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An Integral Mass Balance Formulation of the Constant Concentration Tracer Technique

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Synopsis

Three basic tracer gas techniques for measuring air flow rates in building systems have been developed over the past several decades – the *decay*, *constant injection*, and *constant concentration* techniques. These techniques were originally formulated using differential mass balance equations, or solutions to these equations, that describe the dispersal of tracer in building air flow systems. In recent years alternate formulations of the decay and constant injection techniques based on integral mass balance equations have been considered [1, 2]. These integral formulations have led to new variants of these traditional techniques and have provided means to improve the accuracy of these methods.

This paper extends the integral mass balance approach to the remaining constant concentration technique. An integral formulation of the constant concentration problem is presented that accounts for the possibility of variation of tracer concentration. This approach leads, in principle, to data reduction strategies that may be expected to improve the accuracy of the constant concentration technique and that may be used to isolate those portions of a given constant concentration data set that are likely to be most reliable. The method is applied to the reduction of constant concentration data sets measured at the National Swedish Institute for Building Research and the results of this application are reviewed.

List of Symbols

C_i concentration in zone i expressed in terms of mass fraction (mass-tracer/mass-air)
C_t target concentration
δC_i the variation of concentration about the target concentration in zone i
$\Delta C_i = C_i(t_2) - C_i(t_1)$ the change of concentration from time t_1 to t_2 in zone i
G_i the mass rate of release of tracer in zone i (mass-tracer/time)
M_i the mass of air within zone i (mass-air)
$[M] = \text{diag}\{M_1, M_2, \ldots\}$ the system capacitance matrix
t time
$\Delta t \equiv t_2 - t_1$ time interval from t_1 to t_2
Tol acceptance tolerance
W_{oi} total outdoor air flow rate into zone i, the diagonal elements of [W] (mass-air/time)
[W] the system transport matrix assembled from unknown system air flows
{ } vector quantities
[ ] matrix quantities

1. Introduction

The movement of air into, out of, and through out building systems determines, to a great extent, the quality of air indoors and the energy requirements needed to condition this air to realize thermal comfort. Ironically, in spite of the importance of air movement in buildings, building designers, operators, and occupants seldom have detailed knowledge of the nature of air flow in building systems and have no practical means to measure these air flows directly. As a consequence, the design and control of building air flow systems leaves much to be desired. To address this problem two indirect approaches to determine air flows in building systems have evolved over the past four decades – building *pressurization* *techniques* and *tracer gas techniques* – following the seminal work of Dick [3]. Building pressurization tests are devised to measure building leakage characteristics that may be used to estimate fresh air infiltration [4, 5].

Tracer gas techniques attempt to deduce the building air flows that disperse one or more tracer gases released within a building system by measuring the concentration variations of tracer and attempting to solve associated equations that describe the conservation of tracer mass. In principle, tracer techniques are relatively straightforward but difficulties arise because the requisite mass conservation relations may only be formulated for relatively idealized circumstances (e.g., steady air flow and perfectly-mixed conditions in hypothetical building zones) and often lead to mathematically *ill-conditioned* problems that are especially sensitive to measurement error.

Tracer gas techniques may be classified by a) the tracer injection strategy, b) the data measurement method, and c) mass conservation formulation used. Three tracer injection strategies are commonly used a) an initial injection to establish an initial tracer concentration for the *decay technique*, b) a constant injection of tracer that is the basis of the *constant injection techniques*, and c) an injection of tracer controlled to maintain constant concentrations of tracers within the building system that is the basis of the *constant concentration techniques.* When applied to buildings that may reasonably be idealized as multiple, well-mixed zones, the decay and constant injection techniques have the potential to determine infiltration, exfiltration, and zone-to-zone air flow rates, but the associated mass conservation equations tend to be ill-conditioned.

The constant concentration technique, on the other hand, can only determine fresh air infiltration into each of the idealized building zones but yields mass conservation equations that are well-conditioned and, as a result, provides the most accurate determination of these air flows [6, 7]. Given this potential, researchers in the field have focused on development of control strategies and instrumentation needed to maintain the constant concentration conditions that are the basis of the method and yield well-conditioned problems [6, 8, 9]. This paper presents an alternative, and complementary, strategy to realize the potential of the constant concentration method.

