STUDIES AND IMPROVEMENTS
TO AN
AIR INFILTRATION INSTRUMENT

A Thesis

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by

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ABSTRACT

An instrument for the measurement of air infiltration into buildings is described. The instrument by sensing the thermal conductivity of an air tracer gas mixture indicates the concentration of a tracer gas in the building being investigated and hence the infiltration.

The instrument (Katharometer) has not been extensively used because of the inherent difficulties in using the instrument. This thesis examines the difficulties. An analysis of the theoretical basis of the instrument is made and the problems created by changes of relative humidity, pressure, and temperature are discussed and solutions are suggested. But with control of these factors satisfactory results were not obtained until a voltage control was used. A satisfactory automatic voltage control was developed that has been successfully used in air infiltration measurements.

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I INTRODUCTION

The infiltration of air into buildings is a component of the heating and cooling load. In heating this may amount to 10 - 25% of the design total heat loss. In air conditioning the effect is somewhat less, but the higher cost of air conditioning means it has about the same economic effect. Futhermore, it is not possible with present methods of calculating infiltration to determine accurate heat and moisture balances in buildings.

The method of evaluating this infiltration rate is to sum up estimated values for the various components; windows, doors, and walls in the building. These values have been determined experimentally in the laboratory by measuring the infiltration per unit length of crack or square foot of area. To find the overall infiltration rate the values of the infiltration for the components are summed and an exposure factor is applied. It has never been clearly established that this method gives accurate results for an actual building. In order to establish the accuracy of this method an absolute measurement of over-all infiltration rate must be made of actual buildings for which individual component values are known.

A practical method of determining the over-all infiltration in buildings is the tracer gas technique. In this method a gas, usually helium, is introduced into the space and the decay of the concentration is measured to determine the infiltration rate. Helium is the usual tracer gas because of its rapid diffusion, inertness, safety, and ease of detection. The detector senses the changes in the thermal conductivity of the air-tracer gas mixture. Two types of detectors have been used, a hot wire element and a thermistor, a recent development.

The purpose of this thesis is to study and improve a thermistor thermal conductivity cell to measure over-all infiltration of buildings. A number of investigators have used the instrument but very few results are reported. It is felt the scarcity of results is caused by the inherent difficulty of operating the instrument. The effects of pressure, temperature, and relative humidity upon the instrument will be investigated.

II THEORY OF TRACER GAS INFILTRATION MEASUREMENT

The method of measuring air infiltration in buildings that appears to have greatest possibilities, and has been used by various researchers, is the decay rate method using a tracer gas.[1,2,3*] The method used so far has been to add the tracer gas to the space where infiltration is to be measured, and by using a detector a record is made of the concentration of the tracer gas. The rate of decay of the concentration of the tracer gas is an indication of the infiltration rate.

Making the assumption that a tracer is available that acts in all respects the same as air; that is, it is not adsorbed by components of the building, it diffuses through building components the same as air, it does not stratify but diffuses evenly and quickly throughout the space; then once the tracer becomes uniformly mixed in a space the following holds:

$$-\frac{dc}{dt} = nc$$

$$\frac{dc}{c} = -n\,dt$$

$$\ln\left(\frac{c_2}{c_1}\right) = -n(t_2 - t_1)$$

$$n = \frac{\ln\left(\frac{c_1}{c_2}\right)}{(t_2 - t_1)} \qquad (2.1)$$

Where:

c = the instantaneous concentration of tracer.

n = the number of air changes per unit time period (infiltration rate)

t = time

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* Superscripts refer to List of References, p. 34

If a plot is made of concentration of tracer against time it will be noted that for a constant "n" the plot will be a straight line on semi-logarithmic paper. See Fig. 1.

