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MEASUREMENT OF ENTRANCE LENGTH AND FRICTION FACTOR OF DUCTS USING TRACER-GAS TECHNIQUES

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SYNOPSIS

We describe the use of constant injection and pulse injection techniques for measurement of airflow in a duct. Tracer-gas measurements were compared with measurements made using a pitot tube and a hot-wire anemometer. Tracer-gas concentration, air velocity and pressure distribution were measured at various distances from the duct wall and inlet. An empirical equation was obtained for the entrance length required to achieve fully-developed turbulent flow and this was compared with measurements made using a pitot tube and hot-wire anemometer. We present a relationship for the friction-factor and Reynolds number derived from tracer-gas measurements.

LIST OF SYMBOLS

C_c	Concentration of tracer gas; constant-injection technique (ppm)
C_p	Concentration of tracer gas; pulse-injection technique (ppm)
F_c	Airflow rate using constant-injection technique (m^3/s)
F_p	Airflow rate using pulse-injection technique (m^3/s)
F_u	Airflow rate using pitot tube (m^3/s)
F_h	Airflow rate using hot-wire anemometer (m^3/s)
q	Injection flow rate of tracer gas (m^3/s)
P_a	Pressure at point a (see Figure 9) (Pa)
P_x	Static pressure at point x (see Figure 8) (Pa)
U_b	Bulk velocity (m/s)
U_m	Maximum velocity (m/s)
U_τ	Shear velocity (m/s)
A_A	Cross-sectional area of the bellmouth (m^2)
A_B	Cross-sectional area of the duct (m^2)
f	Friction factor
Re	Reynolds number
L_e	Entrance length (m)
D_h	Hydraulic diameter of the duct (m)
X	Distance from the duct inlet in the direction of flow (m)
t	Time (s)
g	Acceleration due to gravity (m/s^2)
Z	Intercept of static pressure lines (Pa)
ρ	Air density (kg/m^3)
τ_w	Wall shear stress (Pa)

1. INTRODUCTION

Accurate measurement of airflow in ducts is important but often difficult to achieve using traditional instrumentation such as pitot tubes, hot-wire and vane anemometers. Limited access to the flow passage or short duct lengths could restrict measurements and flow velocities less than 3 m/s could lead to measurement inaccuracies if traditional instrumentation were employed. Tracer-gas techniques such as constant-injection and pulse injection offer an alternative approach for measuring airflow in ducts, and unlike traditional instrumentation, are not limited by the length or complexity of duct configuration. As gas chromatographs can detect tracer gas at low concentrations, tracer gas techniques can be used to measuring airflow over a wide range of values. Furthermore, tracer-gas techniques can be used to measure flow rates directly and do not require determination of the cross-sectional area of the duct or flow profile at the duct wall. One further advantage of tracer-gas techniques is that they can be used to determine the airtightness of ductwork. This is important if energy and noise resulting from air leakage are to be controlled.

The present study describes the use of constant-injection and pulse injection techniques for measuring airflow in a duct and compares the results with those obtained using a pitot tube and a hot-wire anemometer. We present an empirical equation for the entrance length required to achieve fully developed turbulent flow and a relationship for friction factor and Reynolds number.

2. THEORY

The following injection strategies were used to measure airflow in a duct:

2.1 Constant-Injection Technique

Tracer gas is injected into the duct at a constant rate and the resulting concentration response is measured. Assuming that the air and tracer gas are perfectly mixed within the duct, and that the concentration of tracer gas in outside air is zero, the following equation can be used for steady-state conditions[1]:

$$F_c = (q/C_c) \times 10^6 \qquad (1)$$

2.2 Pulse-Injection Technique

This technique is based upon the injection into the duct inlet of a short-duration pulse of tracer gas at a rate $G_{(t)}$. The variation of tracer concentration with time is measured at the duct exit. The amount of injected tracer gas is small, so it does not contribute significantly to the volume flow rate of air in the duct.

If we assume that the tracer gas is well mixed across the section of the duct, then the volume flow rate of tracer gas leaving the duct is equal to the product of the flow rate and the exit concentration. If the tracer gas is assumed to be purged from the duct after some time interval (t_1 to t_2) then the volume of tracer leaving the duct must equal to the amount injected. Applying the integral volume balance of tracer gas, we have:

$$F_P = [\int_{t1}^{t2} C_p(t)\,dt]^{-1} \int_{t1}^{t2} G_{(t)}\,dt \qquad (2)$$

3. EXPERIMENTAL

The experimental work was carried out using the duct system shown in Figure 1. The duct was constructed from galvanised mild steel and was 12m long with an internal diameter of 0.56m. The downstream end was connected to an axial fan by means of a diffuser. The flow rate through the duct was varied using a speed controller made by ABB Stromberg Drives, Finland. The fan was driven by an AC motor of 4 kW and with a maximum speed of 2880 rpm. The fan was manufactured by Elta Fan Ltd, UK.

