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A New Control Algorithm for the Measurement of  
Variable Air Change Rates.

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## ABSTRACT

A new algorithm for the continuous measurement of variable air change rates with tracer gases will be presented. It differs from the constant concentration method by allowing the concentration level to vary according to the air change rate. Also the the mixing process of tracer gas within the room under investigation is considered and limited measurement ranges and injection rates of the tracer gas equipment can be accounted for. The new algorithm has a number of advantages, such as quick response to variations in the air change rate and reduced tracer gas consumption. The development of the control strategy is described and its practical applicability is shown by measurements in a laboratory room.

## 1 INTRODUCTION

Continuous measurements of time varying air change rates are usually performed by the constant concentration tracer gas method [3]. The basic idea is rather simple, the problem, however, lies in the choice of the control algorithm for the determination of the tracer gas injection rate. Up to now, classical controllers of P-, PI-, PID-type or modifications thereof as well as adaptive control algorithms have been used [2,4,9].

This paper presents a new control algorithm, which incorporates the following points:

1. The target level for the tracer gas concentration is allowed to vary according to the air change rate.
2. The mixing process of the injected tracer gas with the room air is taken into account.
3. The limited range of the tracer gas flow rate controller is considered.

Points 1 and 2 allow a quicker response to changes in the air change rate without causing over- or under-shoots. Furthermore point 1 saves tracer gas by lowering the target level for high air change rates. Point 3 prevents "wind-up" effects in linear controllers [6] as reported e.g. in [4]. This control algorithm will be described in section 2. Section 3 presents measurement results to show its application to the measurement of variable air change rates.

## 2 DESCRIPTION OF THE CONTROL ALGORITHM

### 2.1 Determination of Variable Air Change Rates

To determine the time varying air change rate  $n(t)$  of a room with volume  $V$  and tracer gas injection rate  $q(t)$  from the measured tracer gas concentration  $c(t)$ , we start with the differential form of the tracer gas conservation law for a single zone with constant temperature

$$\dot{c}(t) = -n(t)c(t) + \frac{1}{V}q(t) \quad (1)$$

By integration over a time interval  $[t - t_m, t]$  and division by the length  $t_m$ , one obtains

$$\frac{\Delta c(t)}{t_m} = -n(t) \bar{c}(t) + \frac{1}{V} \bar{q}(t) \quad (2)$$

where

$$\begin{aligned} \Delta c(t) &= c(t) - c(t-t_m), \\ \bar{c}(t) &= \frac{1}{t_m} \int_{t-t_m}^t c(\tau) d\tau, \\ \bar{q}(t) &= \frac{1}{t_m} \int_{t-t_m}^t q(\tau) d\tau. \end{aligned} \quad (3)$$

It has been assumed, that the air change rate  $n(t)$  varies slowly, such that it is approximately constant during the time of integration:

$$\int_{t-t_m}^t n(\tau)c(\tau) d\tau \approx n(t) \int_{t-t_m}^t c(\tau) d\tau. \quad (4)$$

Thus, the slowly varying air change rate can be obtained from (2) as (see also [1,2])

$$n(t) = \frac{\bar{q}(t)}{\bar{c}(t) V} - \frac{\Delta c(t)}{\bar{c}(t)} \frac{1}{t_m}. \quad (5)$$

This relation holds in principle for arbitrary time functions for the tracer gas injection  $q(t)$  and concentration  $c(t)$ . The only assumption for its validity is that the air change rate is constant during the integration time  $t_m$ , which may be in the range of a few minutes. The range of the measurement  $t$  time is not limited, thus allowing continuous measurements. Two special cases will be briefly mentioned:

- If the tracer gas injection consists of a single pulse completely released within the integration time  $t_m$ , then  $t_m \cdot \bar{q}(t)$  is the total tracer gas volume. This corresponds to the pulse injection technique [1].