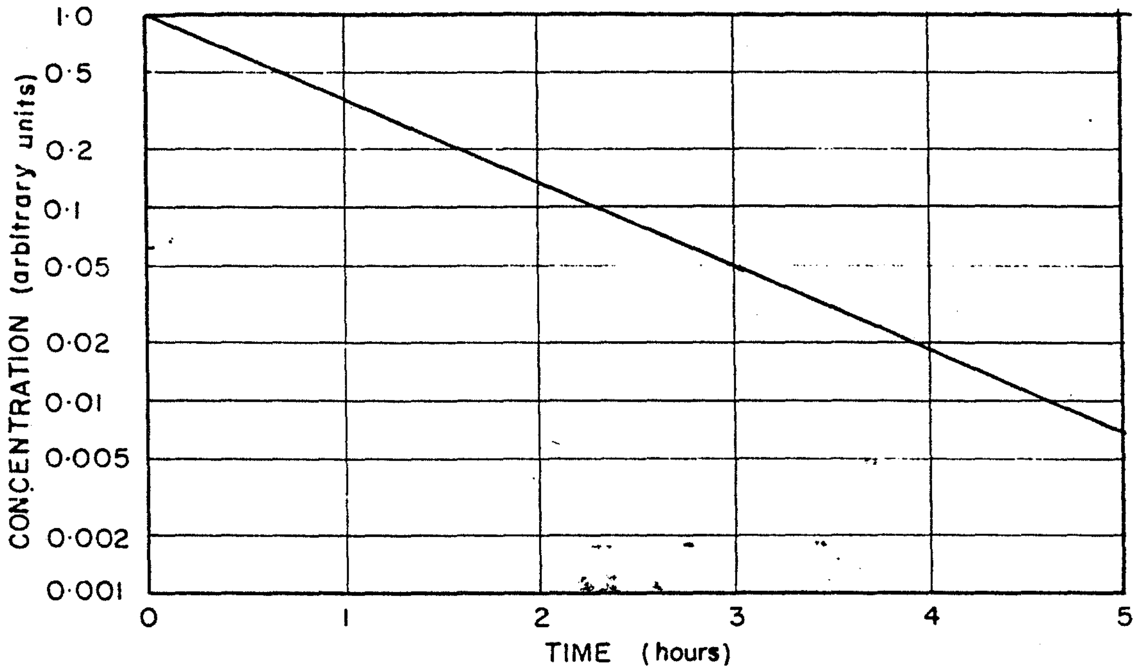


FIGURE 1
CONCENTRATION VERSUS TIME

It should be noted here that the absolute values for concentration are not needed, only relative values, since the ratio of the concentration is used in equation (2.1). Hence the equipment does not need to be calibrated for concentration as long as the indication is linear.

The auxiliary equipment is very simple requiring no special equipment for control or measurement of flow rate of the tracer gas. The disadvantages are: (a) It does not give continuous indication of infiltration rate. (b) It is not a steady state measurement hence there could be problems involving the tracer due to its absorption and adsorption characteristics.

This method is limited to measuring infiltration rates for short periods, since the initial amount of tracer is limited by maximum scale range of recorder and as concentration decreases results are limited by accuracy. The period of measurement then is limited to the time required for the tracer to decrease in concentration from the initial maximum value to the minimum value limited by accuracy; that is

$$(t_2 - t_1) = \frac{1}{n} \ln\left(\frac{C_{max}}{C_{min}}\right) \qquad (2.2)$$

Where

t = time

n = infiltration rate

c = concentration

There are two other methods of measuring infiltration using a tracer gas. Each of these methods would give a continuous record of infiltration rate. See Appendix A.

III THEORY OF THE DETECTOR

The detector senses the thermal conductivity of the mixture of gases and hence a measure of the concentration of the gases. The detector is composed of two or more heated elements each contained within a space with the same constant wall temperature. In a two element system one element is called the active element and the other the reference. The gas sample is passed over the active element and the reference gas is passed over the reference element.

A development by Daynes[4] shown in Appendix B has related the resistance of the element to the thermal conductivity of the gases in the cells. This is below:

$$\frac{r_1}{r_2} = \frac{r_{10}}{r_{20}}\left[1+\alpha b i^2\left\{\epsilon-\beta_2(\epsilon-\delta)\theta + \alpha b i^2\epsilon\left(\frac{r_{20}\epsilon-r_{10}}{r_{10}+r_{20}}\right)-bi^2\left(\beta_2\frac{(2\epsilon-\delta)}{2}\right)\right\}\right]^{*} \quad (3$$

To relate the output of the instrument to the thermal conductivity of the gases in the cells.

let $r_1 = r_2 + \Delta r$

then $\frac{r_1}{r_2} = 1 + \frac{\Delta r}{r_2}$ but $r_{10} = r_{20}$, elements have same cold resistance.

