AIR MOVEMENT & VENTILATION CONTROL WITHIN BUILDINGS

12th AIVC Conference, Ottawa, Canada
24-27 September, 1991

PAPER 26

THE RELIABILITY OF INFILTRATION AND AIR MOVEMENT DATA OBTAINED FROM SINGLE TRACER GAS MEASUREMENTS IN LARGE SPACES

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SYNOPSIS.

The methods available for the measurement of air infiltration and air movement in large industrial halls are restricted by the size of the building and the nature of the operations which take place within it. Single tracer decay measurements are the easiest to perform and this paper examines the possibility of extracting useful information from them. Using a multi-zone representation of the building volume, the properties of tracer decay curves are considered, and the ease of extraction infiltration and air flow data examined by means of simulations. The results show how the error in the derived infiltration rates grow with error in the tracer gas concentration measurements for various methods of treating the results. The simulations are compared to the results of measurements made in a typical industrial hall. Despite the shortcomings of the multi-zone model and the single tracer decay method, it appears possible to obtain reasonable results for the overall air infiltration rates.

1. Introduction.

In recent years there have been many improvements and new developments in the tracer gas methods available for the measurement of infiltration and air movement in buildings. Nevertheless there are some cases in which the choice of method is limited by the characteristics of the building. This is especially true of industrial halls, which are often characterised by large internal volume, continuous operation, unusual contaminants in the atmosphere, and an unwillingness on the part of management to allow interruption of normal activities. Artificial stirring of the air is often impossible, making it difficult to use constant concentration, constant injection, or any of the pulse methods. In addition the large volume and the presence of other contaminants place restrictions on the choice of tracer. Consequently the single tracer gas decay method is the easiest to use, and a number of reports of its use in large buildings have been published [1, 2, 3]. However, there is little evidence to indicate their reliability, because of the difficulty of validation. Having considered the alternatives, we have returned to the single tracer gas decay method, and attempted to evaluate its reliability and performance in the context of large spaces.

2. Theory.

In the absence of full stirring, there will always be significant spatial variations throughout the air volume of a large building. It is therefore necessary to use some kind of discretisation of the volume into sub-volumes (or zones) in order to model these variations. The greater the number of zones the more closely will the discretisation approach the real case. A system of interconnected zones may be treated by means of the well known multi-zone air movement model. Unfortunately this assumes that the air in each zone is fully mixed, an assumption which is unlikely to hold in the sub-volumes of the real building. Nevertheless, this mismatch may not be serious enough to prevent useful information being obtained, and the agreement between the actual discretisation and its multi-zone representation can always be improved by increasing the number of zones. Assuming that the multi-zone model is acceptable, our problem is the extraction of inter-zone flows from single tracer gas decay experiments in such a model.

The multi-zone model is sufficiently well known to require no special description. Following Sinden [4] and Sandberg [5], the relevant equation (in the absence of a source term) is

$$V\dot{\underline{c}}(t) = F\underline{c}(t) \qquad 1$$

where V is the zone volume matrix, F the flow matrix, and $c(t)$ is the vector of the contaminant concentrations at time t. The solution for a model of N zones is

$$\underline{c}(t) = \sum_{k=1}^{N} a_k \underline{x}_k e^{\lambda_k t} \qquad 2$$

where λ_k and $\underline{x}_k$ are the eigenvalues and eigenvectors of the matrix F-λV, and a_k are a set of constants determined by the initial distribution of tracer gas. In what follows, it is convenient to define:

F_{ij} the volumetric flow rate from zone i to zone j, where zone 0 represents the outside.

S_i the total inflow/outflow to zone i, $\sum_{j=0}^{N} F_{ji}$ or $\sum_{j=0}^{N} F_{ij}$

f_{ij} flow rate per unit volume, $f_{ij} = \frac{F_{ij}}{V_j}$

r_i total flow per unit volume, $r_i = \frac{S_i}{V_i}$

The most relevant features of multi-zone single tracer decay may be listed as follows:

a) The decay curves are governed primarily by the eigenvalues λ_k and $\underline{x}_k$ their eigenvectors $\underline{x}_k$. There as many eigenvalues as there as zones, the largest is real, and some or all of the remainder may be complex. Repeated eigenvalues occur when some or all of the r_i are equal and some of the internal flows are zero. Physically such cases are likely to occur when at least one zone receives fresh air and supplies air to other zones, but receives no recirculated air, a condition similar to displacement flow. The largest eigenvalue, λ_1, becomes dominant as time progresses, and eventually governs the rate of decay in all zones. If the time constant of the process is taken as the reciprocal of λ_1, then whatever the initial tracer gas distribution, λ_1 is dominant after approximately two time constants. Thus if the decay is allowed to proceed long enough, λ_1 can always be determined. However, λ_1 is only equivalent to the fresh air infiltration rate when certain conditions are satisfied. There are two types of condition. One occurs when internal flows, f_{ij}, become very large compared to external flows, f_{i0}. This is obvious because it corresponds to full internal mixing. The other occurs when either all f_{i0} are the same or all f_{0i} are the same, which in turn means that,

$$f_{0i} = -\lambda_1 \text{ for all } i$$

or

$$f_{i0} = -\lambda_1 \text{ for all } i$$

The significance of this is that if either the infiltration or the exfiltration is well distributed between the zones, the dominant eigenvalue will be a good representation of the fresh air infiltration rate.

b) In the region where λ_1 is dominant, the ratios of the zone concentrations approach the components of the dominant eigenvector. Hence, x_1 can be determined. If the concentrations in this region are a_1, a_2, ... then

$$\underline{x}_1 = [a_1, a_2, a_3 \ldots] \qquad 3$$

This must be a solution of the eigenvalue equation, which gives:

$$\begin{aligned} -a_1(r_1+\lambda_1) + \quad a_2 f_{21} \quad + \quad a_3 f_{31} \ldots &= 0 \\ a_1 f_{12} \quad - a_2(r_2+\lambda_1) + \quad a_3 f_{32} \ldots &= 0 \\ &\vdots \end{aligned} \qquad 4$$

If the concentrations in all zones are essentially equal at this stage, ie. $x_1 = [1, 1, 1, \ldots]$, then, remembering that $r_i = f_{0i} + f_{1i} + f_{2i} + \ldots$, the eigenvector equation reduces to

$$\begin{aligned} -(f_{01}+\lambda_1) &= 0 \\ -(f_{02}+\lambda_1) &= 0 \\ &\vdots \end{aligned} \qquad 5$$