2. Integral Formulation of the Constant Concentration Equations

The constant concentration method provides a means to determine the total flow rate of outside air into each zone of buildings that may be idealized as well-mixed, multi-zone systems. This is achieved by injecting a tracer gas into each zone in a carefully controlled manner with the objective of maintaining a constant concentration, the so-called target concentration C_t, throughout the entire building system. If this objective is achieved then the zonal concentrations may be expressed as:

$$\{C\} = C_t\{1, 1, \ldots 1\}^T \equiv C_t\{1\} \tag{1}$$

and the time rate of change of these concentrations will vanish:

$$\frac{d\{C\}}{dt} = 0 \tag{2}$$

where each element of the system *concentration vector* **{C}** corresponds to the concentration within each zone. (Vector quantities will be identified by both bold fonts and braces, { }, and matrix quantities by bold fonts and square brackets, [ ].)

At these constant conditions, then, the instantaneous mass balance equations for the tracer assume the particularly simple form:

$$[\mathbf{W}]\, C_t \{1\} = \{\mathbf{G}\} \tag{3a}$$

where **[W]** is the system *transport matrix*, a square matrix containing terms *assembled* from the unknown system air flows [10], and $\{\mathbf{G}\} = \{G_1, G_2, \ldots\}$ with G_i the mass rate of release of tracer in zone i. As C_t is a scalar this equation may be rewritten as:

$$[\mathbf{W}]\{1\} = \frac{1}{C_t}\{\mathbf{G}\} \tag{3b}$$

The quantity on the left hand side of this equation is a vector of the row sums of the system mass transport matrix which is simply equal to the total outdoor air flow rate into each of the zones (i.e., if the tracer is an passive contaminant, outdoor tracer concentrations are negligible, and each zone is well-mixed [10]). Designating the total outdoor air flow rate into zone i by W_{oi} we may rewrite Equation 3b as:

$$W_{oi} = \frac{G_i}{C_t} \tag{3c}$$

That is to say, by employing the constant concentration strategy the coupled system of mass conservation equations is transformed into a system of simple scalar equations – equations that are inherently well-conditioned.

In the practical application of the constant concentration technique zone concentrations are controlled with periodic injections of tracer so that the time variation of G_i typically varies discontinuously between zero and relatively large pulses. It is not, therefore, reasonable to apply this equation directly using instantaneous values of G_i, instead a mean value over a reasonable averaging time period is used:

$$W_{oi} \approx \frac{\int_{t_1}^{t_2} G_i\, dt \Big/ \Delta t}{C_t} \quad ; \quad \Delta t \equiv t_2 - t_1 \tag{3d}$$

This simple result is the basis of the constant concentration technique. Although this technique is particularly simple in concept it is somewhat difficult to apply due to the instrumental control problems one encounters in attempting to maintain constant concentrations within the building system. Nevertheless, the technique has proven to be reliable and accurate [8, 11], especially when each of the individual zones is well-mixed and when the requisite constant concentration conditions are, in fact, maintained, and provides the only means available, at this time, to make nearly instantaneous determinations of these crucial fresh air flow rates.

It is useful to reconsider the constant concentration technique using an integral, rather than instantaneous, formulation of the tracer mass balance relation. To this end we shall assume that tracer concentrations within each zone of the multi-zone system vary by an amount, $\{\delta\mathbf{C}(t)\}$, about the target concentration, or:

$$\{\mathbf{C}(t)\} = C_t\{1\} + \{\delta\mathbf{C}(t)\} \tag{4}$$

Again, for simplicity, we assume negligible tracer concentrations out-of-doors and substitute this expression for the controlled zonal concentrations into the governing mass balance relation, using, now, an integral form:

$$\int_{t_1}^{t_2} [\mathbf{W}]\{C_t\{1\} + \{\delta\mathbf{C}(t)\}\}\, dt + [\mathbf{M}]\{\Delta\mathbf{C}\} = \int_{t_1}^{t_2} \{\mathbf{G}\}\, dt \tag{5}$$

where $<t_1, t_2>$ is an arbitrary time interval, $\{\Delta\mathbf{C}\} = \{\mathbf{C}(t_2)\} - \{\mathbf{C}(t_1)\}$, and $[\mathbf{M}] = \text{diag}\{M_1, M_2, \ldots\}$, with M_i the mass of air in zone i.

