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A COMPARISON OF DIFFERENT METHODS OF CALCULATING INTERZONAL AIRFLOWS BY MULTIPLE TRACER GAS DECAY TESTS

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Measurement methods based upon multiple tracer gas techniques have become an established branch of the study of air infiltration and interzonal air movements. Three general groups of techniques have emerged, namely constant concentration, constant emission, and decay.

Of the decay type group of techniques, several methods of deriving airflows from measured concentration/time curves have been suggested. Broadly speaking, these techniques can be classified into three types: numerical methods involving the use of concentration gradients; numerical methods involving the use of integration of concentration/time data; and thirdly, techniques based upon analytical solutions for the fundamental tracer gas equations. The favoured method of analysis at UMIST has been that of a simplified analytical solution in which the effects of tracer gas re-circulation are only taken into account if the degree of connection between zones if high. This method analysis has been successfully validated for the cases of two and three interconnected cells under controlled conditions in environmental chambers. However, up until now, no direct comparison with the results generated by other methods using the same raw concentration/time data has been made.

This paper describes an exercise in which site data for two and three zone regimes is analyzed by several different methods, and the results obtained by each method compared. It is demonstrated that, in particular, concentration gradient methods appear to be particularly ill-suited to dealing with site data which exhibits irregularities in concentration-time profiles caused by fluctuations in windspeed and wind direction. Integration techniques only appear to be marginally better.

(1) INTRODUCTION.

Indoor air quality and energy consumption in buildings are affected by air infiltration and exfiltration through the building envelope. The air movements between rooms thus induced will have a significant influence on the performance of the building in terms of occupant satisfaction and running costs. A means of accurately determining these airflows is therefore of great value.

The inherently complex nature of inter-cell airflows and their susceptibility to changes in environmental parameters such as windspeed, wind direction, and internal/external temperature difference makes their prediction by numerical techniques very difficult: however, several noteworthy efforts have been made to predict inter-cell air movements using network analysis.

For existing buildings, inter-cell air movements can be determined using tracer gas measurement techniques. Several comprehensive reviews of existing tracer gas techniques have been published. (For example Perera (1), Lagus (2), Charlesworth (3)) Three distinct groups of tracer gas techniques have emerged in recent years, namely constant concentration, constant emission, and decay. This piece of work concerns itself with the analysis of tracer gas concentration/time data derived from decay techniques.

(2) METHODS OF DATA ANALYSIS.

(a) The fundamental tracer gas equations. At this juncture, it would be useful to summarise the fundamental equations describing the variation of tracer gas concentrations with time in a multi-cell system. Consider the N-cell model in figure 1. It is assumed that:

(i) the system is composed of N cells in which air and tracer gas are perfectly mixed at the start of,and at all times during, the tracer gas decay test;

(ii) Q_{ij} and Q_{ji} are the volumetric airflow rates between cells i and j; (note that it is not neccessarily the case that $Q_{ij}=Q_{ji}$)

(iii) the volume of the i^{th} cell is denoted by V_i, and is given in cubic metres;

(iv) $C_{i(t)}$ is the time variation in the concentration of tracer gas in the i^{th} cell, and is given in parts per million;

(v) $f_{i(t)}$ represents the rate of production of tracer gas in cell i, and is given cubic metres per hour;

(vi) V_0 denotes the volume surrounding the structure, and is taken to be infinite-this implies that $C_{0(t)}$ is zero throughout the tracer gas decay test.

