VENTILATION TECHNOLOGIES IN URBAN AREAS

19TH ANNUAL AIVC CONFERENCE
OSLO, NORWAY, 28-30 SEPTEMBER, 1998

NATURAL VENTILATION BY THERMAL BUOYANCY WITH SEVERAL OPENINGS AND WITH TEMPERATURE STRATIFICATION

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SYNOPSIS
Based on the fundamental flow equations, a set of formulas is derived for air velocities, temperature differences and ventilation rates in relation to number of openings, opening areas, net heat input, building geometry, and temperature stratification. The use of the formulas is illustrated on a three-storeyed office building.

LIST OF SYMBOLS

A	opening area	m^2
C_c	contraction coefficient	
C_d	discharge coefficient	
H	vertical distance between centre of highest and lowest placed opening	m
H_j	vertical distance between neutral pressure plane and centre of opening j	m
H_1^*	weighted height related to bottom opening	m
H_N^*	weighted height related to top opening	m
H_N^{**}	weighted height related to top opening taking temperature differences into account	m
$L_{j,n}$	vertical distance between the centres of opening j and n	m
N	number of openings (or opening levels)	
N_1	number of openings below neutral plane	
R	gas constant (= 287 J/kg K)	J/kg K
T	absolute temperature	K
c_p	specific heat capacity of air (= 1007 J/kg K)	J/kg K
g	gravity acceleration (= 9.82 m/s^2)	m/s^2
p	pressure	Pa
q_v	ventilation rate, volume flow	m^3/s
v	air velocity	m/s
ϵ	temperature factor	
ρ	air density	kg/m^3
ζ	resistance coefficient	
ψ	flow coefficient (= $1 + \zeta$)	
Φ_s	net heat input	W
$\Phi_{s\epsilon}$	modified net heat input	W
Δ	difference in pressure or temperature	

<u>Subscripts</u>

c	contracted
i	indoor
m	mean
o	outdoor
1	lowest inlet
N	highest outlet

1. INTRODUCTION

The formulas available for natural ventilation by thermal buoyancy are usually derived for rooms with two openings and an assumed uniform indoor temperature. However, in practice temperature stratification always occurs, and you have more than two openings when buildings of several storeys are considered.

Few references are available where the temperature stratification is taken into account, but they concern only one-storeyed buildings.

The theory for natural ventilation by thermal buoyancy in buildings with uniform indoor temperature and with two or several opening levels is treated in detail in Andersen (1995). Thermal buoyancy with indoor temperature stratification and with two openings is treated in Andersen (1998).

This paper describes the theory for natural ventilation by thermal buoyancy in buildings with several opening levels and with indoor temperature stratification.

2. THEORETICAL CONSIDERATIONS

By thermal buoyancy, the indoor and outdoor pressures are equal at a certain level called the neutral plane. Through the openings below the neutral plane, air flows inward due to a negative indoor pressure, and through the openings above the neutral plane, air flows outward due to a positive indoor pressure. By using the mass balance equation, the position of the neutral plane can be determined.

2.1 Uniform temperature

For two openings and with uniform indoor temperature the position of the neutral plane can with good approximation be determined by (Andersen, 1995):

$$H_1 = \frac{H}{1 + \left(A_1/A_2\right)^2} \tag{1}$$

where H_1 is the vertical distance from the centre of inlet to the neutral plane, H is the vertical distance between the centres of inlet and outlet, and A_1 and A_2 are the areas of inlet and outlet, respectively.

With uniform temperature and several openings you introduce a "weighted" height H_1^* determined by (Andersen, 1995):

$$H_1^* = H_1 \left[1 + \sum_2^{N_1} \frac{C_{dj}\, A_j\, \sqrt{H_j}}{C_{d1}\, A_1\, \sqrt{H_1}} \right]^2 = H_1 \left[1 + \sum_2^{N_1} m_j n_j q_j^{1/2} \right]^2 \tag{2}$$

where H_1 is the distance from the lowest inlet to the neutral plane, and where the neutral plane position is found by solving the mass balance equation iteratively. Further, N_1 is the number of openings below the neutral plane and:

$m_j = C_{dj}/C_{d1}$, and $n_j = A_j/A_1$, and $q_j = H_j/H_1 = (H_1 - L_{1,j})/H_1$

where again C_d is the discharge coefficient and $L_{1,j}$ is the vertical distance between the bottom opening and opening number j.