Although the time interval, $<t_1, t_2>$, is, in principle, arbitrary we prefer to select it so that it is small enough to assure that during this interval the system air flows and, hence, $[\mathbf{W}]$ remain practically constant allowing Equation 5 to be rewritten as;

$$[\mathbf{W}]\int_{t_1}^{t_2} \{C_t\{1\} + \{\delta\mathbf{C}(t)\}\}\, dt + [\mathbf{M}]\{\Delta\mathbf{C}\} = \int_{t_1}^{t_2} \{\mathbf{G}\}\, dt \tag{6}$$

If, then, the time interval is chosen so that:

$$\int_{t_1}^{t_2} \{\delta\mathbf{C}(t)\}\, dt = \{0\} \tag{7}$$

then Equation 6 simplifies to yield:

$$[\mathbf{W}]\{1\} = \frac{\int_{t_1}^{t_2} \{\mathbf{G}\}\, dt - [\mathbf{M}]\{\Delta\mathbf{C}\}}{C_t(t_2 - t_1)} \tag{8a}$$

Recognizing $[\mathbf{M}]$ is a diagonal matrix we obtain the final result:

$$W_{oi} = \frac{\int_{t_1}^{t_2} G_i\, dt - M_i\,(C_i(t_2) - C_i(t_1))}{C_t\,(t_2 - t_1)} \tag{8b}$$

(Compare to Equation 3c.)

The practical application of the constant concentration technique involves the periodic measuring of zonal tracer concentrations followed by a burst injection of tracer, when necessary, to maintain the desired target concentration. The time period between concentration measurement and burst injections is typically on the order of one or two minutes although shorter sampling times may be possible [8]. During a sampling time interval the integral $\int G_i dt$ is simply equal to the amount of tracer released to zone i.

To account for the variation of tracer concentration about the target concentration one could, then, monitor both the zone concentrations and the integral of their variation about the target value during the test. If during a given time period of, say, five or more sampling intervals the integral of the variation is observed to be negligibly small then one may apply Equation 8b directly to the data to obtain an estimate of the total outdoor air flow into each zone, W_{oi}, whether the zonal concentrations remain on target or not as the MΔC term provides an appropriate correction. If the integral of the variation is not observed to be negligibly small then one may either note this inadequacy in the data and make no attempt to determine air flows during the time period or, perhaps, search the time period for an interval when the integral of variation is, in fact, negligibly small, and compute air flows using Equation 8b. When tracer concentrations are well-controlled the integral of the variation and the MΔC correction will be negligibly small, as a result, Equation 8b will simplify to the conventional form, Equation 3c.

What criteria may be used to determine if the integral of the variation is negligibly small? Clearly, if the integral of the zone concentrations, over the time interval chosen, is well-approximated by the integral of the target concentration:

$$\int_{t_1}^{t_2} \{C_t\{1\} + \{\delta \mathbf{C}(t)\}\}\, dt \approx \int_{t_1}^{t_2} C_t\{1\}\, dt = C_t \Delta t\{1\} \tag{9a}$$

the objective will be met and Equation 6 will simplify to the desired form. Rearranging Equation 9 we obtain a more convenient relative form:

$$\frac{\int_{t_1}^{t_2} \{\delta \mathbf{C}(t)\}\, dt}{C_t \Delta t} \approx \{0\} \tag{9b}$$

For computational purposes it is proposed that an acceptance criteria or *tolerance* be based on an maximum norm of the absolute value of each term of this criteria:

$$\mathrm{Tol} \equiv \left\| \mathrm{Abs}\left\{ \int_{t_1}^{t_2} \{\delta \mathbf{C}(t)\}\, dt \Big/ C_t \Delta t \right\} \right\|_{max} \tag{10}$$

3. Application

Sandberg and Blomqvist conducted a number of tests, using an indoor test house contained within their laboratory at the National Swedish Institute for Building Research, to investigate the ability of the constant concentration technique to follow sudden changes in air flow and to identify optimal control algorithms for the constant concentration test equipment used [6]. Total fresh air flow into the test house was mechanically controlled to vary in a step-wise manner while tracers were injected into each of five different rooms in an attempt to maintain a target concentration in all rooms. A total of eight different control algorithms were considered, identified as ALGO1 through ALGO8. We shall consider result obtained for a poorly-controlled case, ALGO5, and a well-controlled case, ALGO6. The results obtained using the six other cases were similar.

All tests were conducted in a similar manner. Tracer concentrations were measured in each of the five rooms at 15 second intervals, pulse injections of tracer were then applied to each room in an attempt to control concentrations at the target value of 50 ppm and, after a 60 second delay, the procedure was repeated. As a result, concentrations were measured in each of the five rooms on a 120 second interval. The measured concentration histories for two representative rooms, identified as room 1 and 5, for the cases discussed here are plotted below, Figures 1 and 2. The relative success of control algorithm ALGO6 is evident from these results.