Hence,

$$1+\frac{\Delta r}{r_2} = 1+\alpha b i^2\left\{\epsilon-\beta_2(\epsilon-\delta)\theta + \alpha b i^2\epsilon\left(\frac{r_{20}\epsilon-r_{10}}{r_{10}+r_{20}}\right)-bi^2\left(\frac{\beta_2(2\epsilon-\delta)}{2}\right)\right\}$$

$$\frac{\Delta r}{r_2} = \alpha b i^2\{\qquad\qquad\}$$

$$\Delta r = r_2\alpha b i^2\left\{\epsilon-\beta_2(\epsilon-\delta)\theta + \alpha b i^2\epsilon\left(\frac{r_{20}\epsilon-r_{10}}{r_{10}+r_{20}}\right)-bi^2\left(\frac{\beta_2(2\epsilon-\delta)}{2}\right)\right\} \quad (3.1)$$

*notation is given in Appendix B

A Wheatstone bridge circuit as shown in Fig. 2 was used to transform the ratio of resistances of the elements into a voltage signal that was recorded by a self-balancing potentiometer. This is shown below.

$$mv = E\left[\frac{r_1}{r_1+r_3} - \frac{r_2}{r_2+r_4}\right]$$ Thevenin's theorem[5]

Consider the case where $r_3 = r_4$

and $r_1 = r_2 + \Delta r$

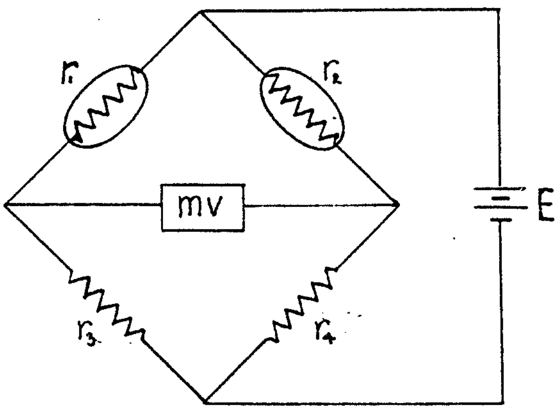
then if $r_1 \gg \Delta r$

$$mv = \frac{E\ \Delta r}{r_2 + r_4} \qquad (3.2)$$

but $$\Delta r = r_2 \alpha b i^2 \left\{\epsilon - \mathcal{B}_2(\epsilon - \delta)\theta + \alpha b i^2 \epsilon\left(\frac{r_{20}\epsilon - r_{10}}{r_{10}+r_{20}}\right) - b i^2\left(\frac{\mathcal{B}_2(2\epsilon-\delta)}{2}\right)\right\} \qquad (3.1)$$

therefore

$$mv = \frac{E\ r_2 \alpha b i^2}{r_2+r_4}\left\{\epsilon - \mathcal{B}_2(\epsilon - \delta)\theta + \alpha b i^2 \epsilon\left(\frac{r_{20}\epsilon - r_{10}}{r_{10}+r_{20}}\right) - b i^2\left(\frac{\mathcal{B}_2(2\epsilon-\delta)}{2}\right)\right\} \qquad (3.3)$$



mv = signal volts

r_1 = active thermistor

r_2 = reference thermistor

r_3 & r_4 = fixed resistors

E = bridge voltage

FIGURE 2
WHEATSTONE BRIDGE CIRCUIT

Hence for a constant current, signal is proportional to

$$\alpha b i^2 \left\{ \epsilon - \beta_2(\epsilon - \delta) \cancel{- \theta} + \alpha b i^2 \epsilon \left(\frac{r_{20}\epsilon - r_{10}}{r_{10} + r_{20}} \right) - b i^2 \left(\frac{\beta_2 (2\epsilon - \delta)}{2} \right) \right\}.$$

Daynes[4] shows by numerical calculation that the main effect is represented by the term $\alpha b i^2 \epsilon$; the other terms being negligible. In other words, signal is proportional to $\epsilon = \frac{K_1 - K_2}{K_1}$ the difference in thermal conductivity of the gas in the reference cell and the gas in the sample cell for small changes.

system was operated with E constant not I constant.

