NATURAL VENTILATION DESIGN USING LOOP EQUATIONS

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ABSTRACT

The design of natural ventilation systems – the configuration and sizing of system components – assumes one of two generic forms: the *nasty form* based on thermal comfort criteria or the *nice form* based on specified airflow rates. The nasty form demands consideration of the complex coupled interaction of a building's airflow and thermal systems – a difficult and often intractable challenge. The nice form, on the other hand, is quite tractable, yet it is commonly approached using iterative and approximate techniques.

This paper presents an 'exact' approach to the nice design problem based on *pressure loop equations*. It allows direct sizing of a variety of airflow components, unambiguous consideration of stack- and wind-driven airflows without simplifying approximations, and nontechnical constraints and operational strategies to be included in the design process.

INTRODUCTION

The benefits of ventilation to displace stale or contaminated air and to cool have been known to humankind for all time. Why then do we now consider the design of natural ventilation systems such a challenge? The stakes are now higher – there is an energy penalty associated with over-ventilating [1, 2] and comfort and health problems with under-ventilating buildings. Hence we now seek to ventilate buildings with greater control and precision than in the past.

How should a designer approach the design of building ventilation systems to achieve these more demanding objectives? Five distinct design tasks can be identified:

- *Establish Global Geometry* – the siting of the building and landscape configuration, overall building form, and positions of fresh air inlets and stale air exhausts.
- *Establish System Topology* – layout airflow paths from inlet to outlet and select the types of airflow components that will control the airflow rates.
- *Component Sizing* – components of the airflow system must then be sized considering reasonable and relevant *design conditions* and appropriate *design criteria*.
- *Operational Strategy* – a strategy to control the ventilation flow rates must be devised to achieve the objective design criteria.
- *Detail and Assembly* – the designer must develop detail and assembly drawings.

This paper addresses only the sizing task. The *nasty* form of this task was considered in an earlier paper [3]. The more tractable *nice* form is addressed here and in two related papers [4, 5]. Guidance for the first and second tasks may be found in recent British publications [6, 7].

Irving and his colleagues developed an approximate approach to the *nice* design problem [6] that is similar to an approach the author has presented to students at MIT and Yale. Both approaches are based on fundamental theory formulated as *approximate loop equations*. An approach based on an exact formulation of these *loop equations* is presented here.

BASIC THEORY

A building system may be idealized as a collection of control volumes linked by discrete airflow components. For the building illustrated in Figure 1, discrete airflow components are identified by alphabetic labels (i.e., components a, b, c, … g) and four control volumes are considered (i.e., zones 1, 2, 3 and the stack "s").

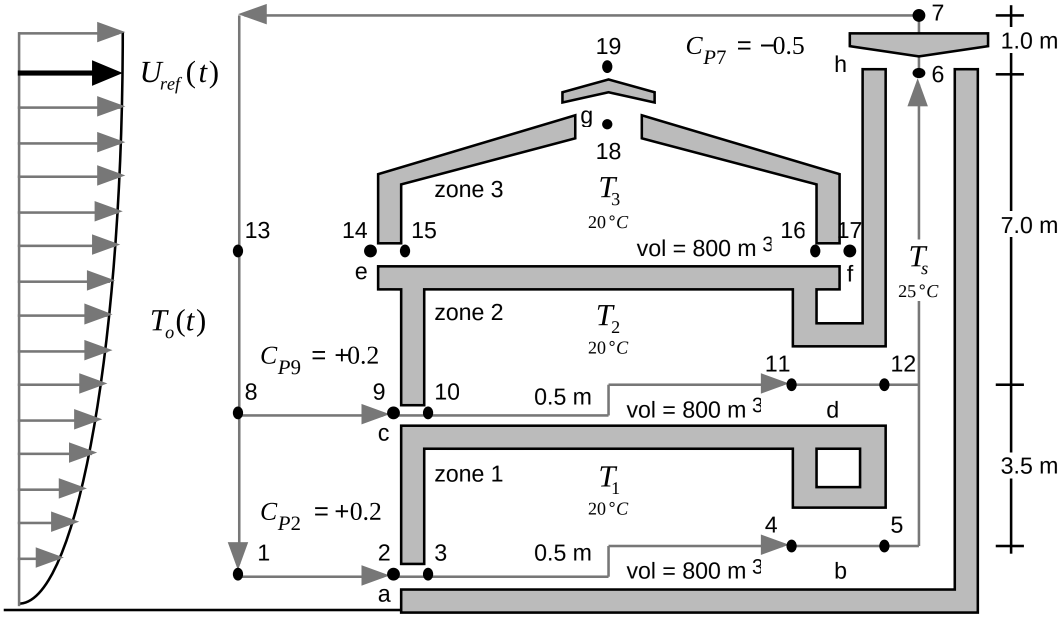

Figure 1 Building idealization based on the Inland Revenue Headquarters, England [6].