Static, velocity pressure and tracer gas tappings were positioned along the duct. The velocity tappings allowed insertion of a pitot tube or a hot-wire anemometer which could be traversed across the duct cross-section in order to measure velocity at various distances from the duct wall. Velocity and static pressures were measured using an EMD 2500 micromanometer, made by Airflow Development, UK.

For the constant-injection technique (see Figure 2), SF_6 tracer gas was supplied at a constant rate into the duct inlet using a mass flow controller which had a maximum flow capability of 3.9 L/min. The measurement accuracy of the mass flow controller was ± 1%.

For the pulse-injection technique, tracer gas was injected at the inlet of the duct using a syringe (see Figure 3). Multipoint injection was necessary for the approximation of a uniform concentration across the cross-section of the duct at the measurement point. It was necessary to measure the concentration of tracer gas at the downstream point to determine the integral of the concentration. This was achieved by filling an air sample bag by means of a small pump. Sampling was begun 10 seconds before the pulse was injected, and continued until the pulse was completely purged from the duct.

The concentration of tracer gas was measured using an Infra-red gas analyser, type BINOS 1000, made by Rosemount GmbH & Co (RAE), Germany. The accuracy of analyser was estimated to be within ± 2%.

4. RESULTS AND DISCUSSION

4.1 Friction-Factor and Reynolds Number

The wall shear stress for steady, incompressible fully-developed flow in a duct is given by:

$$\tau_w = \frac{-D_h}{4}(\Delta P/\Delta X) \qquad (3)$$

The friction factor may be defined as:

$$f = 2\,\tau_w/\rho U_b^2 \qquad (4)$$

Measurement of airflow rate in the duct was carried out by means of the constant-injection and pulse injection techniques as well as using a pitot tube and hot-wire anemometer. SF_6 was injected at $X/D_h = 0.625$, and the concentration of tracer gas was monitored at various positions downstream. Figures 4 and 5 show the variation of tracer-gas concentration with X/D_h for Reynolds numbers in the range 76220 to 392850. The concentration of tracer gas was found to be large close to the injection point, and decreased as X/D_h increased. The tracer-gas concentration remained constant when X/D_h was greater than 15 (for constant-injection) and 8 (for pulse injection technique).

Figures 6 and 7 compare measurements of duct airflow rate made with the tracer-gas techniques, and a pitot tube and a hot-wire anemometer. General agreement was observed, and the best linear relationships were:

$$F_c = 1.05\,F_u + 0.112 \qquad (5)$$

$$F_p = 1.062\,F_u - 9.12 \times 10^{-2} \qquad (6)$$

$$F_c = 0.998\,F_h + 0.223 \qquad (7)$$

$$F_p = 1.01\,F_h + 1.75 \times 10^{-2} \qquad (8)$$

The above results indicate that the flow rate obtained using the pulse-injection technique is in closer agreement with values obtained using the pitot-tube and hot-wire anemometer than the flow rate obtained using the constant-injection technique.

The friction factor f of the duct was calculated using average velocity (based on the pulse-injection technique) and pressure gradient for the fully-developed flow (see Figure 8). The following relationship between f and Re was obtained:

$$f = 0.0124\,Re^{-0.13} \qquad (9)$$

The friction factor obtained using equation (9) differs slightly from that obtained using Blasus[2] equation, probably as a result of the difference of the pipe characteristics and the inlet condition.

4.2 Entrance Length for Fully-Developed Flow

The formation of a boundary layer in a duct is shown in Figure 9. Air enters the duct at point a with a velocity U_a. At point b the velocity is uniform across the duct. At point f the boundary layer is completely formed. Further downstream from point f the boundary layer has a constant thickness. Here the influence of the entrance shape upon the airflow pattern has disappeared and fully-developed flow is said to exist.

We carried out measurements of tracer gas concentration and pressure distribution along the duct for a range of Reynolds numbers. Figures 8 and 10 were used to find an empirical expression (equation 10) for the entrance length required to achieve fully-developed flow (see Appendix for full derivation).

$$L_e/D_h = 2.315\, Re^{0.13} \tag{10}$$

The entrance length derived from equation (10) is similar to that given by Hinze[3], i.e., $L_e/D_h = 0.693\, Re^{0.25}$, using the 1/7th power law approach.

5. CONCLUSIONS

The following conclusions are drawn:

1. Results indicate that the flow rate obtained using the pulse-injection technique is in closer agreement with values obtained using the pitot-tube and hot-wire anemometer than the flow rate obtained with the constant-injection technique.

2. The friction factor for the duct is given by $f = 0.124\, Re^{-0.13}$ and the entrance length required to achieve fully-developed flow is given by $Le/Dh = 2.315\, Re^{0.13}$.