IV EQUIPMENT

For a small portion of the work hot wire elements were used. Hot wire elements have positive temperature resistance characteristics and thermistors have negative characteristics.

Hot wire elements are made of metal wire drawn very fine and made into a helical coil, see Fig. 3. Depending on the characteristics desired in the elements they are made of tungsten, platinum, gold plated tungsten or some other resistance metal. The elements are welded to heavier support wires that extend through a glass seal that can be mounted in the cell block.

The active material for a thermistor is fused into a small bead between two lead wires. It is composed of a mixture of metal oxides and has the property of having a high negative temperature resistance coefficient. See Fig. 4.

The element is enclosed in the cell block which besides acting as a heat sink must perform some other important tasks. These include enclosing the gas to be analysed in a suitable space, conducting the gas to this space, and controlling the gas temperature. Since the heat transferred by conduction is most important it is desirable to make this as large as possible in comparison to other methods of heat transfer. Convection can be reduced by keeping the space small and this is usually done by using small cells, 3/16" to 1/4" in diameter and from 1/2" to 1 1/4" long.

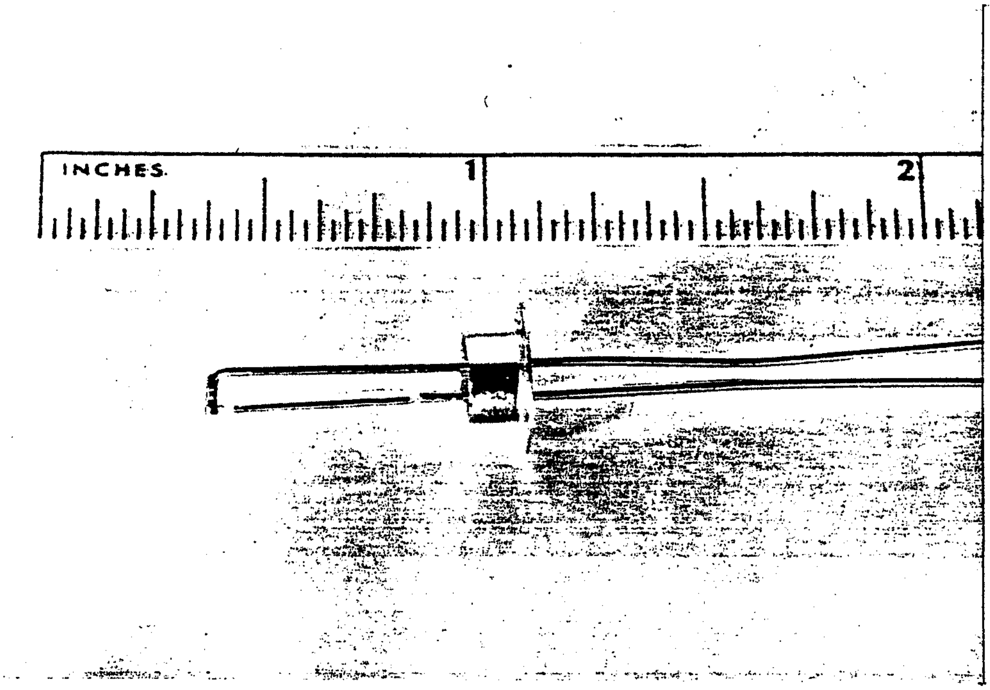


FIGURE 3 HOT WIRE ELEMENT

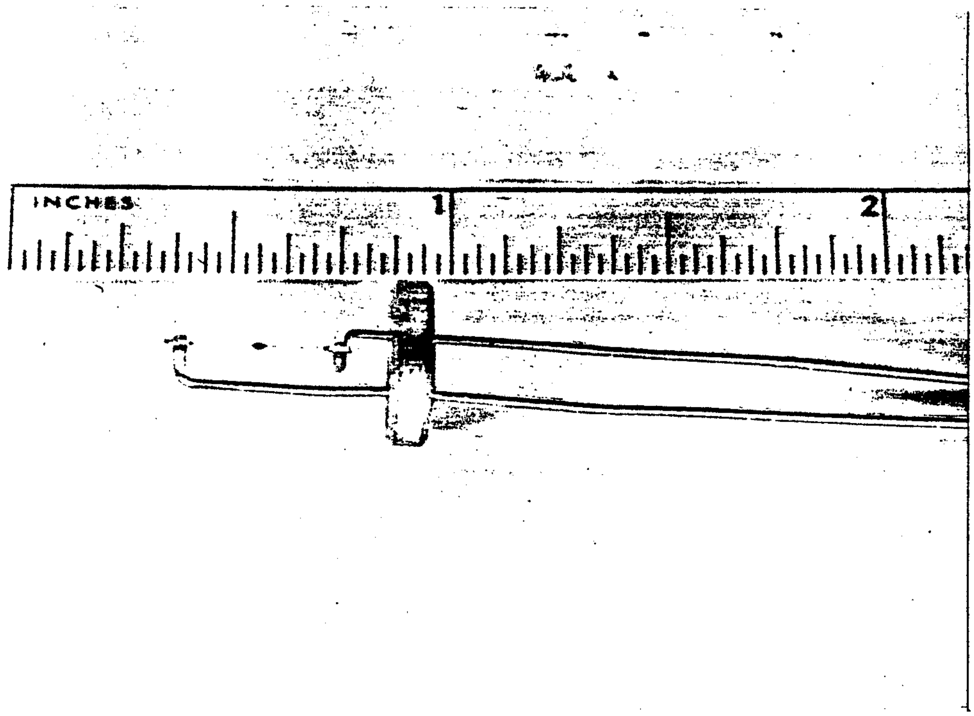


FIGURE 4 THERMISTOR ELEMENT

For a conductivity cell to work, a continuous supply of fresh gas must be brought to the analysing chamber. There are two methods of doing this; first, by direct flow where the gas enters one side of the analysing chamber and flows out the other, second, where the interchange of gas is