Consideration of the conservation of mass of tracer gas give

$$V_i dC_i/dt=f_i+[\sum_{j=1}^{N}Q_{ji}C_j(1-\delta_{ij})]-[Q_{i0}C_i+\{\sum_{j=1}^{N}Q_{ij}(1-\delta_{ij})C_i\}] \quad (1)$$

for i = 1, 2, ... N and j = 1, 2 ...... N

The Kronecker delta function (δ_{ij}) is defined as

$\delta_{ij} = 0$ when i<>j

or $\delta_{ij} = 1$ when i = j

A second set of equations is derived by consideration of the mass of air within the multi-cell system:

$$Q_{0i}+\sum_{j=1}^{N}Q_{ji}(1-\delta_{ij})=Q_{i0}+\sum_{j=1}^{N}Q_{ij}(1-\delta_{ij}) \quad (2)$$

If we define S_i as the outflow of air from cell i to the outside, by substitution for S_i in equation (2) we obtain

$$S_i = Q_{i0} + \sum_{j=1}^{N}Q_{ij}\ (1-\delta_{ij}) \quad (3)$$

There are (N^2-N) unknown values of inter-cell airflows Q_{ij} and Q_{ji}, plus 2N unknown values of Q_{io} and Q_{oi}. We therefore have (N^2+N) unknown values of airflows, and only N equations from equation (1) plus N equations from equation (3) from which to solve for them. Using the N equations of the form of equation (3), N^2 unknown airflows are left, with only N equations of the form of equation (1) remaining from which to solve for them. This means that (N-1) independent sets similar to equation (1) have to be generated in order to be able to solve for all airflows.

(b) Methods of solving the problem.

There are several methods of generating the required (N-1) independent sets of equations. Three methods will be considered:

numerical differentiation, numerical integration, and the derivation of analytical solutions.

(i) Numerical Differentiation

The method of numerical differentiation, adapted to the analysis of equations (1) and (2), can be applied to any number of interconnected cells, the limiting factor being the number of suitable tracers available. The corresponding sets of equations derived from equation (1) will get considerably larger as the number of interconnected cells under consideration increase, and the use of a computer becomes essential for data manipulation.

Equation (1) is solved using the matrix method suggested by Sinden (4). If a single pulse of tracer gas is injected into each cell in the multi-cell system,then it can seen that $f_i=0$: substituting for equation (3) into equation (1) and expressing in matrix form,

$$[Y] = [A].\ [X] \qquad (4)$$

$$\text{where } [Y] = \begin{bmatrix} V_i dC_{Ai}/dt \\ V_i dC_{Bi}/dt \\ \cdots \\ \downarrow \end{bmatrix} \qquad \text{for } i = 1 \text{ to } N$$

$$[A] = \begin{bmatrix} C_{Aj}\ (1 - \delta_{ij}),\ C_{Ai}\ \ldots\ldots\ldots \\ C_{Bj}\ (1 - \delta_{ij}),\ C_{Bi}\ \ldots\ldots\ldots \end{bmatrix}$$

for j = 1 to N

$$(\delta_{ij} = 0,\ i <> j)$$

$$(\delta_{ij} = 1,\ i = j)$$

$$[X] = \begin{bmatrix} Q_{ji} \\ -S_i \\ Q_{ji} \\ -S_j \end{bmatrix}$$

Airflow vector [X] is found by calculating the inverse tracer gas concentration matrix $[A]^{-1}$

Equation (4) becomes:

$$[A]^{-1}.\ [Y] = [X] \qquad (5)$$

When measurements of tracer gas concentration/time histories are made the quantities C_{Ai}, C_{Bi}, C_{Aj}, C_{Bj}, at time (t) are known. Cell volumes V_i can be attained by site inspection. The remaining unknown concentration gradients dC_i/dt, dC_{Bi}/dt are estimated at a specific time (t).

(ii) <u>Numerical Integration</u>

The fundamental tracer gas equation (1) can be rewritten in the form:

$$V_i\ dC_i/dt = fi + \sum_{j=0}^{N} Q_{ji}\ (C_j - C_i) \qquad (6)$$

A detailed derivation of this equation is given by Penman (5). Integrating equation (6) between a time step (t_1) - (t_2) we obtain:

$$V_i\ [C_i\ (t_2) - C_i\ (t_1)] = (\sum_{j=0}^{N} \int_{t_1}^{t_2} (C_j - C_i)\ dt)\ Q_{ji} \qquad (7)$$

$f_i = 0$ when a single pulse of tracer is injected into each cell.