ACKNOWLEDGEMENTS

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APPENDIX

Considering Figure 9, and applying the continuity equation to sections A and B, we have:

$$A_A U_A = A_B U_B \qquad \text{(A1)}$$

But $U_A = U_a$ and $U_B = U_b$ and so equation A1 can be rewritten as

$$U_a = U_b A_B / A_A \qquad \text{(A2)}$$

Applying Bernoulli's equation between points a and f (along the stream line abf, Figure 9) gives:

$$U_a^2/2g + P_a/\rho g = U_f^2/2g + P_f/\rho g \qquad \text{(A3)}$$

But $U_f = U_m$

Substituting equation A2 into equation A3 and rearranging, we have:

$$(P_a - P_f)/\rho g = [U_m^2 - A_B/A_A)^2 U_b^2]/2g \qquad \text{(A4)}$$

Dividing both sides of equation A4 by U_m^2 we have:

$$(P_a - P_f)/\rho U_m^2 = 0.5 - 0.5(A_B/A_A)^2 (U_b/U_m)^2 \qquad \text{(A5)}$$

But $A_B/A_A = 0.424$

Tracer-gas concentration was measured at different distances from the duct wall at the inlet of the duct and at the region of fully developed flow. This allowed the ratio C_m/C_b to be determined. Applying equation A4, we find that the flow rate ratio is given by:

$$F_b/F_m = C_m/C_b$$

since

$$F = AU$$

the velocity ratio U_b/U_m is given by:

$$U_b/U_m = F_b/F_m = C_m/C_b$$

The average value of U_b/U_m for the range of Reynolds numbers used in these experiments was 0.819.

Substituting the values of A_B/A_A and U_b/U_m into equation A5 gives:

$$(P_a - P_f)/\rho U_m^2 = 0.440 \qquad \text{(A6)}$$

Consider the variation of static pressure with X, Figure 10. The difference between the static pressure at $X = 0$ (i.e. pressure ≈ atmospheric pressure) and the static pressure of fully developed flow, $X = L_e$ is given by:

$$P_a - P_x = (Z + Y) = Z + L_e \tan \theta \quad \text{(A7)}$$

or

$$P_a - P_f = Z - L_e dP/dx \quad \text{(A8)}$$

$$\tau_w = -(D_h/4)dP/dx \quad \text{(A9)}$$

Equation A9 is normally applied to regions of fully developed flow but is also a very good approximation for the entrance region of the duct provided that $L/D_h > 1$. Experimental results (Figure 8) showed that dP/dx was constant along the length of the duct for the range of Reynolds numbers used.

Substituting equation A9 into A8 and dividing both sides of equation A8 by ρU_m^2, we have:

$$(P_a - P_f)/\rho U_m^2 = \rho_w g/\rho U_m^2 + 4 L_e \tau_w / D_h \rho U_m^2 \quad \text{(A10)}$$

$$U\tau = (\tau_w/\rho)^{0.5} \quad \text{(A11)}$$

and

$$U\tau = U_b(f/2)^{0.5} \quad \text{(A12)}$$

Substituting equations A11 and A12 into equation A10 and simplifying, we have:

$$(P_a - P_f)/\rho U_m^2$$

$$= Z/\rho U_m^2 + 2(L_e f/D_h)(U_b/U_m)^2 \quad \text{(A13)}$$

From tracer gas measurements, $U_b/U_m = 0.819$

$$f = 0.0124 \, Re^{-0.13} \quad \text{(A14)}$$

Equation A14 is applicable when the velocity profile is fully developed. If the velocity profile is not fully developed but $L/D_h > 1$, then equation 14 is a good approximation.

Substituting the value of U_b/U_m and equation A14 into A13 and simplifying, we have

$$(P_a - P_f)/\rho U_m^2 = Z/\rho U_m^2$$

$$+ 0.0166 \, (L_e/D_h) \, Re^{-01.3} \quad \text{(A15)}$$

Substituting equation A6 into A15 and simplifying, we have:

$$L_e/D_h = (26.42 - 60.1\, Z/\rho U_m^2)\, Re^{0.13} \qquad \text{(A16)}$$

The intercepts Z of the static pressure lines were found from Figure 8.

$$Z/\rho U_m^2 = 0.401 \qquad \text{(A17)}$$

Substituting equation A17 into equation A16 we obtain:

$$L_e/D_h = 2.32\, Re^{0.13} \qquad \text{(A18)}$$

This equation can be used to determine the entrance length L_e for fully developed turbulent flow.

