Short Communication

THE DESIGN OF SPIRES FOR WIND SIMULATION

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1. Introduction

In this note simple formulae are given for the design of spires for use in simulating the planetary boundary layer. The details of the derivation are omitted in the interests of clarity and conciseness but may be found in full in Ref. 1, including some refinements to account for the presence of a ramp, or blockage caused by a model, and to include the effect of corner fillets. The use of spire arrays combined with floor roughness (Fig. 1) began in the late 1960's [2, 3] when it was discovered that they gave velocity profiles of

Fig. 1. Spires and roughness in a rectangular working-section.

tthe right form and also produced large-scale turbulence with an intensity that matched planetary boundary-layer data. The technique is now widely employed and several different shapes of spire are in use. However, a simple design formula does not appear to exist in the literature. The author's experience has been with spires consisting of a tapered flat plate, normal to the flow, with a splitter plate on the downwind side. This is the kind considered in the present note, although some of the results in Ref. 1 are applicable to spires in general.

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Figure 2 shows a spire with a triangular front face and a triangular splitter plate on the downwind side. Non-triangular front faces have not been found to have any obvious advantage over the simple triangle in the author's experience [1]. The mean velocity profile and turbulence properties downstream of the spire array are largely insensitive to the details of the spire shape provided it is approximately triangular and the correct overall drag is maintained. In addition to the spire drag, the floor friction due to distributed roughness also plays a significant role in generating the desired boundary-layer characteristics, particularly near to the floor.

![Fig. 2. Triangular spire with splitter plate.](image)

2. Design formulae

In Ref. 1, the momentum balance of a rectangular working section is analyzed assuming that uniform flow exists upstream of the spire array and that at some point downwind of the spires a boundary layer is formed with a power-law velocity profile and with thickness $\delta$. The power law is defined as

$$\frac{U}{U_\delta} = (z/\delta)^\alpha$$

where $U$ is the velocity at height $z$, $U_\delta$ the velocity at $z \geq \delta$, and $\alpha$ is the power-law exponent. The deficit of momentum flux in the boundary layer and the pressure drop along the working section are balanced against the spire drag, including blockage effects, and the drag of the floor roughness. The result is an expression for the total frontal area of the spire array needed to produce
a boundary layer with the required values of $\alpha$ and $\delta$. With additional empirical information, such as the spire drag-coefficient and the relation between spire height and boundary-layer depth, expressions are then derived for the height $h$ and base-length $b$ of triangular spires that will produce the required boundary layer at a distance $\delta h$ downstream of the spire array. The distance $\delta h$ has been found [1, 2] sufficient to ensure lateral uniformity of the flow when the spires are laterally spaced with their centre-lines at intervals of $h/2$.

Having chosen the required values of $\delta$ and $\alpha$, the first step is to calculate the spire height using the empirical relation [1]

$$h = 1.39 \frac{\delta}{(1 + \alpha/2)} \quad (1)$$

The base-to-height ratio $b/h$ is obtained from the following expression, which applies when the lateral spacing is $h/2$:

$$b/h = 0.5[\psi(H/\delta)/(1 + \psi)](1 + \alpha/2) \quad (2)$$

where

$$\psi = \beta \left[ \frac{2(1 + 2\alpha)}{1 + 2\alpha} \right] + \beta - \frac{1.13\alpha}{(1 + \alpha)(1 + \alpha/2)} \right\} / (1 - \beta)^2$$

$$\beta = (\delta/H) \alpha/(1 + \alpha)$$

and $H$ is the working-section height.

Figure 3 shows a plot of $b/h$ versus $\alpha$ for various values of $\delta/H$. Strictly, for eqn. (2) to be valid, the working-section width should be an integral multiple of $h/2$, but it does not appear necessary to adhere stringently to this in practice.

Incorporated in eqn. (2) is the drag coefficient of the spire array based on frontal area. For $h/2$ spacing, drag measurements in Ref. 1 indicate that the

![Fig. 3. Relation between the spire base-to-height ratio and velocity-profile exponent.](image)
drag coefficient remains close to 1.45 for the range $0.06 < b/h < 0.2$. This includes the effect of a triangular splitter-plate (with a base-length of $h/4$), which was found to be unexpectedly small, only ~3%.

Also included in eqn. (2) is the effect of the aerodynamic drag of the floor roughness, the specification of which is discussed subsequently. The distance $6h$ downstream of the spires is insufficient for the floor roughness to make a dominant contribution to the overall momentum-deficit in the flow, but its contribution cannot be ignored completely. In these circumstances an approximate expression is sufficient and, in deriving eqn. (2), the average floor-drag per unit area in the interval from the spires to $6h$ downstream was taken to be $\frac{1}{2} \rho U^2 C_t$ where $C_t$ is the skin friction coefficient at the downstream end of the interval ($\rho$ being the air density). Furthermore, it was assumed that, with appropriate roughness, the boundary layer at $6h$ is not far from being in equilibrium, thus justifying the use of Gartshore's relation [4] between $C_t$ and $\alpha$:

$$C_t = 0.136 \left[ \frac{\alpha}{(1 + \alpha)} \right]^2$$

These assumptions lead to the term $-1.13 \alpha / [(1 + \alpha)(1 + \alpha/2)]$ appearing in the expression for the parameter $\psi$.