Again we have the condition

$$f_{0i} = -\lambda_1 \text{ for all } i$$

but NOT $$f_{i0} = -\lambda_1$$

If the concentrations are not all equal, then by noting that $r_i = f_{i0} + f_{i1} + f_{i2} + \ldots$, and summing the eigenvalue equations, we get, after some rearrangement, an equation connecting the outflows:

$$a_1 f_{10} + a_2 f_{20} + \ldots = -\lambda_1 \sum a_i \qquad 6$$

c) It has previously been pointed out [6] that it is sometimes possible to derive information from the intersection of the decay curves from two zones. If it is observed that $c_i(t) = c_j(t)$ when $\dot{c}_i(t) = 0$, and if $c_j(t)$ is the maximum at time t in the subset of zones which have a possible connection to zone j, then it may be inferred that the flows into j, F_{kj}, are all zero except for F_{0j} and F_{ij}.

d) If only one zone, say zone i, contains tracer gas at time zero, the tracer gas balance equations simplify to

$$\begin{aligned} V_i \dot{c}_i(0) &= -c_i(0)\, S_i \\ V_j \dot{c}_j(0) &= F_{ij}\, c_i(0) \end{aligned} \qquad 7$$

This would allow determination of S_i and F_{ij}, except that in practice it is difficult to measure concentration gradients near the origin.

The extraction of the flows F_{ij} from the measured decay curves requires the solution of a set of linear equations of the type

$$V_j \dot{c}_j(0) = F_{0j}(c_0(t) - c_j(t)) + F_{ij}(c_1(t) - c_j(t)) + \ldots \qquad 8$$

In a typical experiment there is usually no difficulty in obtaining more equations than is necessary in order to find the N^2 independent unknowns. A least squares method seems appropriate, but has some drawbacks. Firstly the equations are not truly independent. Secondly the large number of unknowns makes it difficult to achieve satisfactory precision. Thirdly the concentration gradients are difficult to measure with adequate accuracy. The first problem cannot be avoided, the second can be ameliorated by introducing prior knowledge and known constraints, and the third can be dealt with either by smoothing techniques or by integration of the equations.

3. The Solution Procedure and the Scope of the Investigation.

Equation 8 is an over-determined set of linear equations of the form

$$Y = Xb \qquad 9$$

where the vector Y contains the values of $V_j.c_j(t)$, the matrix X is assembled from the concentrations, and the vector b contains the solution for the flows. Three methods of filling Y are:

i) for simulated data, using exact gradients,
ii) using gradients obtained by smoothing,
iii) after integration, using $(c_j(t+n) - c_j(t))$ where n is the integration interval.

In general the solution contains a constant, which is expected to be zero because the concentration gradients and the concentrations all approach zero as time progresses. Nevertheless it is better to retain the constant to allow for an offset in the solution. The simple (or straight) least squares solution is

$$b = (X'X)^{-1} X'Y \qquad 10$$

If some of the flows are known, either individually or in combination, they may be expressed in the form

$$C.b = d \qquad 11$$

where C is an identification matrix and d contains the known values. The constrained solution is then

$$B = b + (X'X)^{-1} C' \left[C(X'X)^{-1} C' \right]^{-1} (d - Cb) \qquad 12$$

This assumes that the values in the vector d are known precisely. If they are subject to error, and have a variance of σ, we may write (Sherman [7] and Tarantola [8])

$$B = b + (X'X)^{-1} C' \left[C(X'X)^{-1} C' + \sigma \right]^{-1} (d - Cb) \qquad 13$$

Because the flows F_{ij} cannot be negative, a non-negative constraint may also be introduced. The NNLS method of Lawson and Hanson (9) is well known, and may be applied to the simple solution (equation 10) or to the fixed value constrained solution (equation 12). Already we have four possible solutions:

1. Simple (or straight) least squares.
2. Simple least squares with NNLS applied.
3. Constrained least squares.
4. Constrained least squares with NNLS applied.

The quality of the solution may be examined by the usual analysis of variance table. Other indicators are the variance-covariance matrix, and a suitable condition number of the *XX'* matrix. Where the original data was produced by a simulation, a root mean square error may be formed from the difference between the derived flows and their original values. It is also of interest to compare the fresh air infiltration rate found by summing F_{i0} with the dominant eigenvalue and with the true value.

The main purpose of this study, therefore, was to apply the solution procedures to simulated data sets in order to investigate the error in the solution due to each of the following:

1. Error in the original tracer gas concentrations (0 to 5%).
2. The method of treating the concentration gradient (exact, smoothed or by integration).
3. The time scale of the experiment (one time constant, two time

constants or the full data set).

4. The number of zones (2 to 6).
5. The choice of solution procedure.

4. Results and Discussion.

Table 1 shows the flow matrices used in the simulations. Simulated data for a time period of approximately 2.5 time constants were prepared for each of these flow regimes, and for a range of error levels in the tracer gas concentrations. In most cases, the starting condition corresponded to the seeding of one zone only. The results were analysed by the four LSQ methods listed in section 4, and for each of the three methods of treating the gradient. Table 2 shows the results for the two zone model when analysed by means of the 'straight' LSQ. In the table the codes for the data sets are:

F = analysis using the full data set.

1 = analysis using the first time constant of data only.

2 = analysis using the first two time constants of data.

The codes for the condition numbers refer to the norms on which they are based:

1 = Chebysev

2 = Manhattan

3 = Frobenius

4 = Eigenvalue

The RMS error in the output is given in absolute units of flow, and the fresh air infiltration rate is given in air changes per hour. The results for the data sets with zero input error and analysed using exact gradients confirm the correctness of the procedures. The effect of particular parameters may be highlighted by grouping the results into subsets, and finding the average RMS error in that subset. This has been done for the two zone model in tables 3, 4 and 5. Inspection of these tables leads to some useful conclusions:

1. From tables 3 and 5, using only the first time constant of data creates significantly higher RMS errors than using two time constants. Going beyond two time constants to the full data set produces only

a slight improvement. Thus it is better to use two time constants of tracer concentration data. This agrees with the theoretical prediction that in single zone seeded decay mode, uniform decay is established after approximately two time constants.