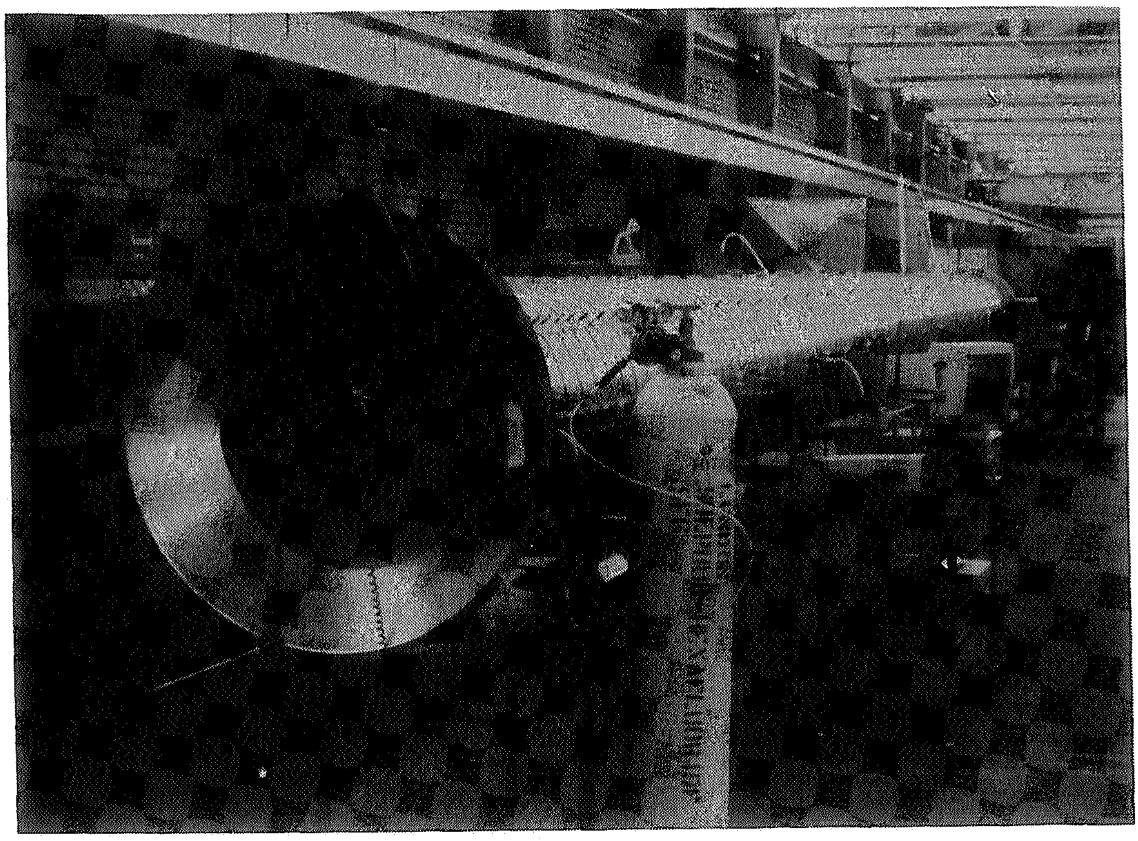

Figure 1 Experimental system for testing tracer-gas techniques

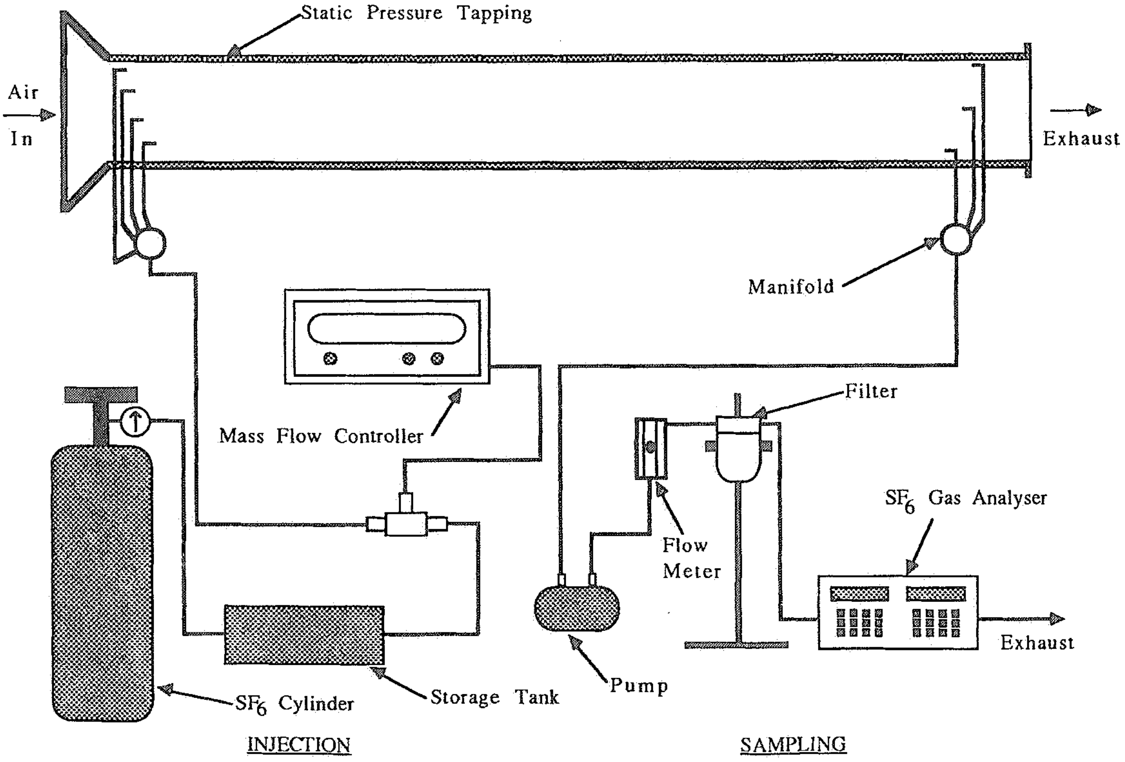

Figure 2 Instrumentation for the constant-injection technique