2.2 Temperature stratification and several openings

By temperature stratification, the neutral plane moves upward compared to the position by uniform temperature. But the pressure differences and thereby the air velocities in the openings stay approximately unchanged because of the increasing temperature difference above the neutral plane and the decreasing temperature difference below the neutral plane. The approximation is best for the top and bottom openings and less good for the openings closest to neutral plane. However, these openings contributes the least to the ventilation rate. Therefore, the position of the neutral plane can be found approximately from the mass balance equation assuming uniform indoor temperature.

An approximate solution of the basic flow equations is shown in Table 1. The formulas in column I of the table are identical to those valid for a uniform temperature equal to the mean indoor temperature T_{im}, and with a temperature difference $\Delta T = \Delta T_m = T_{im} - T_o$. The weighted height H_1^* is determined by Equation 2.

In column II, a modified net heat input $\Phi_{s\epsilon}$ is the independent variable (cf Appendix A):

$$\Phi_{s\epsilon} = \frac{\Phi_s}{\Delta T_N / \Delta T_m} = \frac{\Phi_s}{\epsilon} \tag{3}$$

where Φ_s is the net heat input, and ΔT_N is the temperature difference at the top opening. The factor ϵ takes into account that a smaller ventilation rate is required compared to the uniform temperature situation because of the bigger temperature difference at the outlet. Similar to H_1^* one has:

$$H_N^* = H_N \left[1 + \sum_{N_1+1}^{N-1} \frac{C_{aj} A_j \sqrt{H_j}}{C_{aN} A_N \sqrt{H_N}} \right]^2 = H_N \left[1 + \sum_{N_1+1}^{N-1} m_j n_j q_j^{1/2} \right]^2 \tag{4}$$

where H_N is the vertical distance from the neutral plane to the centre of the top opening, N is the total number of openings (or opening levels), and:

$m_j = C_{dj}/C_{dN}$, and $n_j = A_j/A_N$, and $q_j = H_j/H_N = (H_N - L_{1j})/H_N$

Moreover a new weighted height H_N^{**} is introduced defined by (cf Appendix A):

$$H_N^{**} = H_N \left[1 + \sum_{N-N_1}^{N-1} \frac{C_{dj} A_j H_j^{1/2} \Delta T_j}{C_{dN} A_N H_N^{1/2} \Delta T_N} \right]^2 = H_N \left[1 + \sum_{N-N_1}^{N-1} m_j n_j q_j^{1/2} r_j \right]^2 \tag{5}$$

where $r_j = \Delta T_j / \Delta T_N$.

The resistance and the contraction properties of the openings are given by the discharge coefficient C_d, and a flow coefficient ψ is introduced. The relationship between the coefficients is (Andersen, 1996):

$$C_d = C_c\, C_v = C_c/\psi^{1/2} = C_c/(1 + \zeta)^{1/2}$$

2.2.1 Four openings. For a building with four openings as shown in Figure 1, the neutral plane position is determined by the mass balance equation. The indoor temperature is assumed to be uniform, the neutral plane position is assumed to be somewhere between openings 2 and 3 and the air velocities are dependent on the square root of the distance to the neutral plane, cf Table 1. One gets:

$$A_1 H_1^{1/2} + A_2 (H_1 - L_{12})^{1/2} - A_3 (H - L_{34} - H_1)^{1/2} - A_H (H - H_1)^{1/2} = 0 \tag{6}$$

Table 1. Formulas by serveral openings and temperature stratification.