Discrete nodes are identified with which values of pressure are specifically defined. The form of the variation of temperature and pressure within the control volumes is assumed and directly related to the nodal values. Finally, one of several pressure-flow relations is then associated with each discrete flow component to complete the building idealization task.

Equations describing the behavior of the airflow system may then be formed by summing the changes of pressure as one traverses a continuous loop that follows possible airflow paths from node to node of the building idealization returning to the original starting node. With reference to Figure 1, one such loop follows the nodal path 1-2-3-4-5-6-7 and back to 1. These changes of pressure must, of course, add up to zero upon completion of the loop.

Wind & Hydrostatic Pressures: At surface locations external to a flow component "*e*" wind-driven pressures P_{eo} are related to the ambient pressure $P_o(z_e)$ at the component level z_e and the dynamic pressure of the wind defined in terms of a wind pressure coefficient C_{Pe}, an associated dynamic pressure of a reference wind speed U_{ref}, and the outdoor air density ρ_o:

$$P_{eo} = P_o(z_e) + C_{Pe}\,(\rho_o U_{ref}{}^2 / 2) \tag{1}$$

Pressure changes $\Delta P_i(\Delta z_{i,j})$ due to elevation changes $\Delta z_{i,j}$ are defined by the discrete form of the hydrostatic equation for the usual assumption of uniform air density $\rho_{i,j}$ distributions or the integral form for nonuniform density $\rho_{i,j}(z)$ distributions:

$$\Delta P_i(\Delta z_{i,j}) = -\rho_{i,j}\, g \Delta z_{i,j} \quad \text{or} \quad \Delta P_i(\Delta z_{i,j}) = -\int_{z_i}^{z_j} \rho_{i,j}(z) g dz \tag{2}$$

Flow Component Relations: Finally, the pressure change ΔP_e, along a component "*e*" is related to the volumetric air flow rate $\dot{V}_e$ and an associated design variable ϕ_e as:

$$\Delta P_e = f(\dot{V}_e, \phi_e) \tag{3}$$

Here, three component flow relations will be considered. The first, useful for larger openings such as windows and doors experiencing unidirectional flow, is based on the orifice equation:

Orifice Component $$\Delta P_e = \frac{\rho \dot{V}_e^2}{2\, C_d^2 A_e^2} \quad \textit{where} \quad \phi_e \equiv A_e \tag{4}$$

The discharge coefficient C_d may be expected to have a value close to 0.60 for flow intensities of interest in most situations. The second relation is based on the equation for flow in ducts:

Duct Flow Component $$\Delta P_e = \frac{f\,L}{D_h}\frac{\rho\,\dot{V}_e^2}{2A_e^2} \quad \textit{where} \quad \phi_e = D_h A_e^2 \tag{5}$$

where f is a friction factor that varies from 0.01 to 0.05 for likely flow intensities encountered, L is the length, D_h is the hydraulic diameter, and A_e is the cross-sectional area of the duct.

Finally, a third empirical relation that approximates the behavior of new self-regulating vent components – components that are central to the design of controllable natural systems – is:

Self-Regulating Vent Component $$\Delta P_e = -\Delta P_o \ln\left(1 - \dot{V}_e / \dot{V}_o\right) \quad \textit{where} \quad \phi_e \equiv \dot{V}_o \tag{6}$$

where ΔP_o corresponds to the pressure difference at which control begins and $\dot{V}_o$ is the limiting (nominal) volumetric flow rate of the device. This relation is compared to measured data for Belgium and French vents reported by de Gids in Figure 2 [8].

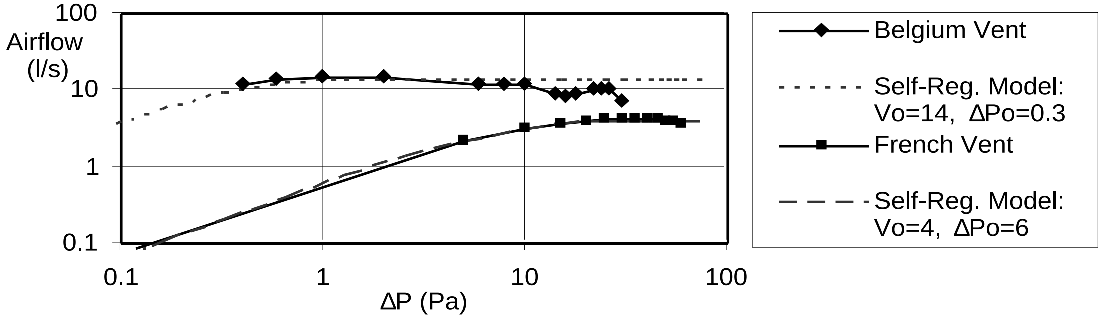

Figure 2 Comparison of self-regulating vent mode with measured data [8].