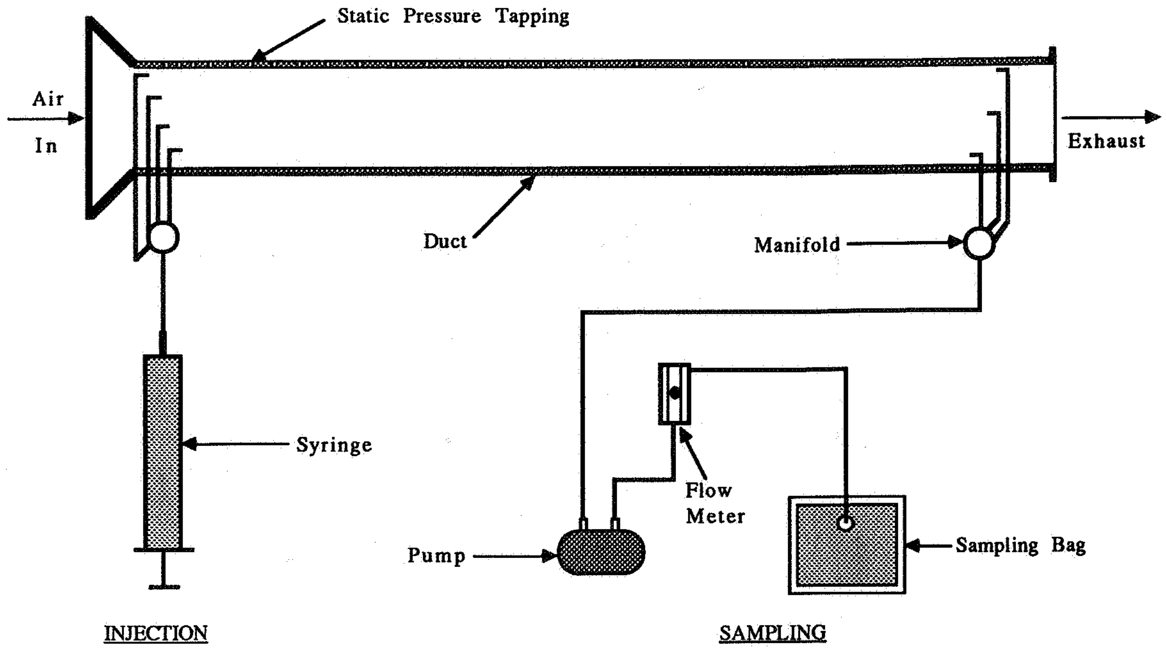

Figure 3 Instrumentation for the pulse-injection technique

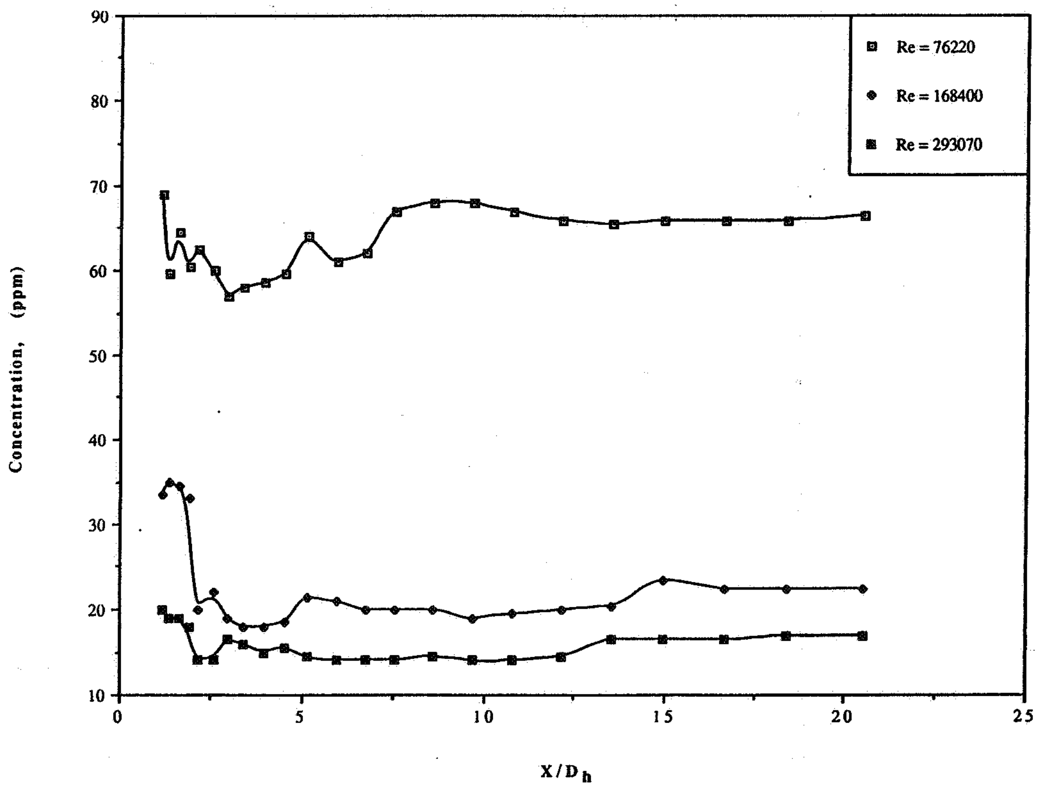

Figure 4 Variation of tracer-gas concentration with X/D_h, constant-injection technique