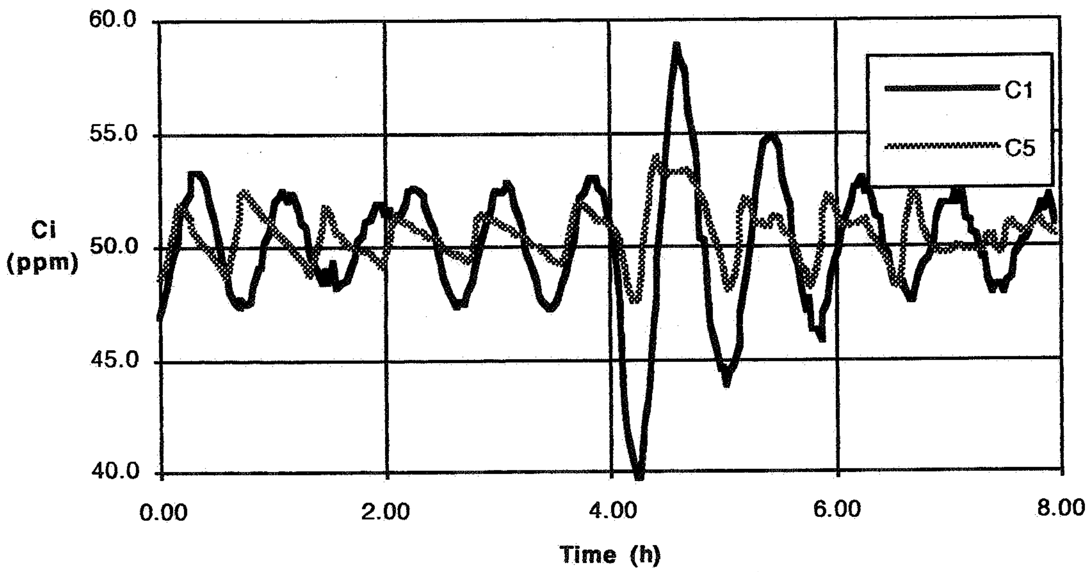

Figure 1 Representative Concentration Time Histories for Test ALGO5

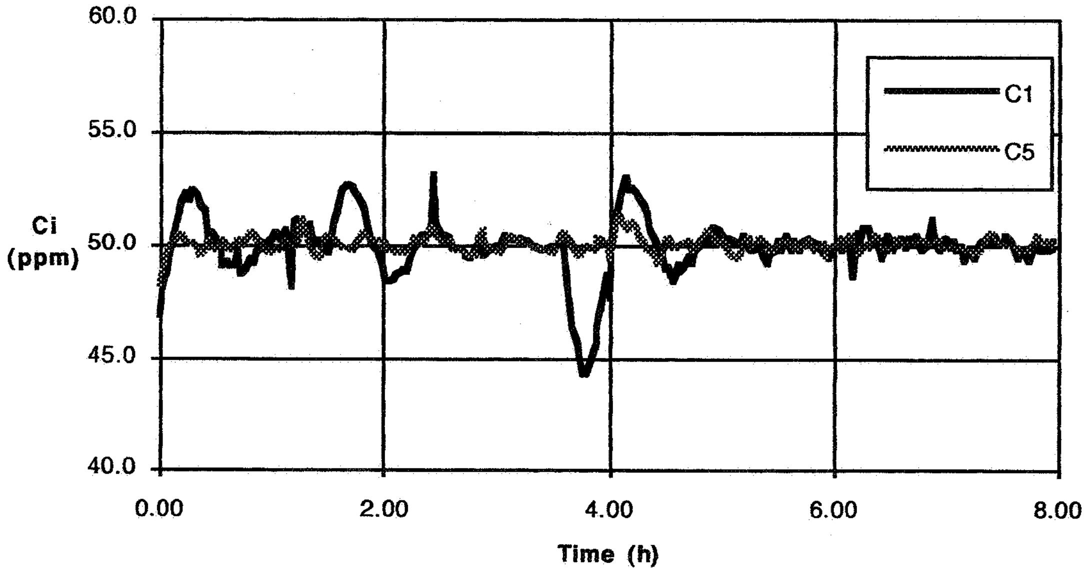

Figure 2 Representative Concentration Time Histories for Test ALGO6

The integral form of the constant concentration theory, Equation 8b, was applied to this, and all other, data using an integration time interval of 30 minutes (i.e., numerically integrating the discrete measured concentration data using 15 data values for each room). This resulted in a *30-minute moving estimate* of fresh air flow into each room (i.e., W_{oi}, i = 1, 2, ... 5) at 2 minute intervals. Summing these results, an estimate of the total fresh air flow into the house was determined and, using the target concentration of 50 ppm, the acceptance criteria was computed, Equation 10, at each of the 2 minute intervals. The results