- If the tracer gas injection is controlled such that the concentration remains at the constant level of a prescribed target concentration  $c_T$ , then  $\bar{c}(t) = c_T = \text{const}$ ,  $\Delta c(t) = 0$ . One can even omit the integration and obtains the familiar form of the constant concentration technique

$$n(t) = \frac{q(t)}{c_T V} . \quad (6)$$

In general, eqn. (5) could be used for continuous air change rate measurements for (almost) arbitrary values of  $q(t)$  and  $c(t)$ . For practical measurements, however, two restrictions have to be observed:

- The tracer gas injection rate is determined by some kind of flow controller, which can only release gas at a rate between a minimum and maximum value such that  $0 \leq q_{\min} \leq q(t) \leq q_{\max}$ .
- Similarly, the measurement device for the tracer gas concentration operates properly only for a given concentration range  $0 < c_{\min} \leq c(t) \leq c_{\max}$ .

This requires that the limited injection rate has to be controlled in such a way, that the concentration remains in the measurement range of the concentration measurement device.

It is important to note, that this control problem differs from the usual constant concentration method. The control of the tracer gas concentration is not a prerequisite for the measurement method, but merely a matter of practical instrumentation considerations. For the constant concentration method, deviations of a few percent of the target concentration are critical. Here, the tracer gas concentration may vary by a factor of 1:10 or more depending on the dynamic range of the concentration measurement device.

There are control algorithms that require an a-priori-knowledge of the air change rate for the determination of the coefficients (e.g. proportional control). Of course, these algorithms are not well suited for air change rate measurements with the constant concentration method. They require additional measures like an integrating term (PI-control) or coefficient adaption. However, these measures tend to slow down the response to changes in the air change rate. With the method suggested by eqn. (5), a dependency of the tracer gas concentration on the air change rate is not a problem, as long as the operating range of the concentration measurement device is not exceeded. It is therefore possible to tune the control algorithm for faster response rather than for minimal deviation from some target concentration. This trade-off will be addressed in the following subsections.

## 2.2 Mixing Model

The core of the proposed method is a control algorithm for the tracer gas injection rate  $q(t)$  to keep the concentration  $c(t)$  within the measuring range of the concentration measurement device. This algorithm has to be adapted to the room under investigation. Therefore a mathematical model for the tracer gas injection and exfiltration is required.

The control algorithms described in [2,4,9] consider the room as a black box and use the measured concentration as the only source of information. Depending on the mixing process of the injected tracer gas within the zone, changes in the injection flow rate will lead to concentration changes only after a certain delay time. Consequently, all these algorithms show a rather slow response to variations in the air change rate. After a step-up or step-down of the air change rate, it takes one hour or more until these methods show the new value with acceptable accuracy, as reported in [2,4,9].

The mixing process of tracer gas with a fan has been described by an additional time constant in [10]. It is used, however, only for the characterisation of the controlled system and has not been incorporated into the controller. This model will be elaborated here in more detail to serve as the basis of a control algorithm.

Usually, tracer gas is distributed in the room under investigation by a mixing fan at the location of the tracer gas injection. The resulting time and space dependent concentration field can be described in great simplification by a two-zone-model of a room with volume  $V$  according to figure 1. The mixing fan and the injection source are separated by a fictitious zone with volume  $V_2$  from the rest of a room with the volume  $V_1 = V - V_2$ . Between both zones exists an air flow  $F_2$  which is generated by the mixing fan. The room exchanges air with the exterior at a rate  $F_1$ . Zone 1 (room without injection zone) and zone 2 (injection zone) are assumed to be well mixed. Their respective tracer gas concentrations are  $c_1(t)$  and  $c_2(t)$ . The tracer gas balances for both zones (isothermal case) are given by (see [5])

$$\begin{aligned} \dot{c}_1(t) &= -\frac{F_1+F_2}{V_1} c_1(t) + \frac{F_2}{V_1} c_2(t) \\ \dot{c}_2(t) &= \frac{F_2}{V_2} c_1(t) - \frac{F_2}{V_2} c_2(t) + \frac{1}{V_2} q(t) . \end{aligned} \quad (7)$$

The zone of instantaneous mixing of the injected tracer gas is confined to the immediate surroundings of the mixing fan, such that  $V_2 < V_1$  and  $V_1 \approx V$ . Then, the relation between infiltration rate  $F_1$  and volume of zone 1  $V_1$  is approximately given by the air change rate  $n$ :

$$\frac{F_1}{V_1} \approx n \quad (8)$$

Under this assumption, the two-zone model given by (7) is related to the one-zone model (1) by

$$\dot{c}_1(t) = -nc_1(t) + \frac{1}{V}q(t) - \frac{V_2}{V}c_2(t) \quad (9)$$

This is obtained by rearranging eqns. (7) and (8).