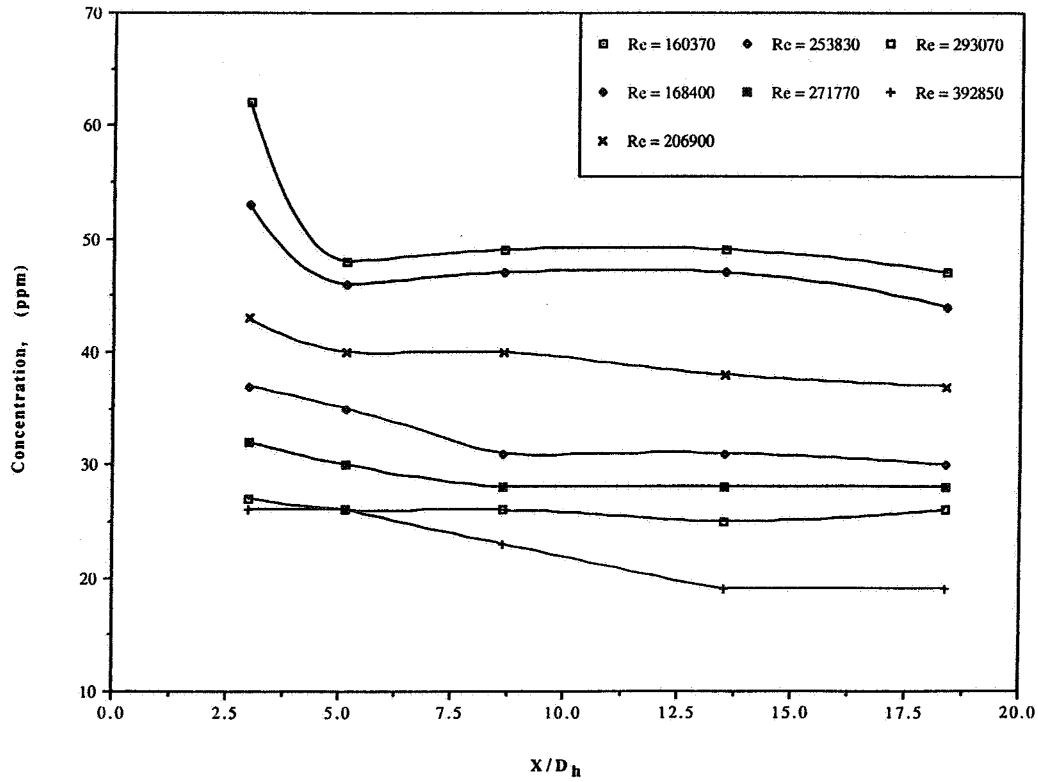

Figure 5 Variation of tracer-gas concentration with X/D_h, pulse-injection technique