Row no.		Column I Formulas based on the mean temperature difference ΔT_m [1) 3)]	Column II Formulas based on the modified net heat input Φ_{se} [1) 2) 3) 4) 5) 6) 7)]
1	Inlets pressure dif., Δp_j (Pa)	$\rho_u \Delta T_m g H_j / T_{im}$	
2	air velocity, $v_{k,J}$ (m/s)	$\left[\frac{2 \Delta T_m g H_j}{\psi_j T_{im}}\right]^{1/2}$	$0{,}038 \left[\frac{\Phi_{se}}{C_{dN} A_N}\right]^{1/3} \left[\frac{1}{H_N^{**}}\right]^{1/6} \left[\frac{H_j}{\psi_j}\right]^{1/2}$
3	Outlets pressure dif., Δp_j (Pa)	$\rho_i \Delta T_m g H_j / T_u$	
4	air velocity, $v_{k,J}$ (m/s)	$\left[\frac{2 \Delta T_m g H_j}{\psi_j T_u}\right]^{1/2}$	$0{,}039 \left[\frac{\Phi_{se}}{C_{dN} A_N}\right]^{1/3} \left[\frac{1}{H_N^{**}}\right]^{1/6} \left[\frac{H_j}{\psi_j}\right]^{1/2}$
5	Temperature dif., ΔT_m (K eller °C)		$7{,}3 \cdot 10^{-5}\, T_o \left[\frac{\Phi_{se}}{C_{dN} A_N}\right]^{2/3} \left[\frac{1}{H_N^{**}}\right]^{1/3}$
6	Ventilation rate, q_V (m^3/s)	$C_{d1} A_1 \left[\frac{2 \Delta T_m g H_1^*}{T_{im}}\right]^{1/2}$	$0{,}039\, (\Phi_{se})^{1/3} (C_{dN} A_N)^{2/3} (1/H_N^{**})^{1/6} (H_N^*)^{1/2}$
7	Top outlet area, A_N (m^2)		$6{,}2 \cdot 10^{-7} \frac{\Phi_{se}}{C_{dN}} \left[\frac{1}{H_N^{**}}\right]^{1/2} \left[\frac{T_u}{\Delta T_m}\right]^{3/2}$
8			$140 \frac{q_V^{3/2}}{(\Phi_{se} H_N^{**})^{1/2} C_{dN}}$ [6)]
9	Bottom inlet area, A_1 (m^2)		$\frac{q_{V_{II6}}}{C_{d1}} \left[\frac{T_{im}}{2\Delta T_m g H_1^*}\right]^{1/2}$ [7)]

1) ΔT_m and T_{im} are the temperature difference and the indoor temperature, respectively, equidistant from top and bottom openings.
2) Φ_{se} is determined by Equation 3.
3) H_1^* is determined by Equation 2.
4) H_N^* is determined by Equation 4.
5) H_N^{**} is determined by Equation 5.
6) q_V is a required ventilation rate.
7) $q_{V,II6}$ is determined by Formula II6 of the table. (Column II, row 6)

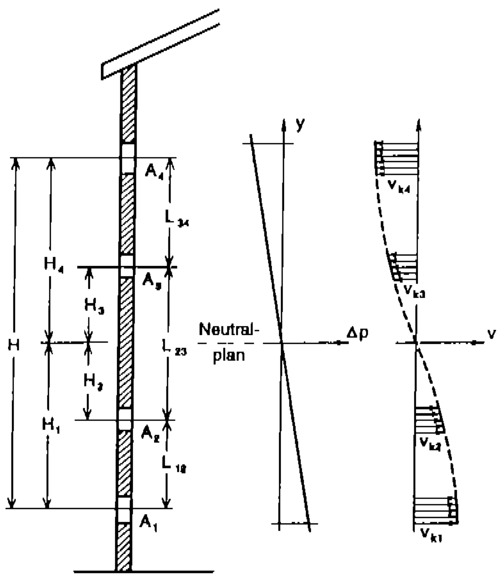

Figure 1. Pressure differences and air velocities by natural ventilation through four openings.