caused by diffusion through the entrance. The rate of response is increased by a larger opening to the analysing chamber but the effect of fluctuation in flow rate is also more pronounced. The choice of a geometry for the cell block is therefore a compromise between response time and flow rate sensitiveness. The design shown in Fig. 5 was used in this study and it was found that for the flow rates used the signal was not affected.

Because of its high thermal conductivity, inertness, high diffusion rate, and because it is not easily absorbed by construction materials, helium was used as the tracer gas.

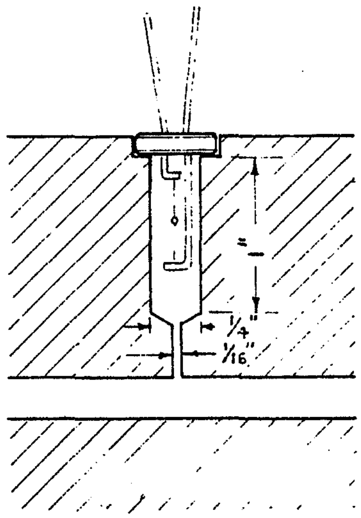


FIGURE 5
CELL BLOCK CONFIGURATION

V INVESTIGATION OF FACTORS AFFECTING PERFORMANCE OF THE DETECTOR

(1) Air-helium Mixture Properties

The assumption of previous investigators (Daynes)[4] that the thermal conductivity of air-helium mixtures was linear was investigated. The equation of Lindsay and Bromley[6] shown in Appendix C was used to calculate the thermal conductivity curve for air-helium mixtures. The general assumption has been that helium in small concentrations in air has a thermal conductivity 6.1 times as great as air. However, when the concentration of helium is low, due to the sag in the curve (see fig. 6A) the apparent thermal conductivity of helium is only 2.8 times as great as air. This means that the sensitivity of the instrument will be less than anticipated, and for higher concentrations of helium the signal will be nonlinear, with the result that absolute calibration of the instrument will be necessary when using higher concentrations. To eliminate the effect of nonlinearity of the thermal conductivity of the air helium mixture the concentration of all mixtures of air-helium used was less than 1%.