Equations (7) give a state space description of the room under investigation (see [6,7,10,11]). The injection rate  $q(t)$  is the input function and  $c_1(t)$  and  $c_2(t)$  are the state variables. The room concentration  $c_1(t)$  is the output function and equal to the first state variable. This state space model will be used for the development of the control algorithm in the next section.

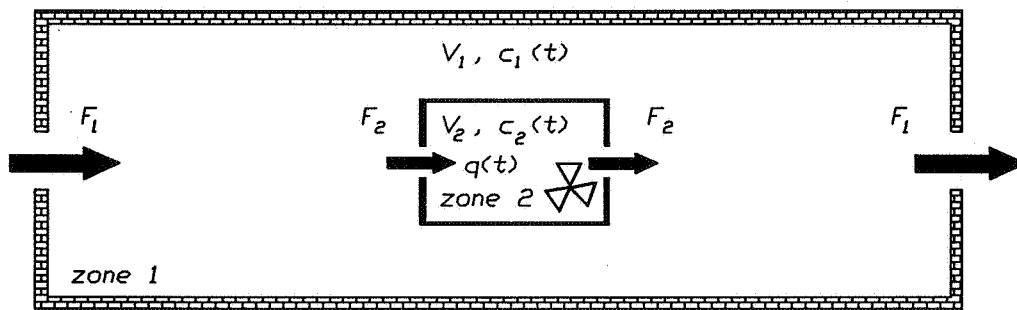


Fig. 1: Two-zone mixing model

### 2.3 Control Algorithm

A first approach for the control of the tracer gas concentration would be a simple control loop, which computes the injection rate  $q(t)$  from the current concentration in the room  $c_1(t)$ . It has been already discussed that the use of classical control algorithms leads to a slow response, since the influence of the tracer gas injection can only be observed by its retarded effect on the concentration  $c_1(t)$ .

A faster response could be expected, if not only the room concentration  $c_1(t)$  but also the concentration in the injection zone  $c_2(t)$  would be used as an input to the controller, because an injection acts directly on  $c_2(t)$  before  $c_1(t)$  is affected. This leads to the concept of state feedback [6,7,10,11], since all state variables are fed into the controller. The problem is that the concentration in the injection zone is not available for real measurements, since the injection zone itself is only a model and not a well defined item.

This problem can be overcome by a state observer [6,7,10,11]. That means here that the two-zone mixing model is included in the control algorithm in order to estimate the non-measurable quantity  $c_2(t)$  from the measurable quantities  $c_1(t)$  and  $q(t)$ . This estimate is used for the state feedback. Since the first state variable  $c_1(t)$  is known, a reduced-order state observer of first order is sufficient for the estimation of  $c_2(t)$ . The design of state observation controllers can be found in standard text books on control systems (see e.g. [6,7,10,11]) and will not be outlined here.

Before a concise description of the control algorithm will be given, it is necessary to consider the limited range of the flow control device for the tracer gas injection. While the control algorithm may compute some desired injection rate  $q(t)$ , the flow controller can only release gas at a rate  $q_b(t)$  with limited range, such that

$$q_b(t) = \begin{cases} q_{\min} & q(t) < q_{\min} \\ q(t) & q_{\min} \leq q(t) \leq q_{\max} \\ q_{\max} & q_{\max} < q(t) \end{cases} \quad (10)$$

where  $q_{\min}$  and  $q_{\max}$  are determined by the type of flow controller. This nonlinearity has to be included into the control algorithm, in order to provide the necessary information on the actual injection rate.

A block diagram of the control algorithm is shown on the left side of figure 2 (dashed box labelled controller). It consists of four components:

- A factor  $l$ , which computes the injection rate required to maintain a given target concentration  $c_T$  under standard conditions defined in advance. They are based on the two-zone mixing model and include an estimate of the mean air change rate during the measurement. Under- and over-estimation does not effect the accuracy of the measurement but only the tracer gas concentration level that is actually reached.
- A block  $F_u$  which computes the partial injection rate  $r_u(t)$  from the current injection rate  $q_b(t)$  as released by the flow controller.
- A block  $F_y$  which computes the partial injection rate  $r_y(t)$  from the (smoothed) tracer gas concentration.
- A nonlinear block which computes the tracer gas injection rate as actually released by the flow controller according to (10).