The integrals in equation (7) are evaluated using numerical integration of time variations in tracer gas concentration, obtained from site data for the period (t_i) - (t_2).

The tracer concentration curves can be divided into several time periods (t_j = 1, 2, ... k), the unknown airflows are found using a least squares approximation for $k \geq N + 1$ time periods. Where multiple tracer gas measurements are used, separate estimates of airflows are possible for each tracer gas.

(iii) <u>Analytical Methods</u>

Several attempts have been made to derive an analytical solution of the fundamental tracer gas equations. (See for example Sinden (4) and I'Anson.(6)) Problems occur because of the unknown time variations of tracer concentration $C_{(t)}$ in the connected cells under consideration.

However, from the work of Dick (7) a simplified analytical solution is available which enables estimates of intercell airflow to be made.

Equation (1) can be rewritten in the form:

$$dC_i/dt = \frac{1}{V_i} \sum_{j=1}^{N} Q_{ji}\ (1 - \delta_{ij}) - \frac{S_i\ C_i}{V_i} \qquad (8)$$

By use of integrating factors, this first order differential equation can be solved for unknown airflows Q_{ji}, S_i, and becomes:

$$C_i\ e\left[\frac{S_i}{V_i}\ t\right] = \sum_{j=1}^{N} \frac{Q_{ji}}{V_i} \int_{0}^{t} C_j\ (1 - \delta_{ij})\ e\left[\frac{S_i}{V_i}\ t\right] dt \qquad (9)$$

A fully detailed account of the equations resulting from the solution of equation (9) is given by Irwin. (8)

(3) <u>Experimental Details</u>

The test house used is a two storey terraced property of low energy design. (Figure 2) Both two and three cell measurements were carried out in this house. For the two cell measurement, upstairs and downstairs were taken as the two cells, whilst for the three cell measurement, the kitchen was taken as a cell in its own right.

Tracer gas concentration measurements were made using the rapid response multiple tracer gas system developed at UMIST: this system is well documented (9, 10) and will not be described in detail here.

(4) <u>Results and Discussion</u>

Table 1 summarises the variations in estimated airflows for a typical two cell case, whilst table 2 shows variations in estimated airflows for a three cell case, using the three methods of data analysis previously discussed. Figure 3 shows the "goodness of fit" with a set of concentration/time data for the two cell case. As can be seen the predicted time variation (equation (1)), using the airflows estimated by numerical differentiation has a poor correlation with the measured data. This is hardly surprising as the method of analysis is reliant upon the estimate of concentration gradients, from two distinct tracer gas concentration/time data points. The uncertainties in the data and their effect on airflow estimates can be clearly seen in figure 3.

There are similar problems with the airflow rates estimated using numerical integration techniques. The "goodness of fit" between measured data and predicted concentration time histories (shown in figure 3) are little better than for the numerical differentiation case. The reason for this lies in the tracer gas concentration difference term shown on the left hand side of equation (7) ie.

$$(C_i\ (t_2) - C_i\ (t_1)) \qquad (10)$$

The method of analysis is reliant upon pairs of site data points in constructing the linear equations to solve for the unknown airflows.

The third method using a simplified analytical solution of the fundamental tracer equations does reflect a better comparison between predicted and measured tracer concentration/time histories. This occurs because the analytical method uses all data collected rather than discrete pairs of data points. As a word of caution, the analysis and consequently the estimates of airflows from the three methods discussed in this paper are all vulnerable to extraneous variables.

(5) Conclusions

Comparison of the three methods of analysis for concentration/time data shows that, of the three methods, the simplified analytical solution as described by Irwin et al (9, 10) gives the closest fit to site measurements of concentration variations with time. It should be noted that the sets of data presented have been deliberately selected so that the effects of fluctuations in extraneous variables are minimal. As these fluctuations become more pronounced, the differentiation and integration techniques become even more inadequate, whilst the simplified analytical solution can still be used satisfactorily. This whole issue will be more fully discussed in reference (11).