(2) Relative Humidity Changes

The effect of changes in relative humidity of the sample gas can be determined by an analysis of the thermal conductivity of air vapor mixtures. With most gases the thermal conductivity of the mixture of the gases is less than

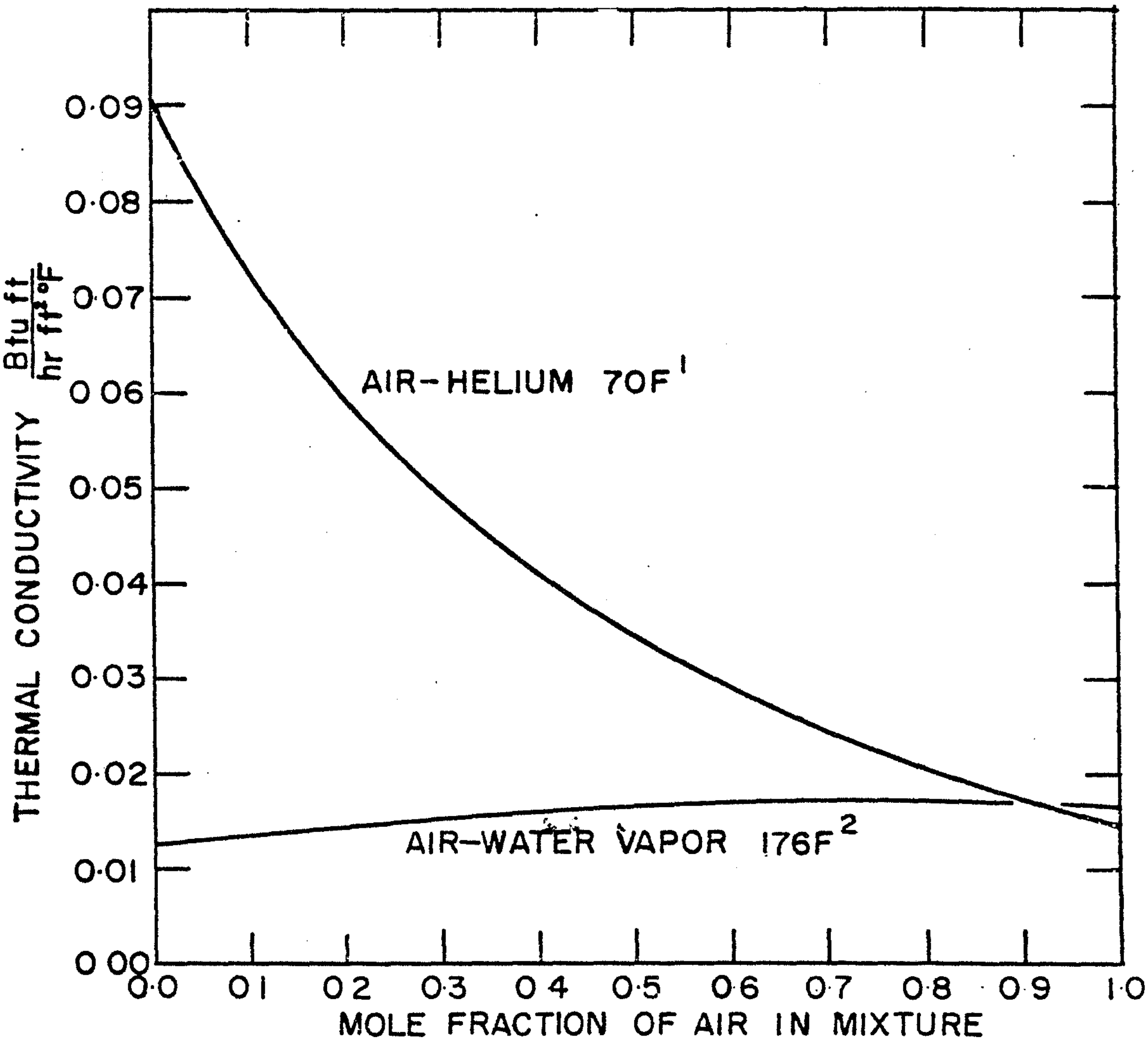


FIGURE 6 A

THERMAL CONDUCTIVITY OF AIR–HELIUM, AIR–WATER VAPOR MIXTURES

1 EQUATION OF LINDSAY AND BROMLEY
IND ENG CHEM 42:1508 (1950)

2 INTERNATIONAL CRITICAL TABLES

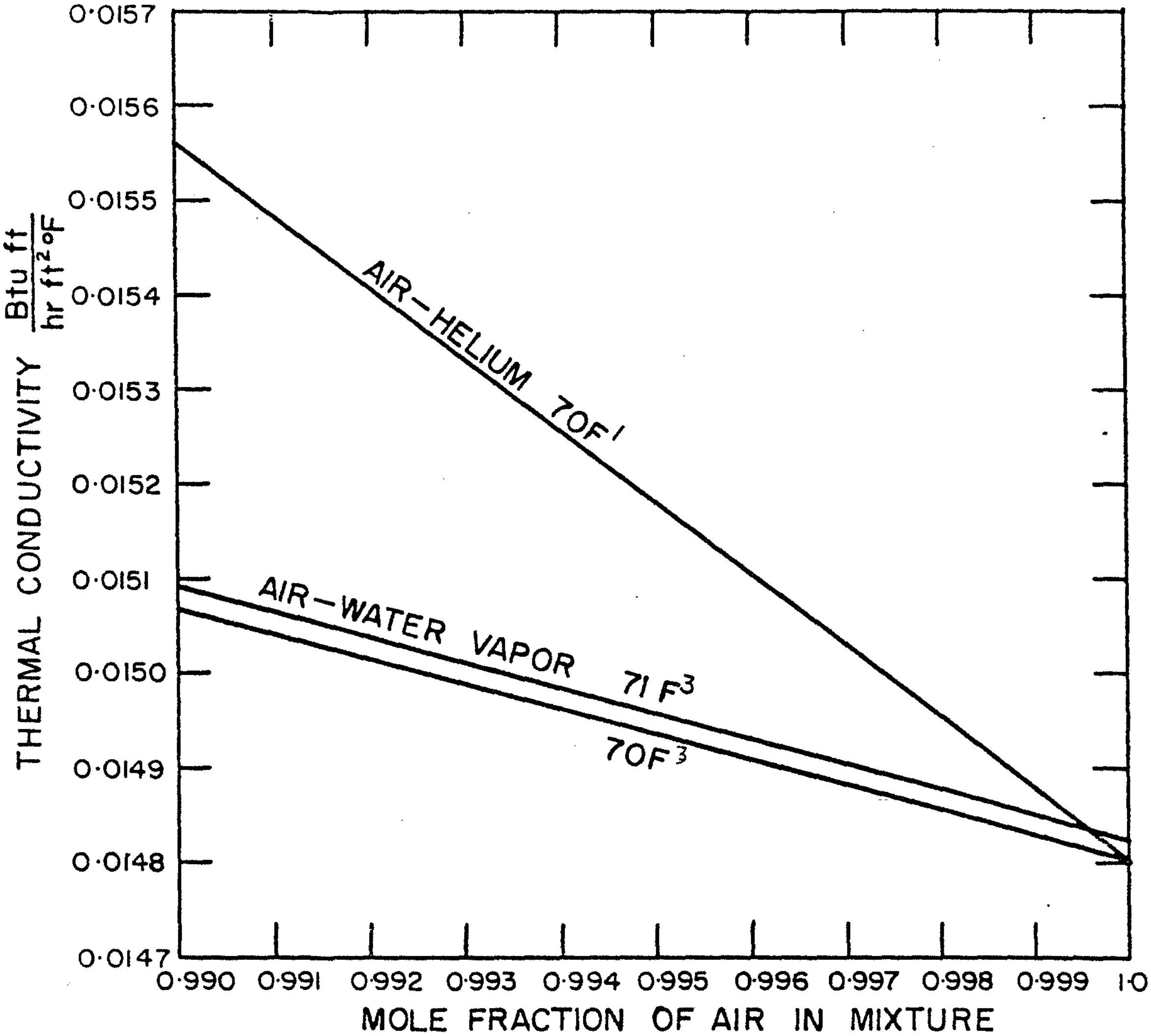


FIGURE 6B

THERMAL CONDUCTIVITY OF AIR-HELIUM, AIR-WATER VAPOR MIXTURES

3 DRAWN PARALLEL TO 2

the weighed average of the thermal conductivities. However, with air vapor the reverse is true. A plot of thermal conductivity versus mole fraction of air in mixture, Fig. 6A & 6B shows that the thermal conductivity for water vapor is less than that for air. However, the conductivity for the mixture increases with increasing concentration of water vapor for low concentrations of water vapor in air. From the available data[7] it would appear that .01 mole fraction water vapor in air (relative humidity 82%) would give a signal the same size as .26% helium in air. If it is required to measure 0.1% helium in air to an accuracy of 1% this means water vapor must be controlled to .00396 mole fraction or the relative humidity be controlled to .326%. In houses with normal occupancy, relative humidity changes of 5% can be experienced in periods of 1/2 hour or less. This would give the same reaction as 0.015% helium change which is equal to a 40% error at minimum signal.