Each of the blocks  $F_u$  and  $F_y$  simulates a first order differential equation.

## 2.4 Total System for the Determination of the Air Change Rate

The total system for the determination of the air change rate is shown in figure 2. It consists of the blocks controller, room, smoothing, and evaluation.

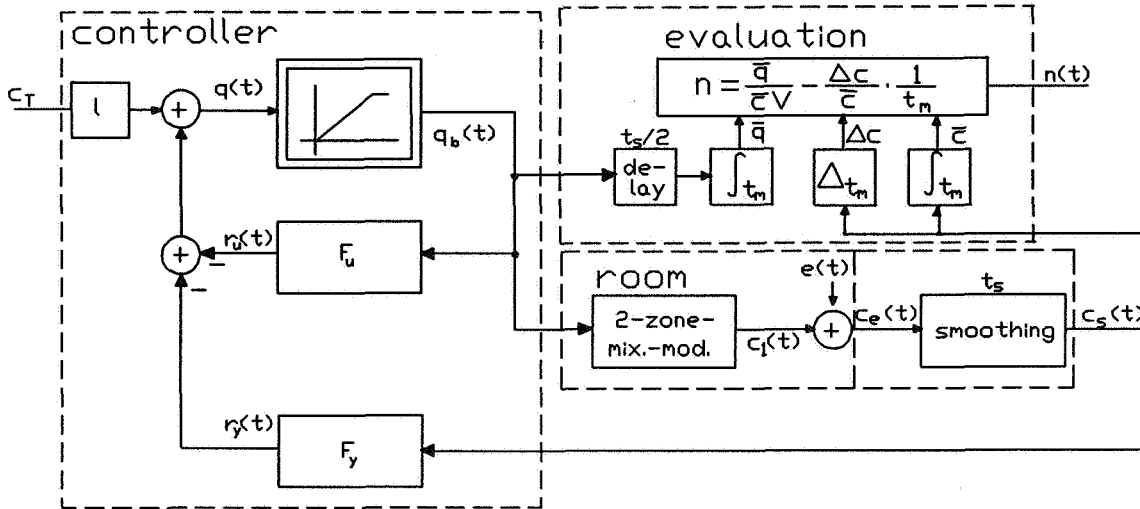


Fig. 2: Total system for the determination of the air change rate

The controller has been described in the previous subsection. The room under investigation responds to the injection rate  $q_b(t)$  with the tracer gas concentration  $c_e(t)$ . Its measurement values can be modelled as the output  $c_i(t)$  of the ideal two-zone mixing model and an additive noise signal  $e(t)$ , which accounts for concentration fluctuations due to incomplete mixing and inhomogeneities in the flow field.

If the control algorithm is to be designed for a fast response to changes in the air change rate, then care has to be taken that short term fluctuations in the measured concentration  $c_e(t)$  do not lead to an injection rate  $q_b(t)$  with similar fluctuations. To avoid this situation, the measured concentration is smoothed by averaging  $c_e(t)$  over a time span  $t_s$ . The smoothed concentration  $c_s(t)$  is used as input signal to the control algorithm as well as for the determination of the air change rate in the evaluation block.

The evaluation computes the estimate  $n(t)$  for the air change rate according to equation (5), where  $q_b(t)$  is used for the injection rate. Since smoothing the measured concentration introduces a delay of  $t_s/2$ , the same delay has to be applied to  $q_b(t)$  as well. One may argue, that equation (5) based on the single-zone model (1) is not consistent with a control algorithm based on the two-zone model (7). An evaluation formula based on the two-zone model could be obtained by integrating (9)



over  $t_m$ . However, eqn. (9) differs from the single-zone model (1) only by  $\dot{c}_2(t)$  scaled by the relation of the injection zone volume to the total volume. Since  $V_2/V < 1$ , this contribution may be neglected and the single-zone model is used instead for the calculation of the air change rate.