2. From tables 3, 4 and 5, the integral treatment of gradient produces substantially lower errors than smoothing.
3. From table 4, increasing the applied error increases the RMS error. However, an increase in the applied error from 0.1% to 1% often produces a proportionally smaller increase in the RMS error.
4. From table 5, the RMS error is significantly lower when zone 1 is seeded. This agrees with the prediction that it is better to seed the zone with the smallest component of the dominant eigenvector. In this case $x_{11} = 1$ and $x_{12} = 2$.
5. Referring again to table 2, it can be seen that there appears to be no correlation between any of the condition numbers and the RMS error.

These conclusions were confirmed by similar analyses of the results for the 3, 4, 5 and 6 zone models. In view of the first two conclusions, the more detailed analyses were restricted to using the first two time constants of data and the integral treatment of gradient.

The two zone model was too simple to explore the effect of constraints. As the number of zones increased, both fixed value and NNLS constraints became more important. Table 6 shows the effect of applying these constraints to the 5 zone model. Both the fixed value constrained solution and the NNLS constrained solution reduce the RMS error substantially compared to the straight solution, with the NNLS constraint being the more effective. Applying both constraints reduces the RMS error even further. The five zone model was used to explore further the effect on RMS error of seeding different zones. In this case the dominant eigenvector is

$$x_1 = [0.309, 0.434, 0.650, 0.416, 0.347]$$

suggesting that best results are obtained when zone 1 is seeded. Table 7 shows that this is not so, and that there is little difference between the five possibilities. However, this is not surprising as the components of x_1 are very nearly equal.

When considering the errors in the output, whether they be RMS errors in the flows or errors in the air change rate, it must be borne in mind that the errors in the input data, that is in the original tracer gas concentrations, were generated by a normally distributed random error generator. Thus each data set at any given error level is unique, and the error in the output is just one value in a normally distributed set of possible errors. This can be explored by repeat runs at the same error level. Table 8 shows the result of repeat runs for the six zone model, using two time constants of data and integration, and table 9 shows the average RMS errors together with the standard deviation of the set and the estimated standard deviation of the population. Clearly, it is the spread of the RMS error that is of particular concern. If this is known it is possible to make an estimate of output errors from an estimate of the measurement error in the original tracer gas concentrations. Because of the small number of repeat runs (only six in each case) the Chi-square test was used to estimate the confidence limits of the population standard deviation, as shown in table 10. It appears from this that the spread in the output error distribution could be quite large. Consequently, where there is only one data set available, as is often the case with field measured experimental data, the error in the result could also be large. On the other hand, the spread in the RMS error does not grow with the applied input error as quickly as may have been expected. This is shown in figure 1. Figure 2 shows the same thing for the percentage difference between air change rate found from the calculated flows and the air change rate found from the exact flows. Finally, the increase in the errors as the number of zones is increased is shown in figure 3, which shows the errors found for the data sets including a 1% applied error level when employing a non-negative least squares and a constrained/non-negative least squares analysis where appropriate.

5. Field measurements.

Analysis of data sets from measurements in real buildings introduce some additional difficulties, especially the identification of zones and zone volumes and the positioning of sampling points. Figure 4 is a typical set of decay curves for an industrial hall, and the results of analysing it are shown in table 11. The solution with fixed value and NNLS constraints gave flows which were plausible in terms of the geometry of the building, and it is interesting to compare the air change rates found by the two methods. In this case it may be presumed that the air change found from the slope of the long term decay is the more reliable, because the decay curves approach each other as time progresses. It often transpires in measured data that the readings from two sample points are essentially equal, suggesting that the zones they represent are indistinguishable. The solution will not be able to separate the flows associated with these two zones, in which case it is better to merge the readings and reduce the number of zones in the model by one. The effect of merging down to a three zone representation is shown in the final column of table 11. Note that it was not possible to merge zones for all the cases.

5. Conclusions.

The single tracer gas decay technique is inherently inaccurate for the measurement of inter-zone flows. Nevertheless it can provide useful information on air change rates and the distribution of air, provided that the analysis includes the points discussed above. In particular, the analysis must include all the prior knowledge that is available, it must take advantage of any of the special properties of the decay curves, and it must use at least non-negative constraints.

References.

1. Potter, I. N., Dewsbury, J. and Jones, T. : The Measurement of Air Infiltration Rates in Large Enclosures and Buildings, 4th AIVC Conference, Switzerland, 26-28 Sept. , 1983.

2. Roussel, C., Garnier, G. and Faivre-Pierret, R. : A Tracer Gas System to Evaluate the Efficiency of Ventilation Systems or Simulate the Consequences of an Accident, Ventilation '85, Ed. by H. D. Goodfellow, pp. 291-295, Elsevier, 1986.

3. Lawrance, G. V. and Waters, J. R. : Measurements of Infiltration and Air Movement in Five Large Single Cell Buildings, 8th AIVC Conference, Germany, 21-24 Sept. , 1987.

4. Sinden, F. W. : Multi-Chamber Theory of Air Infiltration, Building and Environment, Vol. 13, No. 1, pp. 21-28, 1978.

5. Sandberg, M. : The multi-chamber theory reconsidered from the viewpoint of air quality studies, Building and Environment, Vol. 19, No. 4, pp. 221-223, 1984.

6. Waters, J. R. and Simons, M. W. : The evaluation of contaminant concentration and air flows in a multi-zone model of a building, Building and Environment, Vol. 22, No. 4, pp. 305-315, 1987.

7. Sherman, M. H. : On the estimation of multi-zone ventilation rates from tracer gas measurements, Building and Environment, Vol. 24, No. 4, pp. 355-362.

General Case.

$$\begin{matrix} -S_0 & F_{10} & F_{20} & \cdots & F_{n0} \\ F_{01} & -S_1 & F_{21} & & \\ \vdots & & \ddots & & \\ F_{0n} & & & & -S_n \end{matrix}$$

F_{ij} denotes flow from zone i to zone j, (m^3/s).