Loop Equations: Using the equations enumerated above, we may directly form the loop equations for a given building idealization. For the building shown in Figure 1, four *forward* loops could be considered. For example, for loop 1-2-3-4-5-6-7-1 we begin at the ambient pressure node 1 and move forward around the loop adding first the increase due to the wind then the pressure drop along the component a, the hydrostatic decrease resulting from the elevation change from 3 to 4, and so on to obtain:

$$\begin{gathered} +C_{P2}\frac{\rho\, U_{ref}^2}{2} - \overbrace{\Delta P_a}^{\text{inlet}} - \rho_{3,4}\, g\Delta z_{3,4} - \overbrace{\Delta P_b}^{\text{room outlet}} - \overbrace{\Delta P_{bd}}^{\text{stack duct}} - \overbrace{\Delta P_{dh}}^{\text{stack duct}} \\ -\rho_{5,8}\, g\Delta z_{5,7} - \overbrace{\Delta P_h}^{\text{stack terminal}} - C_{P7}\frac{\rho\, U_{ref}^2}{2} + \rho_{7,1}\, g\Delta z_{7,1} = 0 \end{gathered} \tag{7}$$

In general, loop equations involve simply a) a summation of hydrostatic changes that define a stack-driven pressure difference ΔP_s, b) windward and leeward pressures that define the wind-driven pressure difference ΔP_w, and c) a summation of the flow component ΔP_e as:

General Form $$\overbrace{\sum f(\dot{V}_e, \phi_e)}^{\Delta P_e} = \overbrace{g \sum \rho_{i,j}\, \Delta z_{i,j}}^{\Delta P_S} + \overbrace{(\Delta C_P)\frac{\rho U_{ref}^2}{2}}^{\Delta P_w} \tag{8}$$

Continuity: The volumetric airflow rates of the first term of Equation 8 must necessarily satisfy continuity. Thus, for the building idealization of Figure 1 the volumetric flow in the upper part of the stack *dh* must equal the inflow to the stack as $\dot{V}_{dh} = \dot{V}_b + \dot{V}_d$, $\dot{V}_a = \dot{V}_b$, etc.

Design Conditions: To apply the loop equations, one must specify environmental conditions – wind speed and direction and outdoor air temperature. For preliminary calculations extreme or representative values may suffice. For final calculations seasonal or annual variation should be considered. Fortuitously, ΔP_s and ΔP_w depend only on global geometry and system topology. Consequently, the seasonal or annual variation of ΔP_s and ΔP_w may be computed *a priori* and statistical distributions of these variations may then be used to size components.

Feasible Design Surfaces: In design all loop equation variables, except the flow component design parameters, will be known and may be substituted directly. In general, however, one will not be able to define a sufficient number of loop equations to determine a unique solution for the design parameters. In natural ventilation design many alternative *feasible design solutions* may be identified – the loop equations thus define *feasible design surfaces*. This apparent problem, we shall see, presents the designer with the opportunity to include nontechnical considerations and operational strategies in the design process if the problem is approach correctly. This may be made clear by consideration of a practical application.

EXAMPLE APPLICATION

Consider the problem of sizing the system components of the simplified idealization of the Inland Revenue Headquarters. For the purposes of this exercise, the design objective will be to provide a ventilation rate of 5 ACH (i.e., $\dot{V}_a = \dot{V}_c = 1.11 \;\; m^3/s$) during a two month summer period. Given the geometry, pressure coefficients, and assumed internal temperatures shown in Figure 1, the design conditions may be evaluated and the loop equations may be formed.