It should be noted that the two-zone model appears twice in the total system:

- It is used to describe the influence of the tracer gas injection into the room on the measured concentration. The model parameters depend on the fan size and the air change rate in the room under investigation and may vary with time.
- It has been used for the design of the controller and is contained implicitly in the blocks  $F_u$  and  $F_y$ . The model parameters were assumed as fixed values.

## 2.5 Properties of the Controlled System

The properties of the controlled system are analyzed under some simplifying assumptions:

- The tracer gas flow control device operates in the linear range, i.e.  $q_b(t) = q(t)$ .
- The room under investigation behaves like an ideal two-zone model, such that  $e(t) = 0$  and no smoothing is necessary, i.e.  $c_s(t) = c_1(t)$ .
- The system is in a stationary state. The parameters and the injection rate are constant and any transient terms in the concentration have decayed.
- The flow rate of the mixing fan assumed for the design of the controller equals the flow rate of the mixing fan in the room.

The air change rate assumed for the design of the controller is denoted by  $n_T$ , while  $n_R$  denotes the actual air change rate in the room under investigation. The measured concentration is  $c_R = c_1(t)$ .

If the air change rate  $n_T$  used for the controller design is equal to the air change rate of the room, then the observer is able to give a correct estimate for the concentration in the injection zone and the measured concentration  $c_R$  will be equal to the target concentration  $c_T$ .

However, if the air change rate in the room is yet to be determined, then only an estimated value  $n_T$  can be used for the design of the controller. The observer based on this value will not produce the correct injection zone concentration, if the actual air change rate  $n_R$  differs from  $n_T$ . Consequently, also the actual room concentration  $c_R$  will differ from the target concentration  $c_T$ . It can be shown that the concentration and the air change rate of the model assumptions ( $c_T, n_T$ ) and of the room ( $c_R, n_R$ ) are related by

$$\frac{c_R}{c_T} = \frac{n_T + a}{n_R + a} \quad (11)$$

where  $a$  is a measure for the DC-gain of the controller. It determines the relation between the concentrations  $c_R$  and  $c_T$  between the limiting cases:

$$\begin{aligned} a = 0 : & \quad \frac{c_R}{c_T} = \frac{n_T}{n_R} \\ a \rightarrow \infty : & \quad c_R = c_T \end{aligned} \quad (12)$$

For  $a=0$  (no control), the relation of the concentrations is inverse to the relation of the air change rates. This corresponds to a constant tracer gas injection. For  $a \rightarrow \infty$ , the measured concentration  $c_R$  is equal to the target concentration  $c_T$  independent of the air change rate  $n_R$ . This is the case of an ideal constant concentration method.

Note, that the DC-gain of the controller determines also the sensitivity to noise in the measured concentration. The noise sensitivity increases with increasing gain. Thus, the controller design is a compromise between sensitivity to noise and deviation from the target concentration. However, the ultimate goal is the determination of the air change rate, where - according to eqn. (5) - a constant concentration is not required. Therefore, the design of the controller may focus on low sensitivity to noise rather than minimal deviation from the target concentration. Choosing a moderate DC-gain will lead to an injection rate  $q(t)$  that is little affected by concentration measurement noise and to a concentration level in the room  $c_R$  that deviates more or less from the target concentration  $c_T$ . A good estimate of the actual air change rate can be expected from these signals (see eqn. (5) and the block "evaluation" in figure 2). The range of possible values for  $c_R$  can be adjusted to the measurement range of the tracer gas analyser by selecting proper values for  $c_T, n_T$ , and  $a$ . A typical value for  $a$  is  $a = 3 \cdot n_T$ .

Inspection of eqn. (11) shows that high air change rates in the room decrease the concentration level and thus the tracer gas consumption for long term measurements. The main advantage of the proposed control configuration becomes apparent in non-stationary situations. It leads to a substantially faster reaction to temporal changes in the air change rate compared to controllers designed for minimal deviation from the target. This will be demonstrated by an example in the next section.