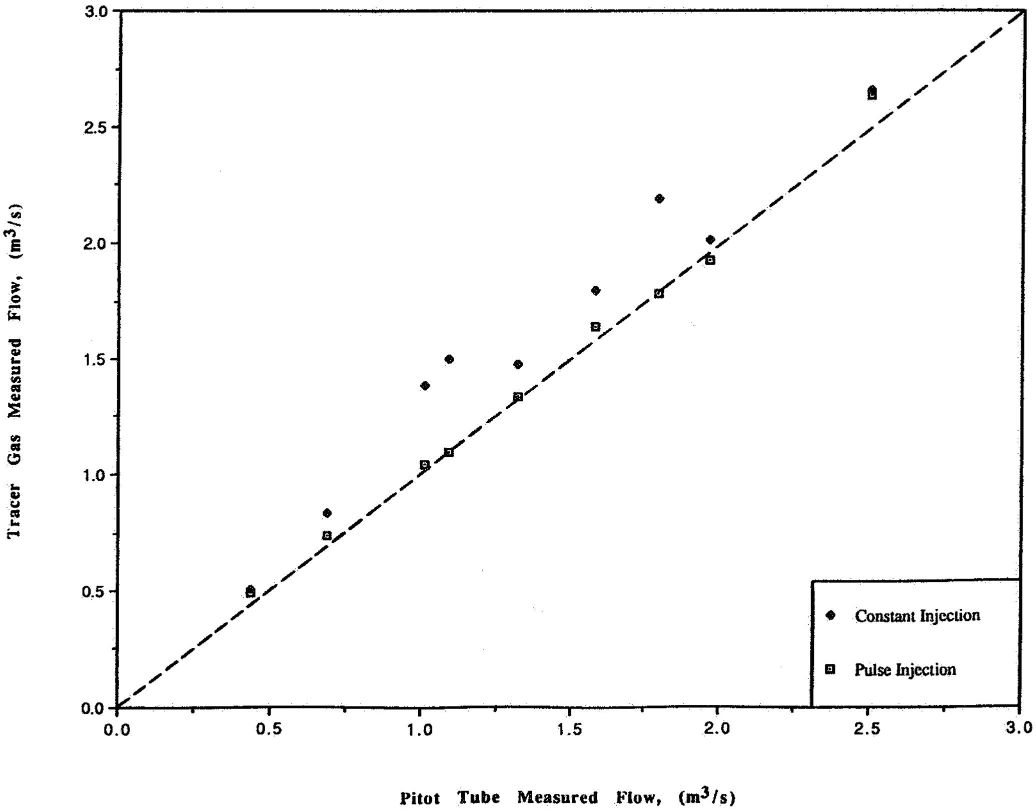

Figure 6 Comparison of tracer-gas airflow measurements with measurements made using a pitot tube

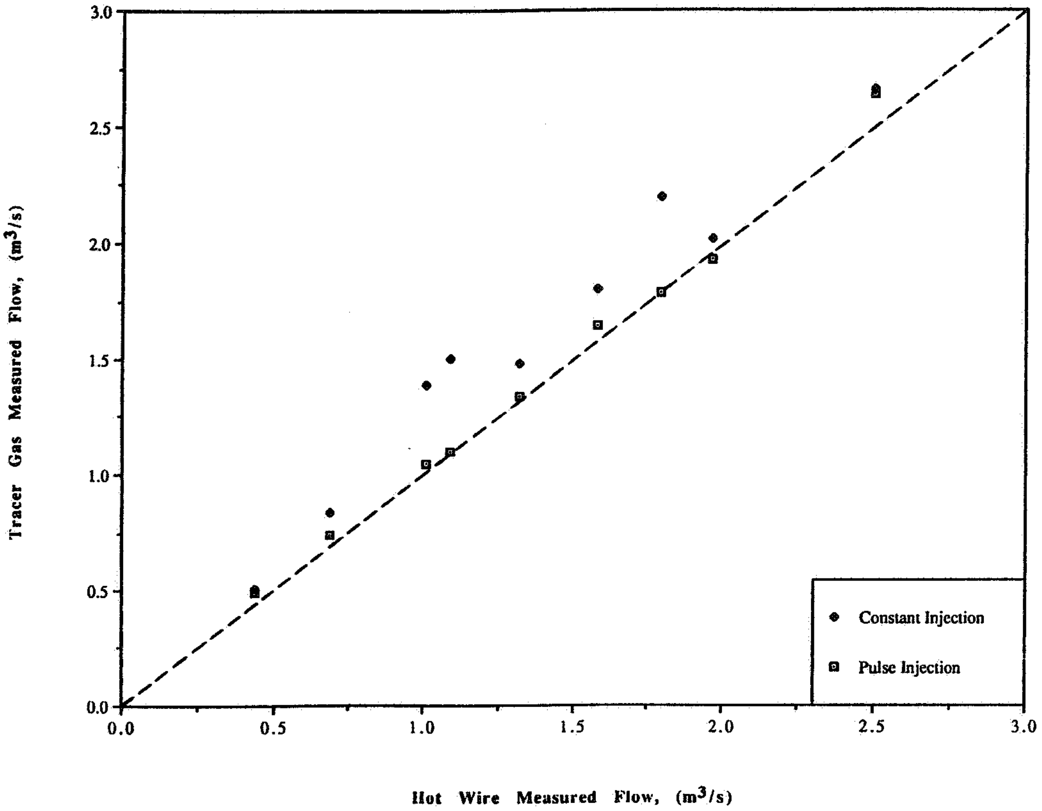

Figure 7 Comparison of tracer-gas airflow measurements with measurements made using a hot-wire anemometer