Modeling the room inlets with the Belgium self-regulating vent model, the room outlets and stack terminal by the orifice model, and the stack by the duct model for a circular cross-section (i.e., $A_s = \pi D_s^4 / 4$) the loop equations may be directly formed:

Loop 1-2-3-4-5-6-7-1:

$$-0.3Ln\left(1-\frac{1.11}{\dot{V}_o}\right)+\frac{2.0535}{A_b^2}+\frac{1.1325}{D_s^5}+\frac{8.214}{A_h^2}=\Delta P_1 \tag{9}$$

Loop 8-9-10-11-12-6-7-8:

$$-0.3Ln\left(1-\frac{1.11}{\dot{V}_o}\right)+\frac{2.0535}{A_b^2}+\frac{1.0067}{D_s^5}+\frac{8.214}{A_h^2}=\Delta P_2 \tag{10}$$

In these equations, the wind and stack pressure terms have been combined into a single *total pressure* term (e.g., $\Delta P_1 = g(\rho_o 12\,m - \rho_1 0.5\,m - \rho_s 11.5\,m) + (C_{P2} - C_{P7})(\rho_o U_{ref}^2 / 2)$).

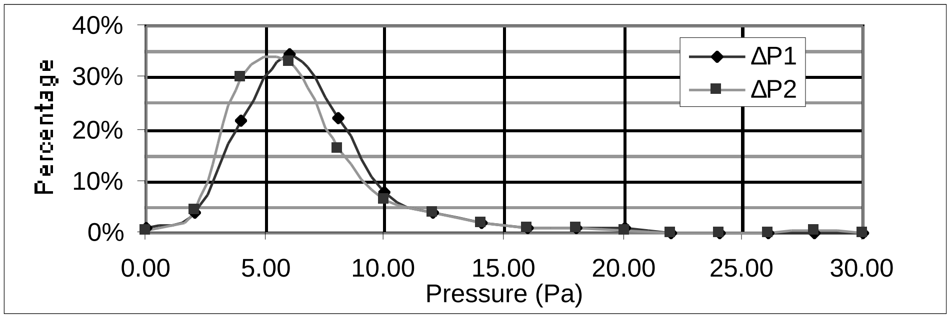

Figure 3 Histogram of stack plus wind pressures for July and August in Boston.

To proceed, *design conditions* must be established. Here, they were evaluated by computing the total pressure on an hour-by-hour basis using Boston WYEC2 weather data [9] and histograms were extracted for the July and August period, Figure 2. From these results it is seen that there is a 90% probability that the total pressure will fall between 2 to 11 Pa for both loops. These (probable) minimum and maximum values will be taken as the design conditions. Importantly, dead-calm conditions are not probable in this case.

Limiting Asymptotes: In general, the loop equations define *feasible design surfaces* bounded by planar asymptotes. These asymptotes define limiting values of design parameters where the resistance of a single flow component governs the flow. They may be determined by systematically considering cases where all but one component is allowed to approach a negligible resistance. For example, the A_b asymptotes of loop 1-2-3-4-5-6-7-1 is determined as:

$$\lim_{\substack{\dot{V}_o \to \infty \\ D_s \to \infty \\ A_h \to \infty}} \left(-0.3 Ln\left(1 - \frac{1.11}{\dot{V}_o}\right) + \frac{2.0353}{A_b^{2}} + \frac{1.1325}{D_s^{5}} + \frac{8.214}{A_h^{2}} \right) = \frac{2.0353}{A_b^{2}} = \overbrace{2.0}^{\Delta P_{1,\min}} \quad or \quad \overbrace{11.0}^{\Delta P_{1,\max}} \qquad (11)$$

Thus, opening *b* must be greater than or equal to 1.013 m^2 for minimum design conditions or 0.432 m^2 for maximum design conditions to achieve the 5 ACH objective.

Table 1 Design surface asymptotes for the example problem.

Design Parameter	**Minimum Design ΔP**	**Maximum Design ΔP**
$\dot{V}_{o,a}$	1.1114 m^3/s	1.11 m^3/s
A_b	1.013 m^2	0.432 m^2
$\dot{V}_{o,c}$	1.1114 m^3/s	1.11 m^3/s
A_d	1.013 m^2	0.432 m^2
A_h	2.027 m^2 loop 1-2-3-… 2.027 m^2 loop 8-9-10-..	0.086 m^2 loop 1-2-3-… 0.086 m^2 loop 8-9-10-..
D_s	0.89 m loop 1-2-3-… 0.87 m loop 8-9-10-…	0.63 m loop 1-2-3-… 0.62 m loop 8-9-10-…

The asymptotes for all design parameters considered are tabulated above. These values offer useful guidance to the designer. For example, component sizes must be equal to or exceed the values in the left column to achieve the ventilation rate of 5 ACH under minimum design conditions. Note that the self-regulating inlet device limits are nearly equal for both minimum and maximum design conditions – i.e., since they *self-regulate*.