2 Zone Model.

-6.00	3.00	3.00
5.00	-7.00	2.00
1.00	4.00	-5.00

3 Zone Model.

-6.00	0.00	2.00	4.00
4.00	-7.00	1.00	2.00
1.00	3.00	-4.00	0.00
1.00	4.00	1.00	-6.00

4 Zone Model.

-12.00	3.00	3.00	1.00	5.00
0.00	-4.00	2.00	2.00	0.00
4.00	1.00	-9.00	0.00	4.00
7.00	0.00	0.00	-11.00	4.00
1.00	0.00	4.00	8.00	-13.00

5 Zone Model.

-14.00	1.00	5.00	0.00	5.00	3.00
3.00	-4.00	1.00	0.00	0.00	0.00
4.00	0.00	-11.00	4.00	0.00	3.00
1.00	0.00	2.00	-5.00	0.00	2.00
1.00	3.00	0.00	0.00	-9.00	5.00
5.00	0.00	3.00	1.00	4.00	-13.00

6 Zone Model.

-17.00	1.00	5.00	0.00	5.00	3.00	3.00
3.00	-4.00	1.00	0.00	0.00	0.00	0.00
4.00	0.00	-11.00	4.00	0.00	3.00	0.00
1.00	0.00	2.00	-5.00	0.00	0.00	2.00
1.00	3.00	0.00	0.00	-9.00	5.00	0.00
5.00	0.00	3.00	0.00	4.00	-13.00	1.00
3.00	0.00	0.00	1.00	0.00	2.00	-6.00

Table 1. Flow matrices for simulated models.

Table 2. Results for the simulated two zone model.

Grad Calc Method	Seeded Zone	Error Applied (%)	Data Set	Condition Numbers 1 ($x10^2$)	2 ($x10^2$)	3 ($x10^2$)	4 ($x10^{-1}$)	RMS Error ($x10^{-1}$)	Diff in ACR from Flows (h^{-1})
E	1	0.0	F	2.045	2.045	5.622	0.516	0.000	0.0000
X			1	11.850	11.850	40.960	1.241	0.000	0.0000
A			2	2.839	2.839	8.475	0.745	0.000	0.0000
C		0.1	F	2.043	2.043	5.614	0.517	0.054	0.0009
T			1	11.950	11.950	41.280	1.243	0.205	0.0128
			2	2.829	2.829	8.440	0.746	0.050	-0.0017
		1.0	F	2.108	2.108	5.815	0.511	0.509	0.0133
			1	10.630	10.630	36.650	1.227	1.817	-0.1368
			2	2.857	2.857	8.527	0.736	0.271	-0.0022
	2	0.0	F	10.320	10.320	28.230	0.745	0.000	0.0000
			1	29.310	29.310	119.400	1.703	0.000	0.0000
			2	10.900	10.900	34.480	1.056	0.000	0.0000
		0.1	F	10.390	10.390	28.440	0.754	0.220	-0.0020
			1	29.460	29.460	120.200	1.703	0.280	-0.0037
			2	11.060	11.060	35.040	1.056	0.203	0.0042
		1.0	F	9.298	9.298	25.350	0.755	2.183	-0.0907
			1	25.630	25.630	103.900	1.703	2.423	-0.1769
			2	10.160	10.160	32.100	1.057	1.770	-0.0858
S	1	0.0	F	2.045	2.045	5.622	0.516	0.169	0.0021
M			1	11.850	11.850	40.960	1.241	0.264	0.0067
O			2	2.839	2.839	8.475	0.745	0.178	0.0027
O		0.1	F	2.043	2.043	5.614	0.517	0.475	-0.0086
T			1	11.950	11.950	41.280	1.243	2.176	-0.2026
H			2	2.829	2.829	8.440	0.746	0.683	-0.0338
E		1.0	F	2.108	2.108	5.815	0.511	5.540	-0.1442
D			1	10.630	10.630	36.650	1.227	20.330	-1.3349
			2	2.857	2.857	8.527	0.736	4.361	0.0854
	2	0.0	F	10.320	10.320	28.230	0.754	0.114	0.0037
			1	29.310	29.310	119.400	1.703	0.267	0.0110
			2	10.900	10.900	34.480	1.056	0.131	0.0047
		0.1	F	10.390	10.390	28.440	0.754	3.841	0.2082
			1	29.460	29.460	120.200	1.703	14.650	1.0009
			2	11.060	11.060	35.040	1.056	5.226	0.3064
		1.0	F	9.298	9.298	25.350	0.755	27.730	-1.6772
			1	25.630	25.630	103.900	1.703	61.410	-4.1935
			2	10.160	10.160	32.100	1.057	30.400	-1.8838
I	1	0.0	F	0.121	0.121	0.412	5.337	0.030	-0.0002
N			1	5.886	5.886	9.747	4.291	0.030	-0.0002
T			2	0.346	0.346	0.897	4.982	0.030	-0.0002
E		0.1	F	0.120	0.120	0.412	5.338	0.048	0.0014
G			1	5.828	5.828	9.645	4.291	0.173	0.0126
R			2	0.344	0.344	0.891	4.982	0.525	-0.0006
A		1.0	F	0.127	0.127	0.436	5.365	1.161	0.0054
L			1	5.121	5.121	8.547	4.287	10.700	-0.7984
			2	0.346	0.346	0.907	4.976	1.744	-0.0855
	2	0.0	F	1.428	1.428	3.941	9.806	0.030	-0.0002
			1	17.560	17.560	41.090	9.827	0.030	-0.0002
			2	2.519	2.519	6.511	9.820	0.030	-0.0002
		0.1	F	1.438	1.438	3.968	9.806	0.116	-0.0059
			1	17.940	17.940	41.990	9.827	0.570	0.0399
			2	2.572	2.572	6.658	9.820	0.138	0.0081
		1.0	F	1.364	1.364	3.747	9.808	1.624	0.0908
			1	22.240	22.240	52.220	9.829	17.860	1.2794
			2	2.578	2.578	6.664	9.823	2.088	0.1282

Table 2. Results for the simulated two zone model.