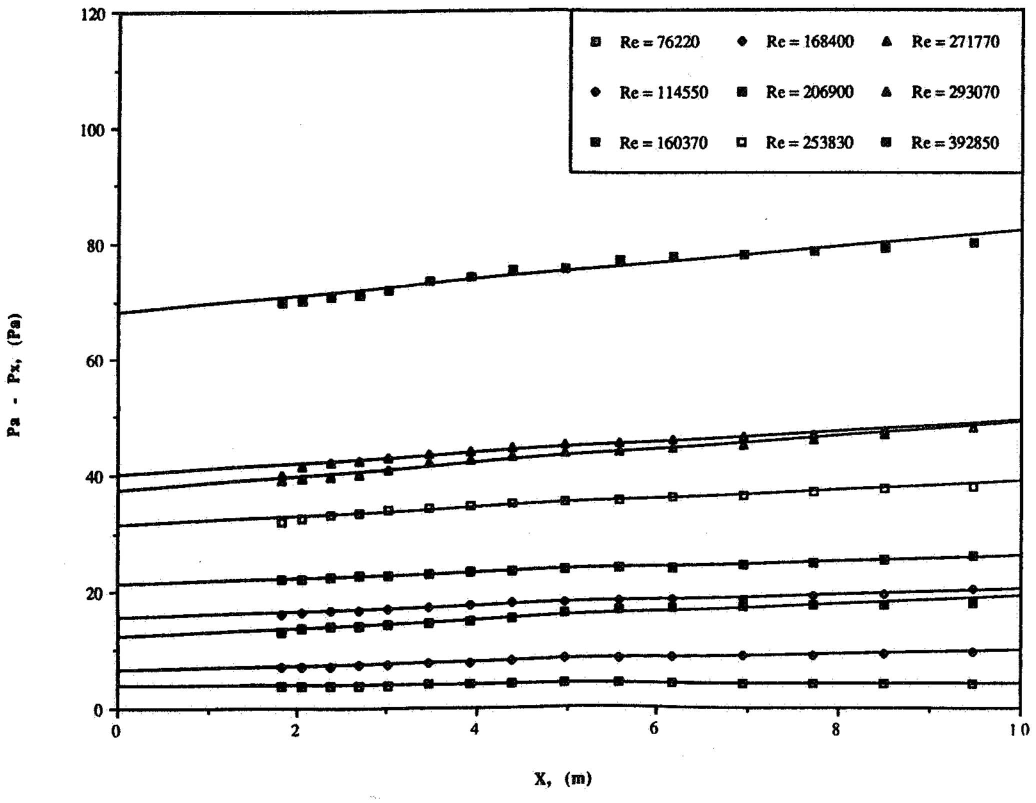

Figure 8 Static pressure distribution along the duct for various values of Re

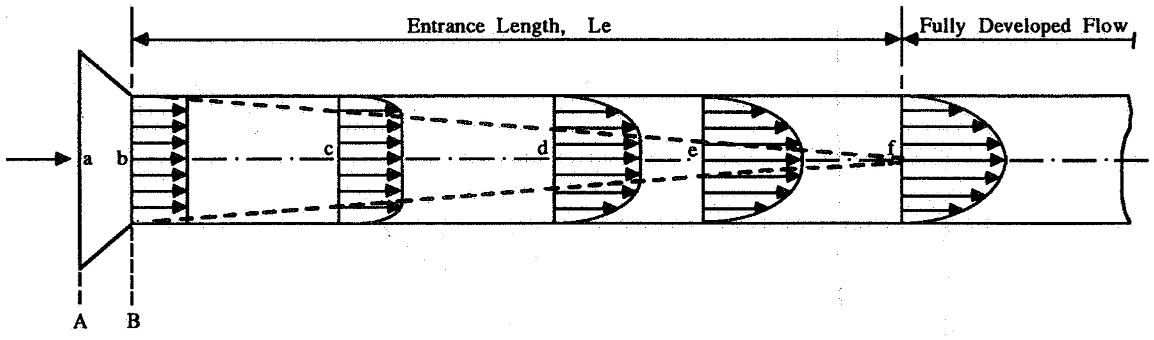

Figure 9 Formation of a boundary layer in a duct

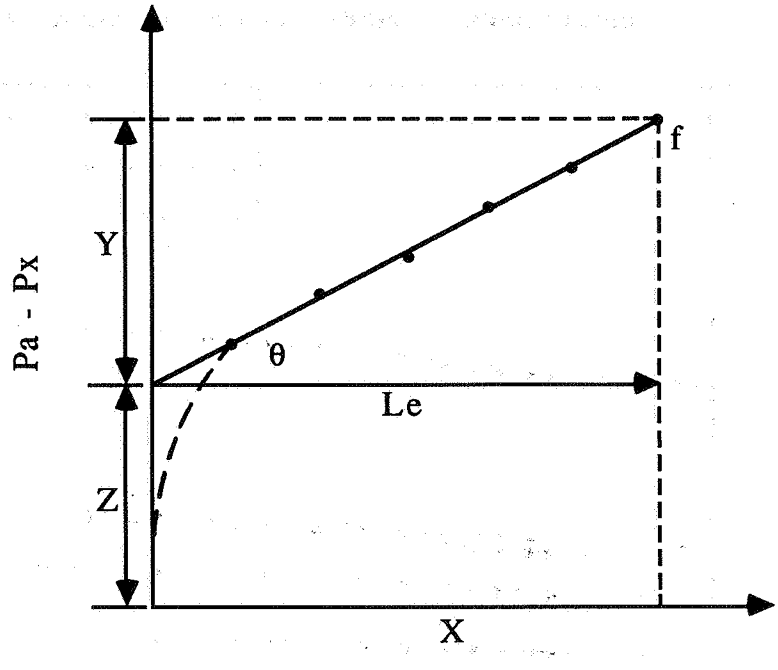

Figure 10 Variation of static pressure with X