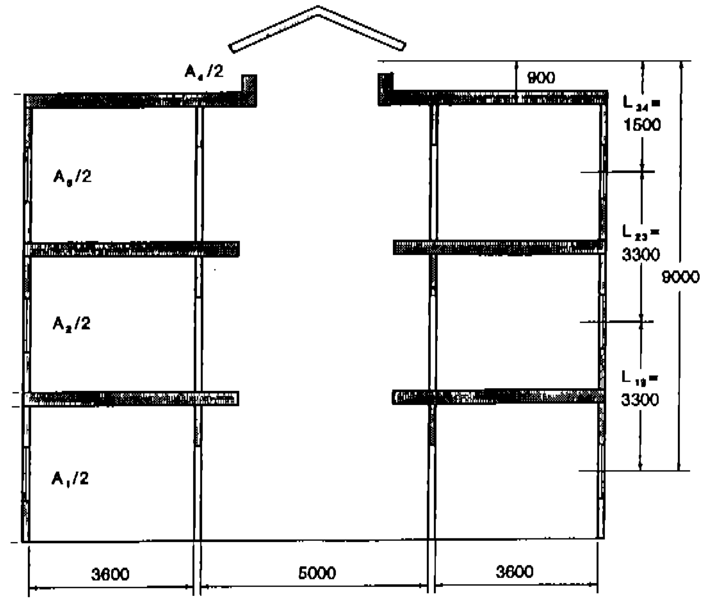

Figure 2. Cross section through three-storyed office building.

where the distance H_1 is the only unknown quantity. The weighted height H_1^* is determined by:

$$H_1^* = H_1\left[1 + \frac{C_{d2}\,A_2\,\sqrt{H_2}}{C_{d1}\,A_1\,\sqrt{H_1}}\right]^2 = H_1\left(1 + m_2\,n_2\,q_2^{1/2}\right)^2 \qquad (7)$$

where H_1 is found from Equation 6, and where:

$m_2 = C_{d2}/C_{d1}$, and $n_2 = A_2/A_1$, and $q_2 = H_2/H_1 = (H_1 - L_{12})/H_1$

The weighted height H_4^{**} is determined by:

$$H_4^{**} = H_4\left[1 + \frac{C_{d3}\,A_3\,H_3^{1/2}\Delta T_3}{C_{d4}\,A_4\,H_4^{1/2}\Delta T_4}\right]^2 = H_4\left(1 + m_3\,n_3\,q_3^{1/2}r_3\right)^2 \qquad (8)$$

where $H_4 = H - H_1$ and where:

$m_3 = C_{d3}/C_{d4}$, and $n_3 = A_3/A_4$, and $q_3 = H_3/H_4 = (H_4 - L_{34})/H_4$, and $r_3 = \Delta T_3/\Delta T_4$

3. EXAMPLE. OFFICE BUILDING

Figure 2 shows the cross section of a three-storeyed office building with an atrium at the centre. It should be ventilated by natural ventilation and the design situation is a calm summer day with an out-door temperature of 25°C and a required indoor temperature in the offices not exceeding 29°C. A 3 m long section of the building is considered.

The indoor temperature stratification is assumed to have a vertical gradient of 0.2 K/m and the openings are sharp-edged with a discharge coefficient of $C_d = 0.62$. Further the net heat input is calculated to $\Phi_g = 2250$ W.

There are openings in four levels, and the necessary opening areas are requested for the following three situations:

- Equally big opening areas at each opening level.
- Graduated opening areas for getting fresh air into all offices.
- Graduated opening areas for getting the same amount of fresh air into all offices.

3.1 Equally large opening areas

With the window opening area of each office equal to $A_1/2$, and with the opening area at the top of the atrium being A_1, the opening area at each of the four opening levels is equal to A_1.

It can be assumed that the neutral plane is placed somewhere between the openings at second and third floors. The position can then be determined by the following mass balance equation, cf Equation 6, with $A_1 = A_2 = A_3 = A_4$:

$$H_1^{1/2} + (H_1 - 3.3)^{1/2} - (6.6 - H_1)^{1/2} - (9.0 - H_1)^{1/2} = 0 \qquad (9)$$

The equation is satisfied for $H_1 = 4.7$ m and this is in accordance with the previously assumed position between openings 2 and 3. With an indoor temperature at third floor of 25 + 4 = 29°C, one gets $\Delta T_m = 3.6$ K and $\Delta T_4 = 4.5$ K, so that $\epsilon = 4.5/3.6 = 1.25$ and $\Phi_{g6} = 2250/1.25 = 1800$ W (cf Equation 3).