Grad Calc Method	Seeded Zone	Error Applied (%)	Data Set	Condition Numbers 1 ($x10^2$)	Condition Numbers 2 ($x10^2$)	Condition Numbers 3 ($x10^2$)	Condition Numbers 4 ($x10^{-1}$)	RMS Error ($x10^{-1}$)	Diff in ACR from Flows (h^{-1})
E	1	0.0	F	2.045	2.045	5.622	0.516	0.000	0.0000
X			1	11.850	11.850	40.960	1.241	0.000	0.0000
A			2	2.839	2.839	8.475	0.745	0.000	0.0000
C		0.1	F	2.043	2.043	5.614	0.517	0.054	0.0009
T			1	11.950	11.950	41.280	1.243	0.205	0.0128
			2	2.829	2.829	8.440	0.746	0.050	-0.0017
		1.0	F	2.108	2.108	5.815	0.511	0.509	0.0133
			1	10.630	10.630	36.650	1.227	1.817	-0.1368
			2	2.857	2.857	8.527	0.736	0.271	-0.0022
	2	0.0	F	10.320	10.320	28.230	0.745	0.000	0.0000
			1	29.310	29.310	119.400	1.703	0.000	0.0000
			2	10.900	10.900	34.480	1.056	0.000	0.0000
		0.1	F	10.390	10.390	28.440	0.754	0.220	-0.0020
			1	29.460	29.460	120.200	1.703	0.280	-0.0037
			2	11.060	11.060	35.040	1.056	0.203	0.0042
		1.0	F	9.298	9.298	25.350	0.755	2.183	-0.0907
			1	25.630	25.630	103.900	1.703	2.423	-0.1769
			2	10.160	10.160	32.100	1.057	1.770	-0.0858
S	1	0.0	F	2.045	2.045	5.622	0.516	0.169	0.0021
M			1	11.850	11.850	40.960	1.241	0.264	0.0067
O			2	2.839	2.839	8.475	0.745	0.178	0.0027
O		0.1	F	2.043	2.043	5.614	0.517	0.475	-0.0086
T			1	11.950	11.950	41.280	1.243	2.176	-0.2026
H			2	2.829	2.829	8.440	0.746	0.683	-0.0338
E		1.0	F	2.108	2.108	5.815	0.511	5.540	-0.1442
D			1	10.630	10.630	36.650	1.227	20.330	-1.3349
			2	2.857	2.857	8.527	0.736	4.361	0.0854
	2	0.0	F	10.320	10.320	28.230	0.754	0.114	0.0037
			1	29.310	29.310	119.400	1.703	0.267	0.0110
			2	10.900	10.900	34.480	1.056	0.131	0.0047
		0.1	F	10.390	10.390	28.440	0.754	3.841	0.2082
			1	29.460	29.460	120.200	1.703	14.650	1.0009
			2	11.060	11.060	35.040	1.056	5.226	0.3064
		1.0	F	9.298	9.298	25.350	0.755	27.730	-1.6772
			1	25.630	25.630	103.900	1.703	61.410	-4.1935
			2	10.160	10.160	32.100	1.057	30.400	-1.8838
I	1	0.0	F	0.121	0.121	0.412	5.337	0.030	-0.0002
N			1	5.886	5.886	9.747	4.291	0.030	-0.0002
T			2	0.346	0.346	0.897	4.982	0.030	-0.0002
E		0.1	F	0.120	0.120	0.412	5.338	0.048	0.0014
G			1	5.828	5.828	9.645	4.291	0.173	0.0126
R			2	0.344	0.344	0.891	4.982	0.525	-0.0006
A		1.0	F	0.127	0.127	0.436	5.365	1.161	0.0054
L			1	5.121	5.121	8.547	4.287	10.700	-0.7984
			2	0.346	0.346	0.907	4.976	1.744	-0.0855
	2	0.0	F	1.428	1.428	3.941	9.806	0.030	-0.0002
			1	17.560	17.560	41.090	9.827	0.030	-0.0002
			2	2.519	2.519	6.511	9.820	0.030	-0.0002
		0.1	F	1.438	1.438	3.968	9.806	0.116	-0.0059
			1	17.940	17.940	41.990	9.827	0.570	0.0399
			2	2.572	2.572	6.658	9.820	0.138	0.0081
		1.0	F	1.364	1.364	3.747	9.808	1.624	0.0908
			1	22.240	22.240	52.220	9.829	17.860	1.2794
			2	2.578	2.578	6.664	9.823	2.088	0.1282

Table 3. Average RMS errors for each portion of data - two zone model.

Least Squares Technique	Gradient Calculation Method	Average Root Mean Square Error for portion of data set		
		full set	first τ_n	second τ_n
Straight	Exact	0.049	0.079	0.038
	Smoothed	0.631	1.651	0.683
	Integral	0.050	0.490	0.076
Non-negative	Exact	0.048	0.066	0.038
	Smoothed	0.410	1.570	0.450
	Integral	0.050	0.348	0.057

Table 4. Average RMS errors for different applied error levels - two zone model.

Least Squares Technique	Gradient Calculation Method	Average Root Mean Square Error for Applied Error Level		
		0%	0.1%	1.0%
Straight	Exact	0.000	0.017	0.150
	Smoothed	0.019	0.451	2.496
	Integral	0.003	0.018	0.586
Non-negative	Exact	0.000	0.017	0.097
	Smoothed	0.019	0.421	1.154
	Integral	0.003	0.018	0.434

Table 5. Average RMS errors for seeding of each zone - two zone model.

Gradient Calculation Method	Seeded Zone	Average Root Mean Square Error for portion of data set		
		full set	first τ_n	first two τ_n
Exact	1	0.019	0.067	0.011
	2	0.080	0.090	0.066
Smoothed	1	0.206	0.759	0.174
	2	1.056	2.544	1.192
Integral	1	0.041	0.363	0.061
	2	0.059	0.615	0.075

Table 6. Average RMS errors for each analysis technique - five zone model.

Least Squares Technique	Average Root Mean Square Error for applied error level		
	0%	0.1%	1.0%
Straight	0.008	5.842	6.048
Non-negative	0.009	1.357	1.760
Constrained	0.959	2.934	3.993
Const/NNLS	0.232	1.101	1.667

Table 7. Average RMS errors for each zone seeded - five zone model.

Analysis Method	Average RMS error levels for each zone				
	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5
LSQ	4.338	3.669	4.530	4.225	3.018
NNLS	1.007	1.050	1.006	1.307	0.841

Table 9. Statistical analysis of average RMS errors - six zone model.