### 3 MEASUREMENT RESULTS

The system for the determination of the air change rate as depicted in figure 2 has been implemented in one channel of the tracer gas measurement system MULTI-CAT (Multi-channel analysis of tracers) [8]. The result of a measurement in a laboratory room is shown in figure 3. Door and windows were opened and closed during the measurement according to the following schedule:

phase 1	0 - 15 minutes	door and windows closed
phase 2	15 - 40 minutes	door closed, two windows tilted
phase 3	40 - 70 minutes	door open, two windows tilted
phase 4	70 - 100 minutes	door closed, one window tilted

Obviously, the tracer gas concentration is not constant. Instead it reaches a new equilibrium in each phase. This is due to the fact, that the DC-gain of the controller was selected for a high noise suppression in the concentration measurement. It can be seen that the injection rate is very smooth, although the concentration curve exhibits quite sizeable short term fluctuations. The air change rate was calculated from the injection and the smoothed concentration by eqn. (5). It varies between ca. 0.5 1/h in phase 1 and up to 14 1/h in phase 3.

The following advantages of this control strategy become apparent from these curves:

- Even with high variations of the air change rate, the variations of the injection rate are rather modest and stay within the linear range of the tracer gas flow control device (here 2 – 100 L/h). Demanding a constant concentration level for all four phases would have led to excessive injection rates ( $> 100$  L/h) in phase 3 or for a reduced concentration level to an injection rate below the minimum value (2 L/h) for phase 1.
- The controller responds quickly to variations of the air change rate. The time series of the air change rate shows, that after each opening or closing of the door or the windows a transition phase of less than 15 minutes is sufficient until a new equilibrium is reached (if it exists at all, see phase 3). Other control strategies, which try to maintain a constant concentration under all circumstances, exhibit a transition phase in the order of one hour and more [2,4,9].
- The concentration level decreases with an increase of the air change rate (see phases 1,2,3). In other words, the exfiltration of tracer gas from the room is under proportional to the