The height H_4^{**} is determined by Equation 8 with $H_4 = 9.0 - 4.7 = 4.3$ m, $m_3 = n_3 = 1.0$, $q_3 = 1.9/4.3 = 0.44$, and $r_3 = 4.0/4.5 = 0.89$. One gets:

$$H_4^{**} = 4.3\,(1 + 0.44^{1/2} \cdot 0.89)^2 = 10.9 \text{ m}$$

The opening areas are determined by Formula II 7 (column II, row 7) in Table 1:

$$A_4 = A_1 = 6.2 \cdot 10^{-7}\, \frac{1800}{0.62} \left[\frac{1}{10.9}\right]^{1/2} \left[\frac{298}{3.6}\right]^{3/2} = 0.41 \text{ m}^2$$

The required opening area of the windows then becomes $A_1/2 = A_4/2 = 0.21$ m^2.

3.2 Fresh air in all offices

By equally big opening areas, one gets an inward air flow in the window openings at the first and second floors, and outward flow in the window openings at third floor. Fresh air into the third floor offices can be obtained by moving the neutral plane up above the third floor openings.

With the neutral plane at third floor ceiling level, one gets $H_4^{**} = H_4 = 1.0$ m. The distance from ceiling to centre of the top opening is increased by 0.1 m to take the bigger top opening area into account. $Q_{g6} = 1800$ W and $\Delta T_m = 3.6$ K are unchanged. The required opening area in the top of the atrium is then determined by:

$$A_4 = 6.2 \cdot 10^{-7} \frac{1800}{0.62} \left[\frac{1}{1.0} \right]^{1/2} \left[\frac{298}{3.6} \right]^{3/2} = 1.4 \text{ m}^2$$

or 0.7 m_2 for each of the two hatches on the atrium roof.

The opening areas of the windows can be determined by Formula II 9 in Table 1, when the ventilation rate has been determined by Formula II 6 with $H_N^{**} = H_N^* = 1.0$ m, and the weighted height H_1^* by Equation 2. For the ventilation rate one gets:

$$q_v = 0.039 \cdot 1800^{1/3} (0.62 \cdot 1.4)^{2/3} \cdot 1.0^{1/6} \cdot 1.0^{1/2} = 0.43 \text{ m}^3/\text{s}$$

For H_1^* one gets, when $H_1 = 9.0 - 1.0 = 8.0$ m, $m_2 = m_3 = n_2 = n_3 = 1.0$, $q_2 = 4.8/8.0 = 0.60$, and $q_3 = 1.5/8.0 = 0.19$:

$$H_1^* = H_1 (1 + m_2 n_2 q_2^{1/2} + m_3 n_3 q_3^{1/2})^2 = 8.0 (1 + 0.60^{1/2} + 0.19^{1/2})^2 = 39.1 \text{ m}$$

Finally one gets:

$$A_1 = \frac{0.43}{0.62} \left[\frac{302}{2 \cdot 3.6 \cdot 9.82 \cdot 39.1} \right]^{1/2} = 0.23 \text{ m}^2$$

or about 0.12 m^2 per window.

3.3 Equal amounts of fresh air

With equal big inlet openings one gets the biggest air flow rate through the windows at first floor. Equally big air flow rates can be obtained by graduating the inlet areas.

The air flow rate through each of the six window openings should be $q_{vw} = 0.43/6 = 0,07$ m^3/s. The neutral plane shall still be placed at the level of third floor ceiling.