Applied Error Level (%)		Analysis Technique	
		LSQ	CONST/NNLS
0.1	$\bar{x}$	7.836	0.734
	σ_n	2.545	0.448
	σ_{n-1}	2.788	0.491
1.0	$\bar{x}$	8.836	1.897
	σ_n	1.305	0.493
	σ_{n-1}	1.429	0.481
2.0	$\bar{x}$	7.795	1.507
	σ_n	1.004	0.387
	σ_{n-1}	1.100	0.424
5.0	$\bar{x}$	11.126	2.047
	σ_n	2.421	0.422
	σ_{n-1}	2.652	0.462

Table 8. RMS Errors for each data set - six zone model.

Applied Error Level (%)	Data Set	Root Mean Square Error for analysis techniques	
		LSQ	CONST/NNLS
0.0	1	0.007	0.016
0.1	1	9.718	0.374
	2	5.343	0.600
	3	3.571	1.018
	4	8.048	0.749
	5	10.251	0.139
	6	10.083	1.523
	Average	7.863	0.734
1.0	1	9.824	2.283
	2	8.330	2.039
	3	7.650	2.259
	4	10.838	2.230
	5	9.356	1.259
	6	7.020	1.314
	Average	8.836	1.897
2.0	1	8.164	1.356
	2	6.447	1.256
	3	7.640	1.937
	4	6.903	1.312
	5	9.590	1.053
	6	8.026	2.126
	Average	7.795	1.507
5.0	1	13.288	1.681
	2	10.478	2.264
	3	10.655	1.987
	4	11.278	1.429
	5	6.714	2.121
	6	14.343	2.801
	Average	11.126	2.047

Table 10. Estimates of population standard deviation for RMS errors.

Applied Error Level (%)	Confidence Limit	Estimates of Population Standard Deviation	
		LSQ	CONST/NNLS
0.1	95	1.743<2.788<6.838	0.307<0.491<1.204
	80	2.051<2.788<4.912	0.361<0.491<0.865
	50	2.420<2.788<3.815	0.426<0.491<0.672
1.0	95	0.894<1.429<3.507	0.301<0.481<1.180
	80	1.052<1.429<2.519	0.354<0.481<0.847
	50	1.241<1.429<1.956	0.417<0.481<0.658
2.0	95	0.756<1.100<2.966	0.265<0.424<1.040
	80	0.890<1.100<2.131	0.312<0.424<0.747
	50	1.050<1.100<1.655	0.368<0.424<0.580
5.0	95	1.658<2.652<6.505	0.289<0.462<1.134
	80	1.951<2.652<4.673	0.340<0.462<0.814
	50	2.302<2.652<3.629	0.401<0.462<0.633

Table 11. Air Change Rates for experimental test runs.

Building	Seeded Zone	Data Set	Openings	ACR from Time Constant (h^{-1})	ACR from calculated flows (h^{-1})	
					6 Zone	3 Zone
Structures Lab	6	1	0	0.496	0.995	1.080
		2	0	0.670	1.496	0.884
		3	0	0.158	1.517	-
	1	4	0	0.390	1.624	1.597
		5	0	0.492	1.325	1.133
		6	0	0.467	1.134	1.658
	6	11	7	1.402	4.125	3.005
		12	7	1.705	2.785	2.275
	1	13	7	2.734	4.135	4.786
		14	7	1.657	1.885	-

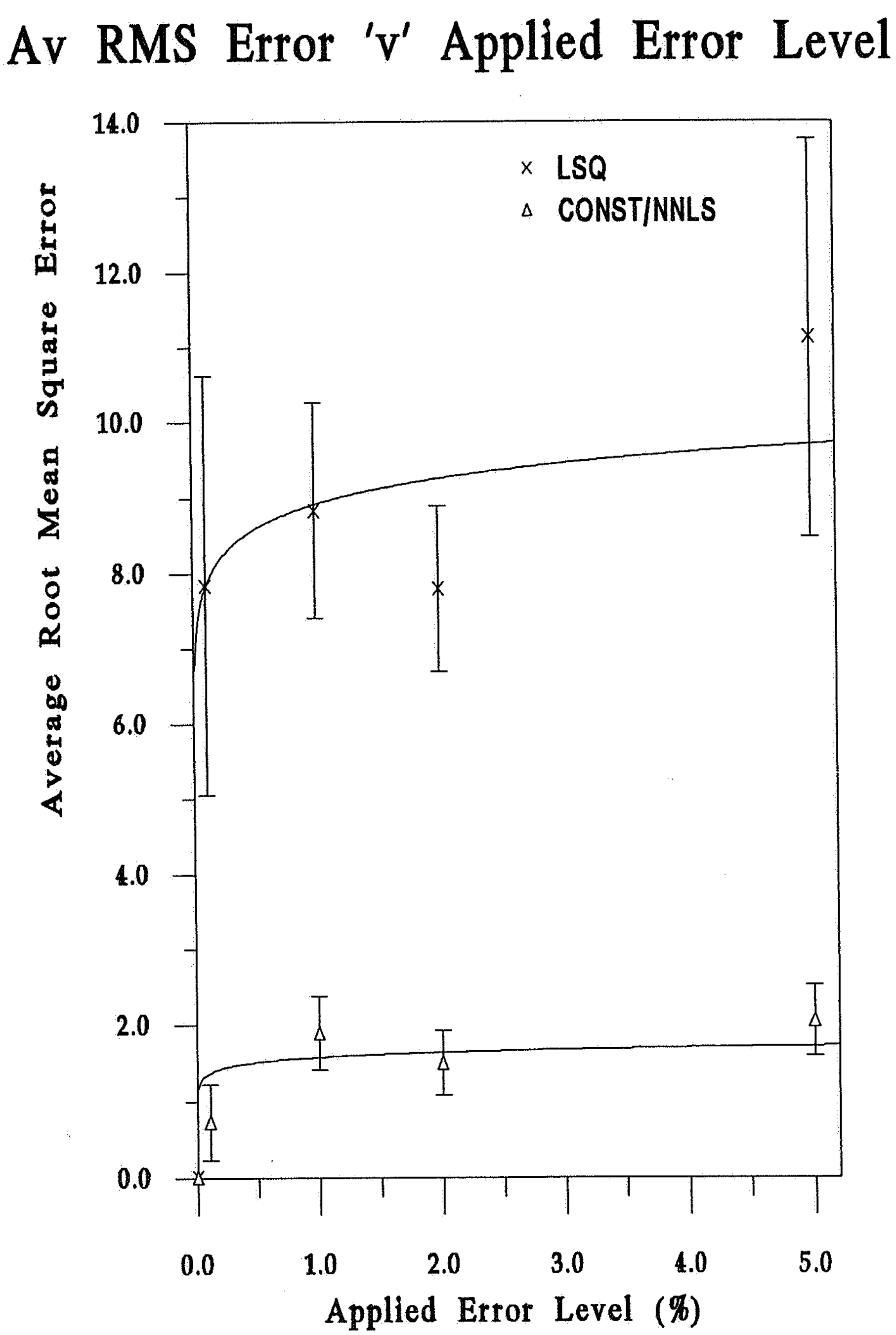

Figure 1. Average values of RMS Error plotted against Applied Error Level.