are compared below to the mechanically controlled total fresh air flow for two acceptance tolerances, 1% Tol and 2% Tol, Figures 3 and 4. The total fresh air flow reported by Sandberg and Blomqvist is also plotted using + markers labeled as ALGO5 and ALGO6, respectively.

Regrettably, while tracer was injected at 120 second intervals in controlled pulses, the amount of tracer injected was not recorded. The recorded data provided only values for the integral of the total tracer injected in each of several consecutive 30 minute intervals. Consequently, to apply Equation 8b the integral of amount of tracer injected:

$$\int_{t_1}^{t_2} G_i \, dt$$

was estimated by linear interpolation between these 30 minute values.

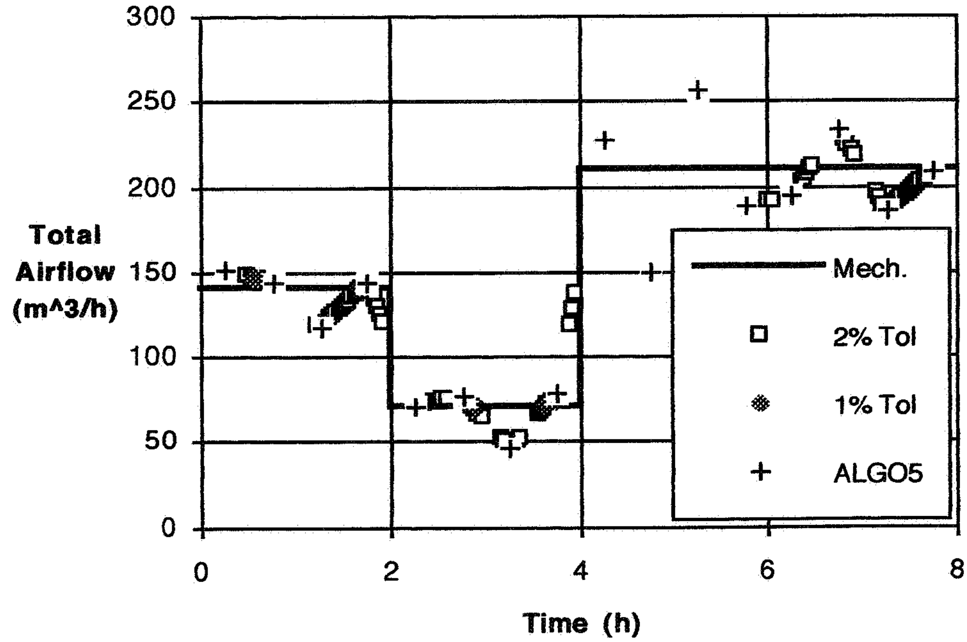

Figure 3 Comparison of Air Flow Estimates with Mechanically Controlled Values for ALGO5