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TABLE 1

Variations in Measured Airflows (2 Cell Case)

	Airflows (m^3/hr)			
	Q_{12}	Q_{21}	S_1	S_2
Numerical Differentiation	113	203	352	138
Numerical Integration	65	157	282	107
Simplified Analytical Solution	190	215	263	254

TABLE 2

(Three Cell Case)

	Q_{12}	Q_{13}	Q_{21}	Q_{31}	Q_{23}	Q_{32}	S_1	S_2	S_3
Numerical Differentiation	122	18	65	88	105	129	258	288	270
Numerical Integration	89	88	188	55	98	111	299	212	241
Simplified Analytical Solution	106	75	113	77	110	82	325	220	234

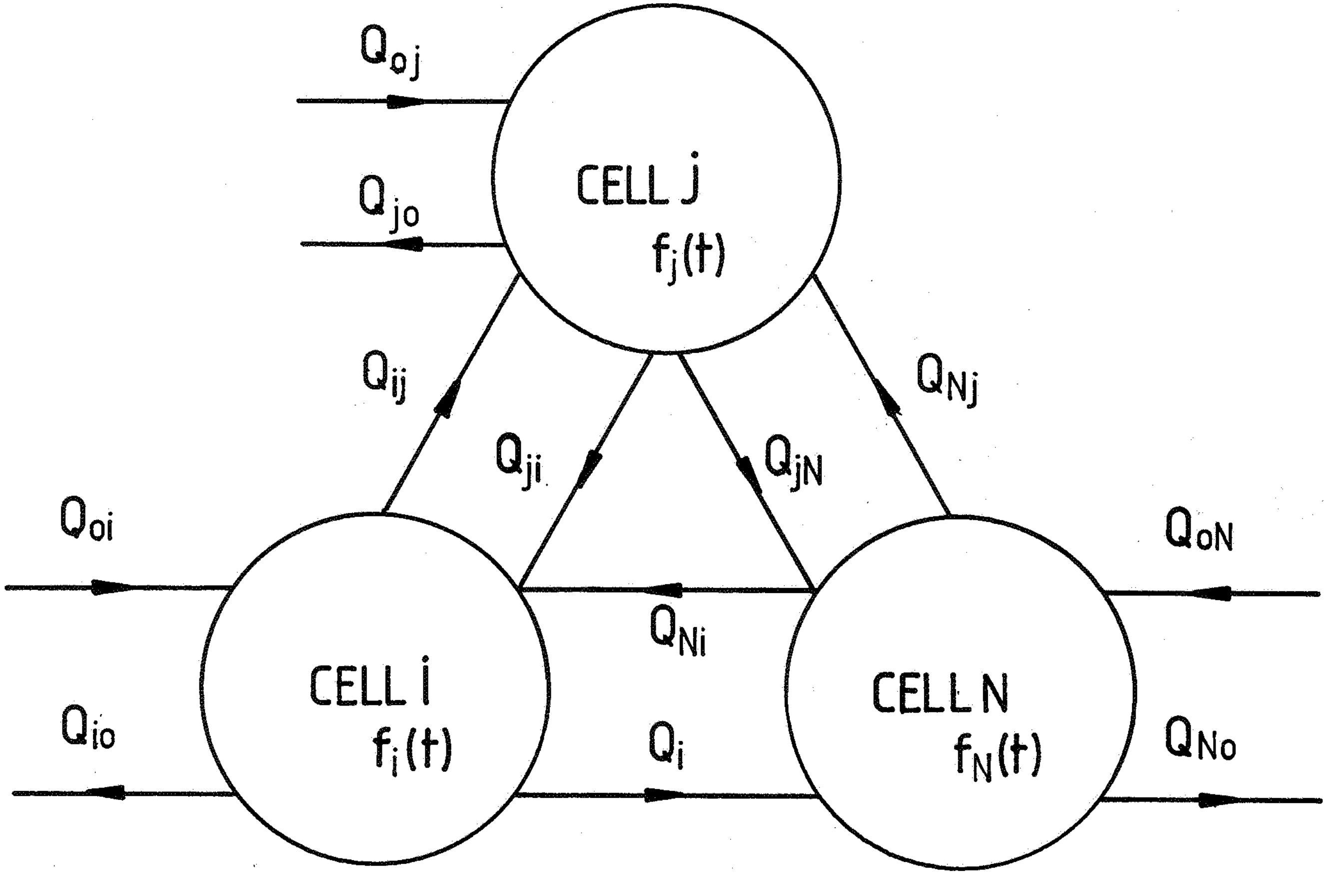

FIGURE 1: Test cell system

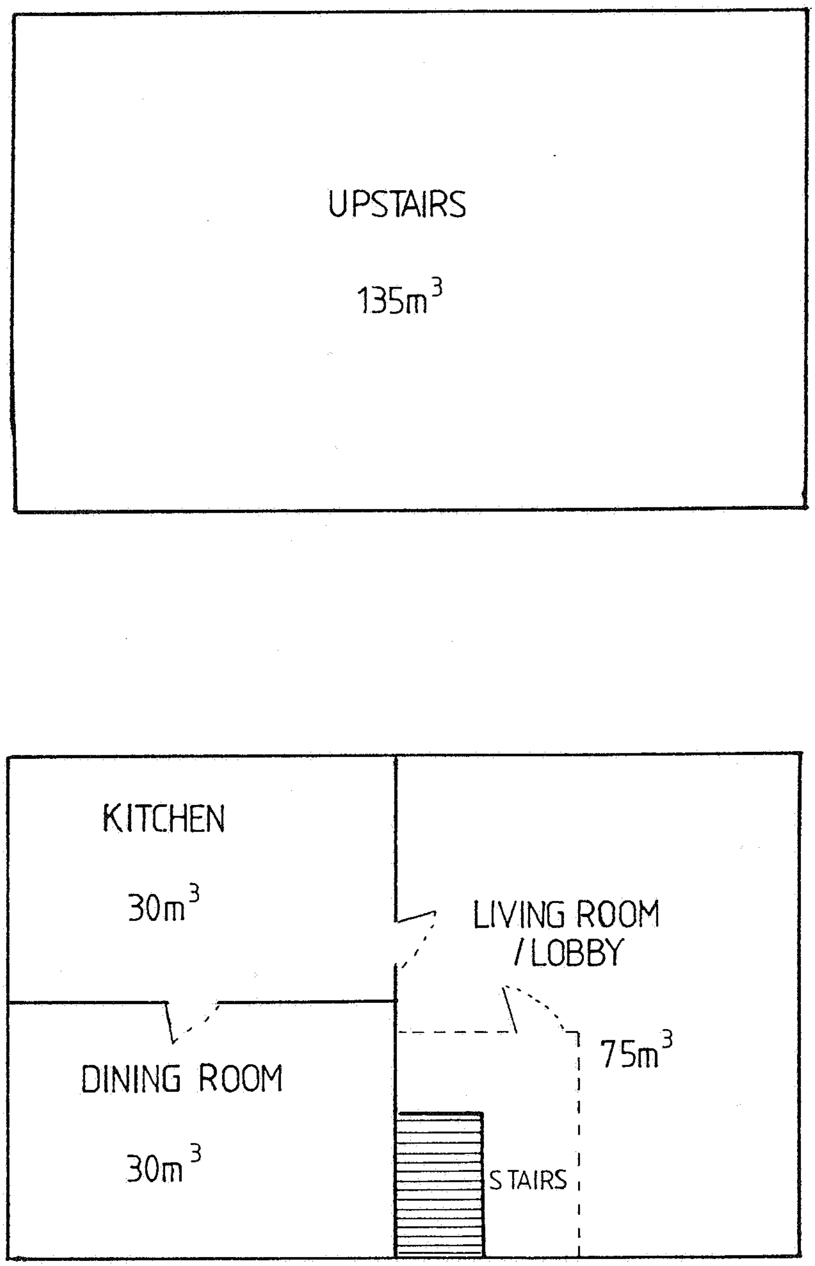

FIGURE2: Plan of test house

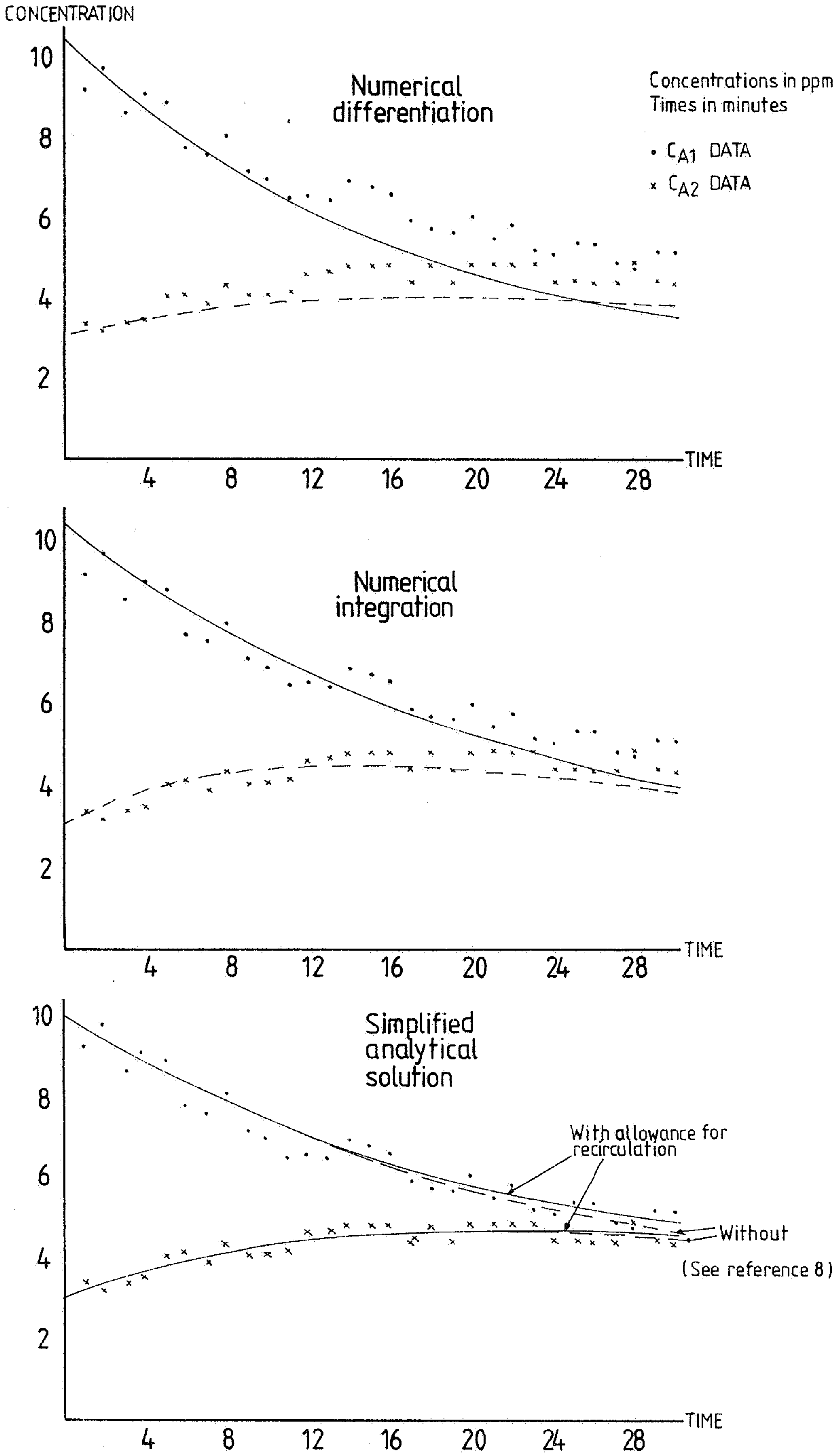

FIGURE 3: GOODNESS OF FIT COMPARISON