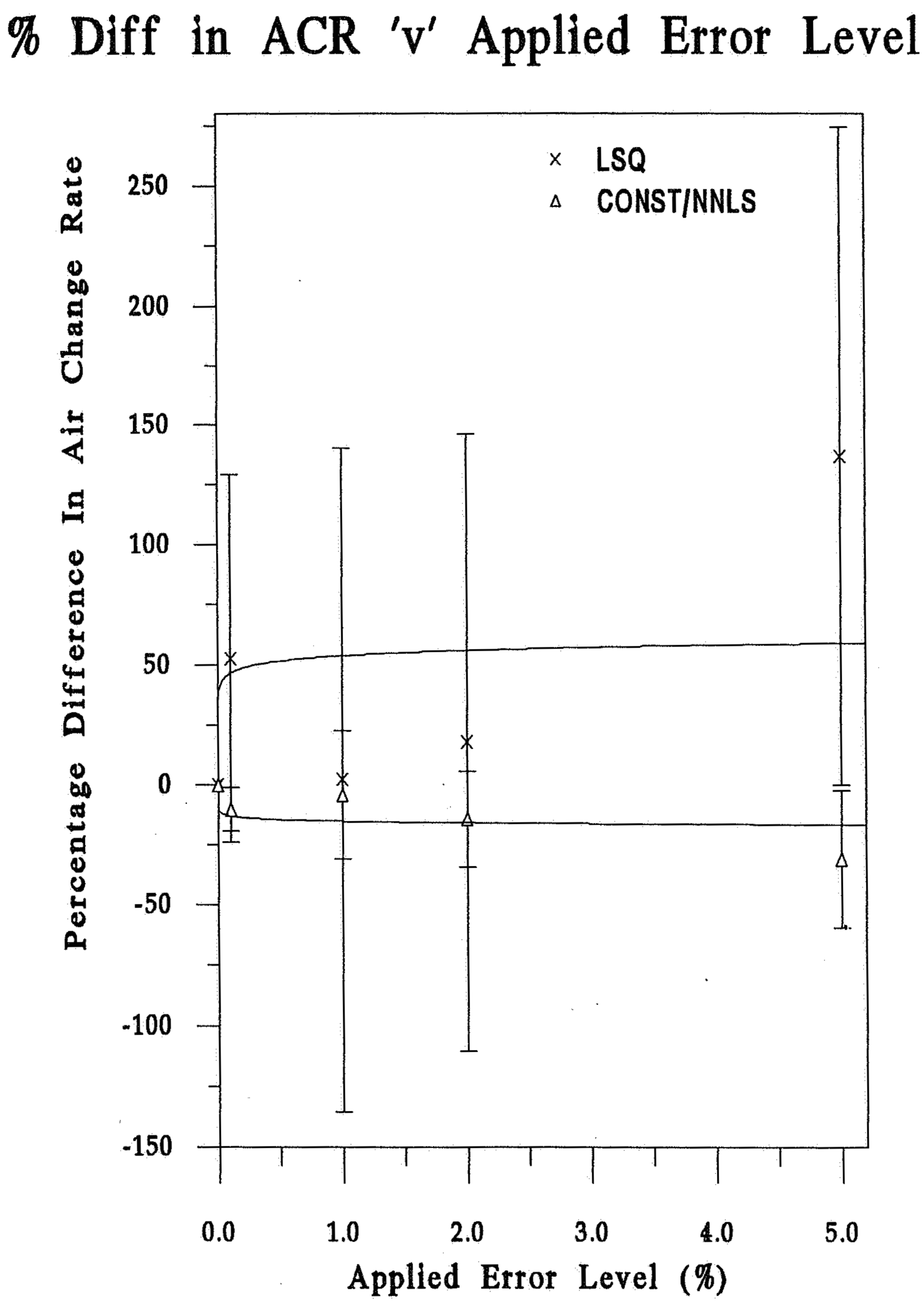

Figure 2. Percentage Difference in Air Change Rate plotted against Applied Error Level.