Nontechnical Design Constraints & Operational Strategies: The designer can then proceed to select specific sizes to satisfy other design constraints. For example, in this case the designer should select self-regulating inlets such that the nominal size $\dot{V}_o$ is slightly greater than 1.11 m^3/s – say $\dot{V}_o$ = 1.12 m^3/s – and might assume openings *b* and *d* are to be doorways with clear areas equal to 2. 5 m^2. Introducing these constrains, we obtain:

$$Loop\ 1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6\text{-}7\text{-}1: \qquad \frac{8.214}{A_h^{2}} + \frac{1.132}{D_s^{5}} + 1.744 = \overbrace{2.0}^{\Delta P_{\min}} \quad or \quad \overbrace{11.}^{\Delta P_{\max}} \qquad (12)$$

Equation 12 is plotted in Figure 4 below.

The final design specification can now be made. If, for example, the designer chooses to use a 1.75 m (effective) stack diameter, then the stack terminal opening A_h would need to be set to 6.62 m^2 for the minimum and 0.95 m^2 for the maximum design condition. This could be realized with an *operational strategy* whereby the stack terminal is opened wider during windier than calmer conditions – this was done in the Inland Revenue building.

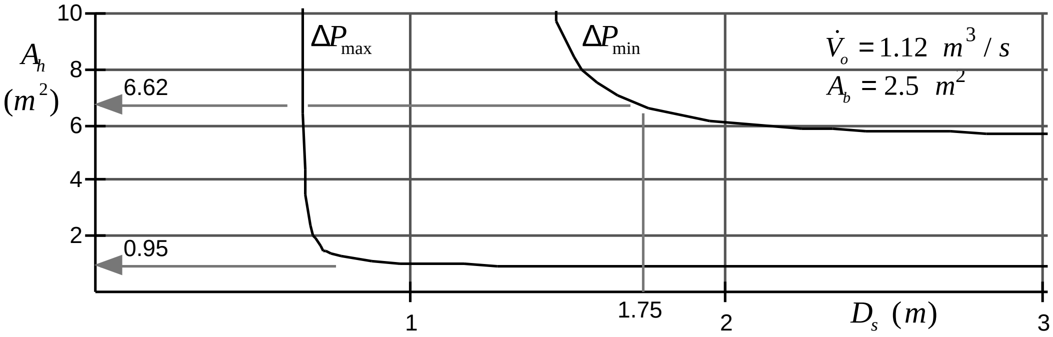

Figure 4 Feasible design curves for loop 1-2-3-4-5-6-7-1 defined by Equation 12.

CONCLUSION

This paper has outlined the basic tasks of natural ventilation design and presented an approach to the sizing of system components based on *loop equations* that is: a) *exact* – based on fundamental theory without simplifying assumptions, b) *complete* – allows the unambiguous consideration of stack and wind effects and specific characteristics of system components, and c) *inclusive* – allows nontechnical constraints and operational strategies to be included. More details will be presented in an AIVC Tech Note “A Practical Guide to Passive Ventilation Air Quality Control in Houses” currently being drafted by the author.

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REFERENCES

1. Orme, M., *Energy Impact of Ventilation: Estimates for the Service and Residential Sectors*, . 1998, AIVC: Coventry.
2. Sherman, M. and N. Matson. *Ventilation-Energy Liabilities in US Dwellings.* in *14th AIVC Conference - Energy Impact of Ventilation and Air Infiltration.* 1993. Copenhagen: AIVC.
3. Axley, J.W. *Macroscopic Formulation and Solution ov Ventilation Design Problems*. in *Ventilation and Cooling: 18th AIVC Conference*. 1997. Athens, Greece: AIVC.
4. Axley, J.W., *Passive Ventilation for Residential Air Quality Control.* (submitted to) ASHRAE Journal, 1998.
5. Axley, J. *Introduction to the Design of Natural Ventilation Systems Using Loop Equations*. in *19th AIVC Conference - Ventilation Technologies in Urban Areas*. 1998. Oslo, Norway: AIVC.
6. Irving, S. and E. Uys, *CIBSE Applications Manual: Natural Ventilation in Non-domestic Buildings*, . 1997, CIBSE: London.
7. Stephen, R.K., L.M. Parkins, and M. Woolliscroft, *Passive Stack Ventilation Systems: Design and Installation*, . 1994, BRE: Watford.
8. de Gids, W.F. *Controlled Air Flow Inlets*. in *18th AIVC Conference - Ventilation and Cooling*. 1997. Athens, Greece: AIVC.
9. ASHRAE, *1997 ASHRAE Handbook – Fundamentals*. SI Edition ed. 1997, Atlanta, GA: ASHRAE.