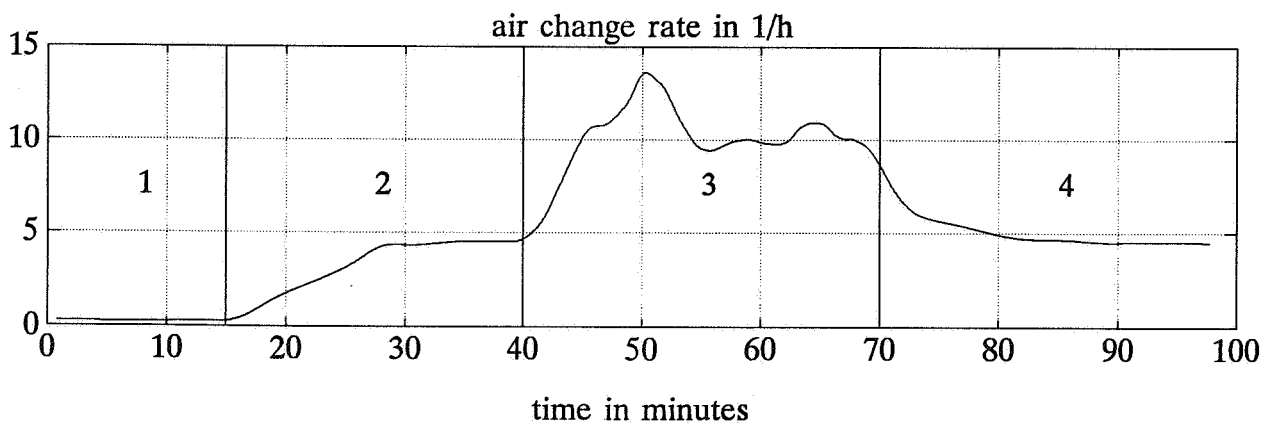
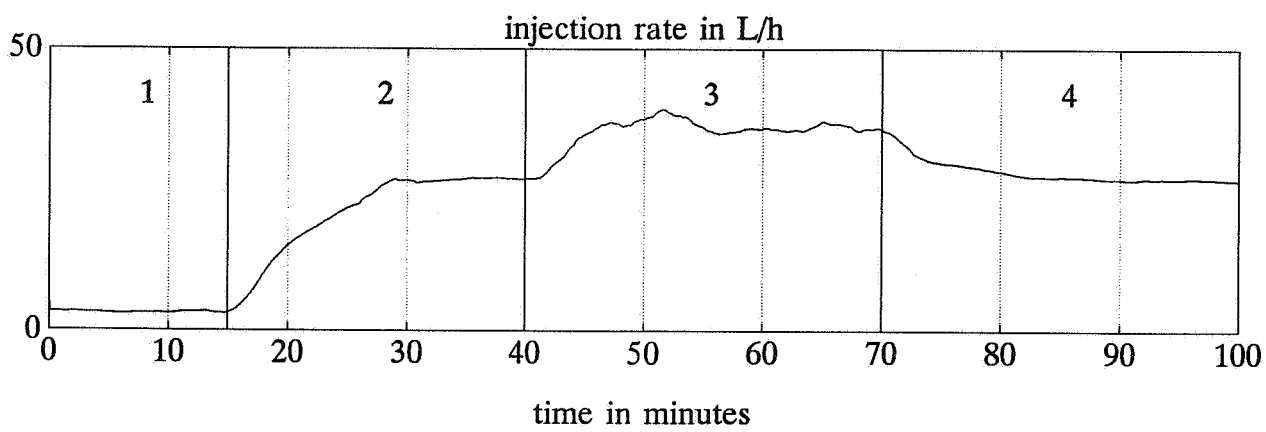
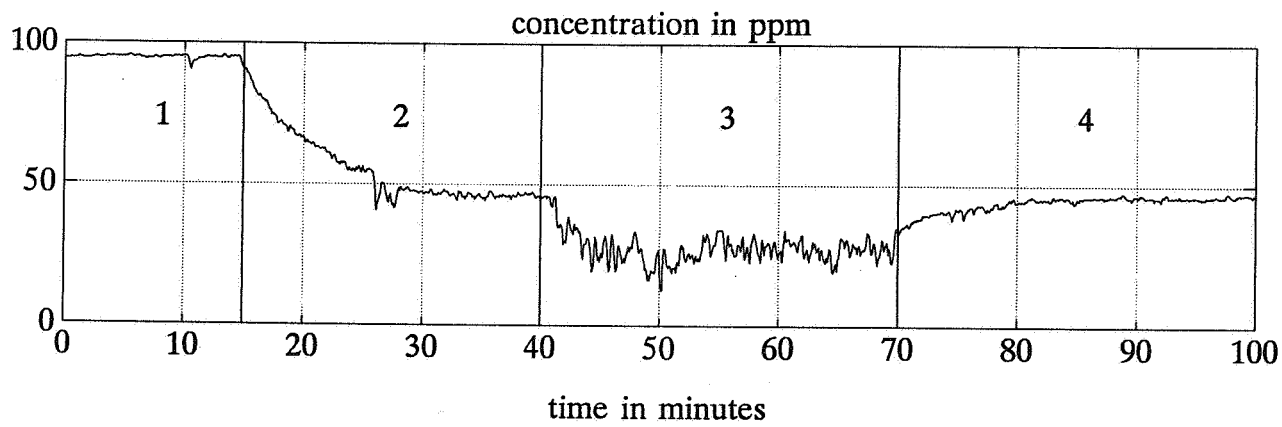


Fig. 3: Measurement of variable air change rate in a laboratory room: measured tracer gas concentration, injection rate, air change rate

exfiltration of air. This leads to a reduced tracer gas consumption for continuous measurements.

## CONCLUSION

A new control algorithm for the determination of variable air change rates has been presented. In contrast to the usual constant concentration method, it incorporates the following points:

- The target level for the tracer gas concentration is allowed to vary according to the air change rate.
- The mixing process of the injected tracer gas with the room air is taken into account.
- The limitations of concentration measurement range and tracer gas flow rate are observed.

This approach is rather flexible since the algorithm can be adapted to the measurement conditions on site and to the properties of the tracer gas equipment. The following advantages over existing constant concentration control algorithms have been confirmed by measurements in a laboratory room:

- The variation range of the injection rate can be confined to the flow range of the tracer gas flow control device at hand, even with high variations of the air change rate.
- The controller responds quickly to variations of the air change rate. The response time to a step change of the air change rate is less than 15 minutes.
- The tracer gas consumption for continuous measurements is reduced compared to the constant concentration method.

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