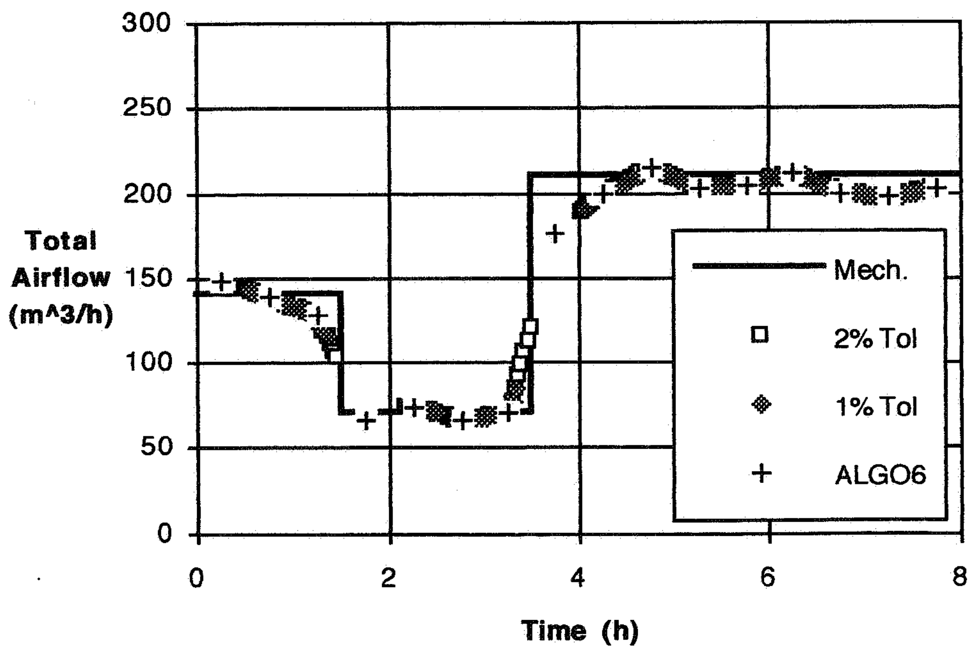

Figure 4 Comparison of Air Flow Estimates with Mechanically Controlled Values for ALGO6

Figures 3 and 4 may be somewhat difficult to read at first. The solid line presents the variation of total airflow as controlled by the mechanical system. Each of the markers represents an estimate of the total air flow during the 30 minute time period centered on the marker. The + markers indicate the estimates based upon the conventional constant concentration approach, Equation 3d, as reported by Sandberg and Blomqvist. The □ markers indicate the estimates based upon the integral approach, Equation 8b, having an acceptance tolerance, Equation 10, less than or equal to 2% and the ◈ markers an acceptance tolerance of less than or equal to 1%.

As expected from the theory, the smaller acceptance tolerance of 1% yields better estimates of total airflow but also results in the rejection of much of the data. In the case with poor control of the zone concentrations, Figure 3, nearly all of the data is rejected when the 1% criteria is enforced. For the well-controlled case, on the other hand, most of the data passes the 1% tolerance test. The results obtained for the six other tests not reported here are similar – a 1% tolerance consistently results in accurate estimates of total air flow and, for these test involving sudden changes of airflow, a rejection of much of the recorded data as unacceptable – although the results from two test, ALGO7 and ALGO8, revealed consistent underestimations of air flows for both conventional and integral approaches indicating a systematic source of error. Accepting integral constant concentration results passing the 2% tolerance test generally provides better estimates of airflow than those reported by Sandberg and Blomqvist and results in the rejection of some of the data as unacceptable (e.g., those + values between the 4th and 6th hours of Figure 3) but overall the success is not as consistent as provided by the 1% acceptance tolerance.

4. Conclusion

An integral formulation of the theory underlying the constant concentration tracer technique has been presented that leads to data reduction strategies that appear to improve the accuracy of the technique and provides the means to isolate those portions of a given constant concentration data set that are likely to be most reliable. When applied to eight data sets provided by the National Swedish Institute for Building Research (NSIBR) the proposed data reduction method lead to consistently better accuracy than that provided by the conventional approach when a data acceptance criteria demanding the integral of variation of zone concentrations remain within a 1% tolerance of the corresponding integral of the target concentration was enforced. While a 2% tolerance generally provided better accuracy than the conventional approach tolerances larger than 2% offer no significant advantage. Finally, it should be noted that the proposed approach offers no remedy for systematic errors that, apparently, resulted in consistent under-predictions of air flows for two of the data sets analyzed.

The application of the proposed data reduction strategy to the NSIBR data sets was compromised by insufficient detail in the record of tracer injection time histories. Presumably, a more complete record of tracer injection – easily obtained using available instrumentation – would have improved results.

The proposed approach involves numerically evaluating integrals of the variation of zone concentrations about the target concentration and integrals of the tracer injection time history. The integration time interval is, in principle, arbitrary although, from a practical point of view, an interval small enough to assure that system air flows remain relatively constant and yet large enough to provide sufficient measured data to realize an accurate numerical integration should be used. A time interval of 30 minutes was used in the present study to yield results that could be compared to those results reported by the investigators at

NSIBR but in other circumstances one may consider searching the data for time periods that result in satisfaction of the acceptance criteria. Following a similar argument, the target concentration may also be considered to be arbitrary and one may search the data set, using a variety of candidate target concentrations, to find combinations of target concentrations and integration time intervals that result in satisfaction of the acceptance criteria.

A formal error analysis of the proposed approach was not considered in the present study. It is believed that such an analysis would be relatively straightforward and could establish the quantitative link between the accuracy of computed fresh air flows and the acceptance tolerance imposed.

Acknowledgement

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