilated. Ventilation will cause the surface heat transfer co-efficients to be increased; for low air speeds this will depend on the surface and air temperatures.

References

1. CLANCY, E. M., HOWARTH, A. T., AND WALKER, R., "Measurement of case study buildings: (1) De Montfort University Auditorium", workshop presentation, Roomvent conference, Tokyo, Japan, 17 - 19 July 1996.

2. CIBSE, CIBSE Guide A, 1986

3. CLANCY, E., "Factors affecting the environmental performance of a naturally ventilated lecture theatre", PhD thesis, De Montfort University, March 2000.

Appendix

Derivation of "summer" internal temperatures

The first order system can be described using the following plant function:

$$G(s) = \frac{K}{s+a} \qquad \text{(i)}$$

where K is the system gain and a is the inverse of the time constant
(s is the Laplace differential operator).

The input (variation in external temperature) is of the form:

$$I(s) = \frac{\omega}{s^2 + \omega^2} \qquad \text{(ii)}$$

where ω = angular frequency of the external temperature.

Hence the output is:

$$O(s) = G(s)\,I(s) = \frac{K}{s+a}\left(\frac{\omega}{s^2 + \omega^2}\right) \qquad \text{(iii)}$$

The inverse Laplace transform is found; this yields the following co-efficients:

$$A_1 = \frac{\left[\left(\frac{1}{\tau}\right)^2 - \omega^2\right]}{\left[\left(\frac{1}{\tau}\right)^2 + \omega^2\right]} \qquad \text{(iv)}$$

$$A_2 = \frac{-k}{2\omega^2} \qquad \text{(v)}$$

$$T_{ai}(t) = T_{base} + T_{\text{var}}\left(A_1 A_2 + A_2\right)\sin\left((15t + 180)\frac{180}{\pi}\right) \qquad \text{(vi)}$$

where $T_{ai}(t)$ = Internal air temperature;
τ = room time constant (s);
T_{base} = average external temperature (^{o}C);
T_{var} = variation of external temperature

Derivation of "stack flow" equation (equation (2))

$$\Delta P = (\rho_c - \rho_h) g (h_2 - h_1) \qquad \text{(vii)}$$

$$\rho_c = \frac{P_{atm}}{R_a T_{ao}}; \quad \rho_h = \frac{P_{atm}}{R_a T_{ai}} \qquad \text{(viii) \& (ix)}$$

where P_{atm} = atmospheric pressure (N m^{-2})
R_a = gas constant for air (N kg^{-1} K^{-1})

Substituting (viii) and (ix) into (vii) gives:

$$\Delta P = \frac{P_{atm}}{R_a}\left[\frac{T_{ai} - T_{ao}}{T_{ai} T_{ao}}\right] g (h_2 - h_1) \qquad \text{(x)}$$

(x) is re-arranged to provide:

$$\Delta P = \rho_h \left[\frac{T_{ai} - T_{ao}}{T_{ao}}\right] g (h_2 - h_1) \qquad \text{(xi)}$$

The above equation then can be inserted into equation (1) to give equation (2)