To specify the roughness size that will produce the required value of $C_t$, empirical correlations for the drag of roughness elements may be used such as those of Wooding et al. [5]. For example, Wooding et al.'s correlation for cube roughness results in the following expression for the ratio of cube height $k$ to boundary-layer thickness $\delta$:

$$k/\delta = \exp\left\{ (2/3) \ln(D/\delta) - 0.1161 [(2/C_t) + 2.05]^{1/2} \right\}$$

(3)

where $D$ is the spacing of the roughness elements. Equation (3) is valid in the range $30 < \delta D^2/k^3 < 2000$.

3. Discussion

Equations (1) and (2) should ideally be validated by comparing them with data obtained from a series of controlled experiments with a range of triangular-spire and roughness configurations. This has not been done. However, Table 1 gives a comparison with data from various ad hoc wind-simulations set up in the past, mostly for model studies. The theoretical values of $h/H$ and $b/h$ were obtained by substituting the experimental values of $\alpha$ and $\delta/H$ into eqns. (1) and (2). The data were not all obtained with triangular spires and the roughness was in most cases adjusted by trial and error rather than by using eqn. (3). Where the spire shape was not exactly triangular, an effective experimental value of $b$ was calculated, defined as the base length of the triangle having the same area and height as the spire. For $\alpha = 0.23$ and 0.25, the boundary-layer data were obtained at $4.5h$ rather than $6h$ from the spires. The values of $b/h$ shown in parentheses for these values of $\alpha$ were obtained by reducing the skin friction term in $\psi$ in proportion to the reduced
TABLE 1

Comparison between experimental and theoretical parameters

<table>
<thead>
<tr>
<th>Spires triangular?</th>
<th>$\alpha$</th>
<th>$s/H$</th>
<th>$h/H$</th>
<th>$b/H$</th>
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<tr>
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</tr>
<tr>
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<td>0.147</td>
<td>0.167</td>
<td>0.162</td>
</tr>
</tbody>
</table>

aData obtained at a downstream distance from the spires of 4.5$h$, rather than 6$h$ as for the other results. The values in parentheses in the final column were obtained by reducing the skin friction term in the expression for $\psi$ (see text, eqn. (2)) in proportion to the reduced distance.

Although it is not an exact comparison, Table 1 indicates that the design formulae are generally in accord with previous data.

It is of interest to estimate the contribution of the floor friction to the overall momentum deficit in the boundary layer. The effect of the floor friction is most easily seen in the simplified case where $\delta/H \ll 1$, implying the spire blockage effect is negligible. Equation (2) then reduces to

$$b/h = \frac{(1 + \alpha/2)\alpha/(1 + \alpha)(1 + 2\alpha)}{1 - 0.56[\alpha(1 + 2\alpha)/(1 + \alpha)(1 + \alpha/2)]}$$

The floor friction effect is represented by the term $0.56 \alpha(1 + 2\alpha)/[(1 + \alpha)(1 + \alpha/2)]$ in the second factor and is roughly proportional to $\alpha$ in the range of interest, $0 < \alpha < 0.5$. For $\alpha = 0.25$, the calculated reduction in the required ratio $b/h$, due to the floor friction term, is 15%. This gives a measure of the contribution of the floor roughness to the overall momentum deficit and it is seen to be significant but not dominant.

Equations (1)—(3) enable a particular boundary layer to be developed 6$h$ downwind of the spires. Further downwind, the boundary layer will grow in thickness because of the continued floor-roughness. An estimate of the increase $\Delta \delta$ in boundary-layer thickness in going from 6$h$ to 6$h + \Delta x$ downwind of the spires is given by

$$\Delta \delta = 0.068 \alpha[(1 + 2\alpha)/(1 + \alpha)] \Delta x \cdot F$$

where $F$ is a correction factor to be applied in order to take account of the pressure drop in the rectangular working-section due to the boundary-layer growth. If no attempt is made to eliminate the pressure drop by moving the tunnel roof, then

$$F = \{1 + (\delta/H)[\alpha(3 + 2\alpha)/(1 + \alpha(1 - \delta/H))]\}^{-1}$$
If the pressure drop is eliminated, which is preferable because a pressure gradient tends to change $\alpha$, then $F = 1$.

The development of the present design approach was prompted by the finding, from wind simulations developed for several model studies, that the earlier method of Campbell and Standen [2] tended to overestimate the amount of spire drag required. The reason for the overestimate is discussed in Ref. 1. Campbell and Standen's method is an adaptation of Cowdrey's method for grids [6] and is based on an energy balance rather than a momentum balance. Both the present method and Campbell and Standen's were derived without direct reference to the required turbulence properties of the boundary layer generated, apart from the knowledge that tapered spires do generate large-scale turbulence. The design methods centre around achieving the correct mean velocity profile. However, the experimental evidence, such as can be found in Refs. 1–3, shows that once the correct mean velocity profile has been achieved, the turbulence intensity and scale tend to fall into line in comparison with full-scale data when using the spire-roughness technique.

References