The air velocities in the windows can be determined by Formula I 2 in Table 1 where $\psi_j = 1 + \varsigma_j = 1 + 0.1 = 1.1$, ($\varsigma_j$ being the resistance coefficient) and one gets:

$$v_{cj} = \left[\frac{2 \cdot 3.6 \cdot 9.82 \cdot H_j}{1.1 \cdot 298} \right]^{1/2} = 0.46\, H_j^{1/2} \qquad (11)$$

For one window (with index j) with the contraction coefficient assumed to be $C_c = 0.65$, the required opening area is determined by:

$$A_{jw} = \frac{q_{vw}}{C_c\, v_{cj}} = \frac{0.07}{0.65 \cdot 0.46\, H_j^{1/2}} = \frac{0.23}{H_j^{1/2}}$$

For each floor, one gets the following opening areas per window:

First floor, $H_j = 8.1$ m: $A_{1w} = 0.08$ m^2
Second floor, $H_j = 4.8$ m: $A_{2w} = 0.11$ m^2
Third floor, $H_j = 1,5$ m: $A_{3w} = 0.19$ m^2

3.4 Summarizing

The calculation results are summarized in Table 2. The table shows the temperatures occurring at the different floors and at roof level as well as the required opening areas by the three different conditions considered in the calculations.

Table 2. Required opening areas in three-storyed office building by various conditions.

Level	1 floor	2 floor	3 floor	Roof
Temperature difference, ΔT_j, K	2.5	3.4	4.0	4.5
Required opening areas by:				
equal opening areas, m^2	0.21	0.21	0.21	0.21
fresh air in all offices, m^2	0.12	0.12	0.12	0.70
equal amount of fresh air, m^2	0.08	0.11	0.19	0.70

APPENDIX A

The theoretical air velocity (i.e. no friction loss) in the openings is determined by:

$$v_{theo\,j} = \left[\frac{2\Delta T_m\, g\, H_j}{T_{im}}\right]^{1/2} \tag{A1}$$

By stationary conditions the net heat input is equal to the heat removed by the air flow through the outlets:

$$\Phi_s = \sum_{N_1+1}^{N} c_p\, \rho_{im}\, q_v\, \Delta T_l = \sum_{N_1+1}^{N} c_p\, \rho_{im}(C_{dj}\, A_j\, v_{theo\,j})\Delta T_j$$

$$= \sum_{N_1+1}^{N} c_p\, \rho_{im}\, C_{dj}\, A_j\, (2\,\Delta T_m\, g\, H_j/T_u)^{1/2}(\Delta T_j/\Delta T_N)\Delta T_N$$

$$= c_p \rho_{im} C_{dN} A_N (2\,\Delta T_m\, g\, H_N/T_u)^{1/2}\Delta T_N \left[1 + \sum_{N_1+1}^{N-1} \frac{C_{dj}}{C_{dN}}\frac{A_j}{A_N}\left[\frac{H_j}{H_N}\right]^{1/2}\frac{\Delta T_j}{\Delta T_N}\right]^2$$

$$= c_p\, \rho_{im}\, C_{dN} A_N \left(2\Delta T_m\, g\, H_N^{**}/T_u\right)^{1/2}(\Delta T_N/\Delta T_m)\Delta T_m$$

or:

$$\frac{\Phi_s}{\Delta T_N/\Delta T_m} = \frac{\Phi_s}{\epsilon} = \Phi_{ss} = c_p\, \rho_{im}\, C_{dN} A_N \left(2\,\Delta T_m\, g\, H_N^{**}/T_u\right)^{1/2}\Delta T_m \tag{A2}$$

This equation can be solved with regard to ΔT_m. When using $\rho_{im} = p_{im}/(R\ T_{im})$ one gets:

$$\Delta T_m = \left[\frac{R}{c_p p_{im}}\right]^{2/3} \left[\frac{1}{2g}\right]^{1/3} \left[\frac{T_{im}}{T_o}\right]^{2/3} \left[\frac{\Phi_{sc}}{C_{dN} A_N}\right]^{2/3} \left[\frac{1}{H_N^{**}}\right]^{1/3} T_o$$

By introducing R = 287 J/kgK, c_p = 1007 J/kgK, p_{im} = 101300 Pa, g = 9.82 m/s^2, and by assuming $T_{im}/T_o \cong 1.03$, one finally gets:

$$\Delta T_m = 7.5 \cdot 10^{-5} T_o \left[\frac{\Phi_{sc}}{C_{dN} A_N}\right]^{2/3} \left[\frac{1}{H_N^{**}}\right]^{1/3} \quad \text{(A3)}$$

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