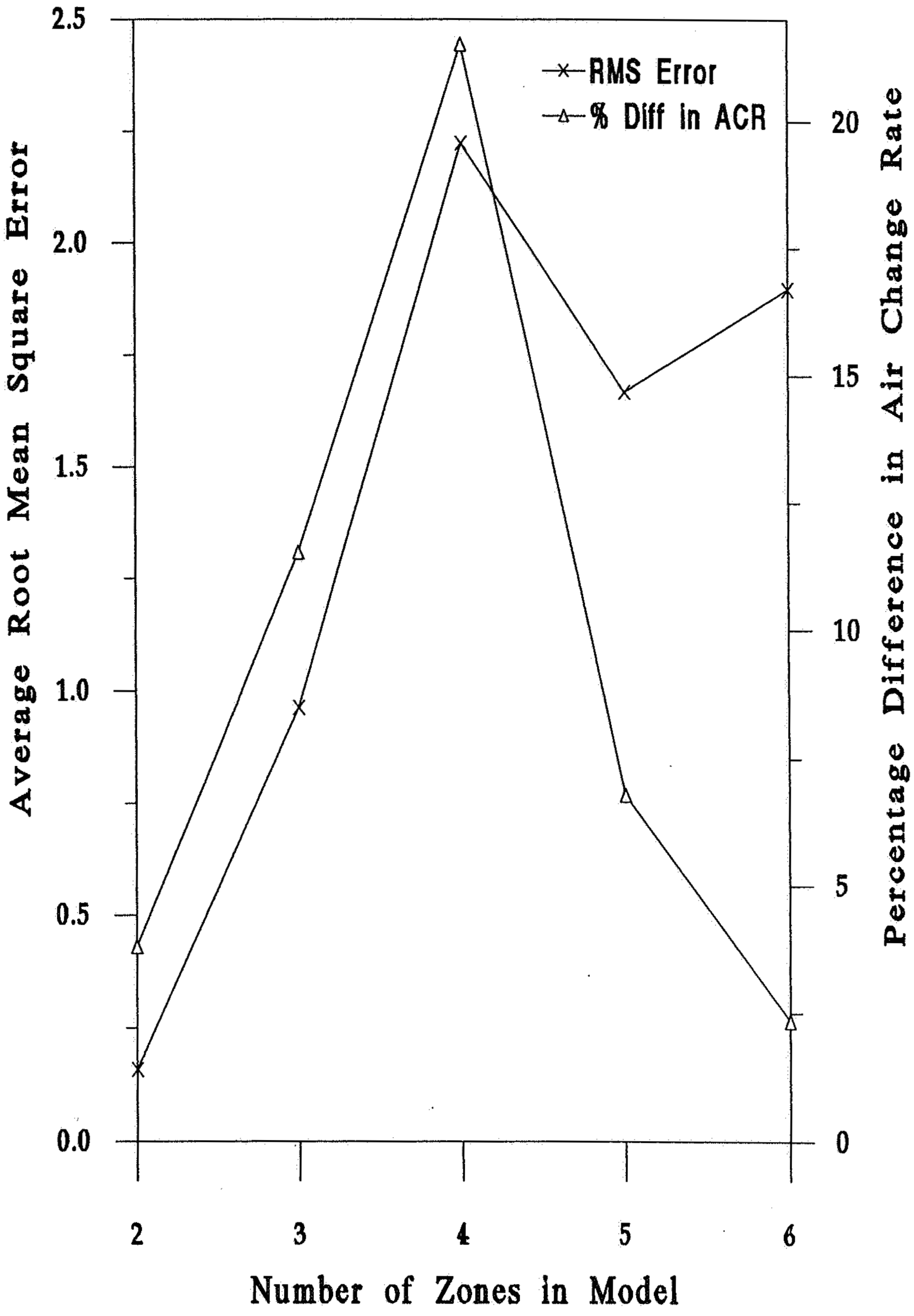

Figure 3. Average values of RMS Error and Percentage Difference in Air Change Rate for each model.

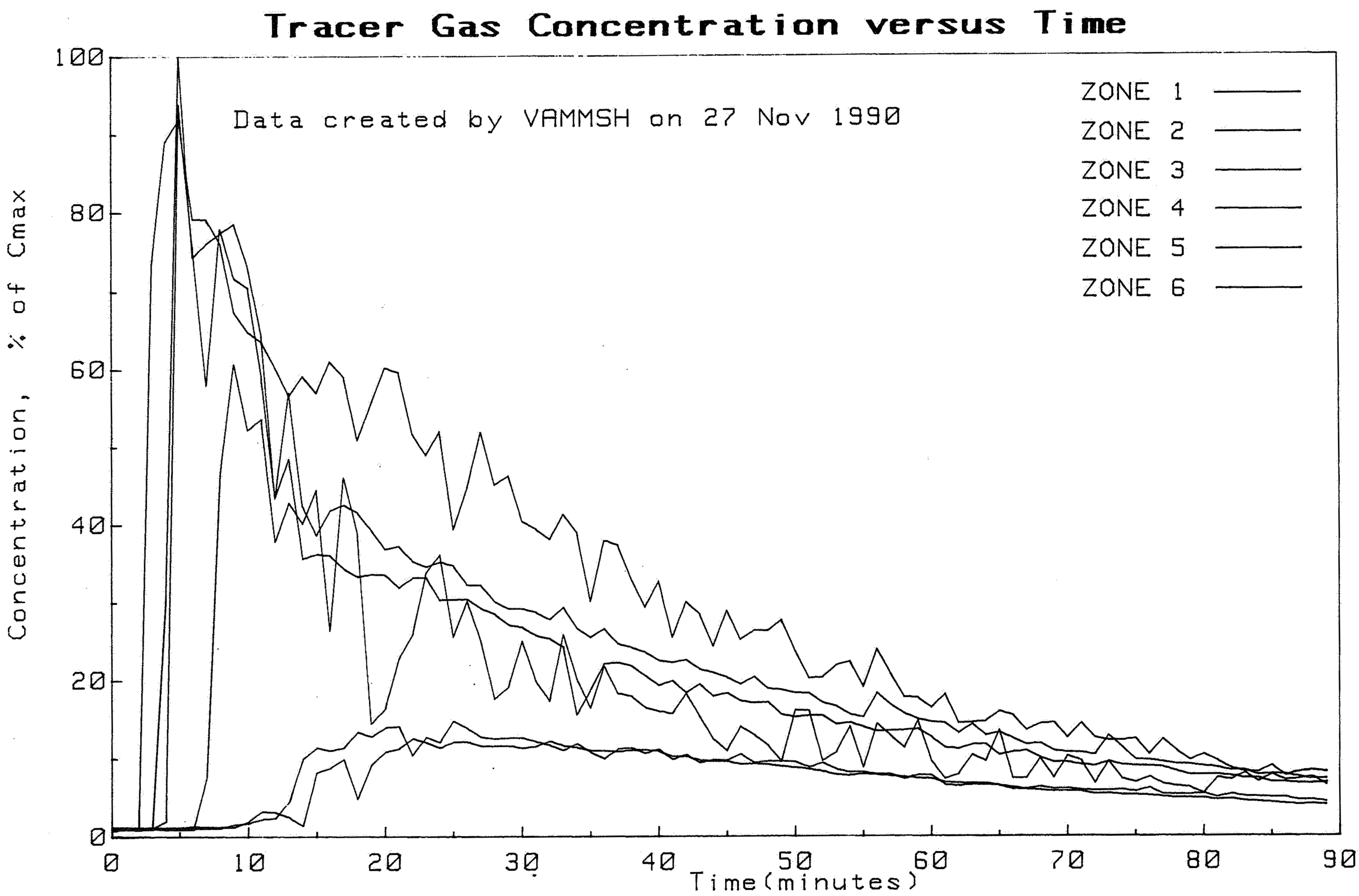

Figure 4 Typical set of decay curves for an industrial hall.