The control of relative humidity of the sample is possible by (1) saturation of the air, (2) by drying. Since saturation presents difficulties due to control it seems desirable to attempt drying.[8] Anhydrous magnesium perchlorate was used to reduce the relative humidity of air to less than 0.01%.[9] This was used in all the experiments carried out with no trouble being experienced as a result of water vapor.

(3) Pressure Effects

Initial investigations were carried out using hot wire elements because of the unavailability of thermistors. It was found that sudden changes in pressure had a marked influence on the signal while gradual changes of the magnitude of barometric changes appeared to have negligible effect. The results of this work is covered in Appendix D under "Pressure Effects of Hot Wire Elements".

It may be concluded that normal barometric air pressure changes will not affect the determination of air infiltration in buildings.

(4) Temperature Effects

An examination of equation (B.3) by Daynes shows that if the resistance of the reference cell remains constant it may be replaced by a standard resistor. This simplifies the instrument since it now has only one element. A standard resistance box was used to replace the reference element in the Wheatstone bridge circuit and with this arrangement an attempt was made to discover the effect of temperature on a single thermistor in a bridge circuit. Since thermistors as resistance elements are very sensitive to temperature changes close control was needed of the block temperature to keep the system stabilized. To examine this experimentally one thermistor was mounted in a Wheatstone bridge with the output from the bridge going to a 5 millivolt recording potentiometer. The physical system consisted of a low thermal mass block with fins to dissipate heat from the unit. The block was constructed of copper as shown in Fig. 7 and was immersed in an ice water mixture. It was found that the electrical system with this arrangement was not stable. The instability was thought due to the heat transfer characteristics of the block to the ice mass and an attempt was made to improve this by stirring the ice water mixture. The stirring improved the stability slightly but it was still not good enough as a detection system. To avoid further delay, calculations were made to try and determine the order of temperature control required. This has been done in Appendix E. The temperature of the thermistor must be controlled to approximately 0.047 F in order to keep the displacement on the potentiometer to one milli-volt deflection. Since it would be desirable to keep the full scale deflection of a recorder or indicator for helium concentration to approximately one milli-volt and since temperature effects should be kept to less than 1% of this, this would be a temperature fluctuation less than 4.7×10^{-4} F. It was felt that this was an unobtainable value and that some method must be used to compensate for temperature fluctuations.

In a Wheatstone bridge circuit with legs of equal value, if two adjacent legs contain resistors that have resistance changes that are equal, the bridge will remain balanced in spite of the changes. This is the basis of temperature compensation with a second thermistor. The second thermistor

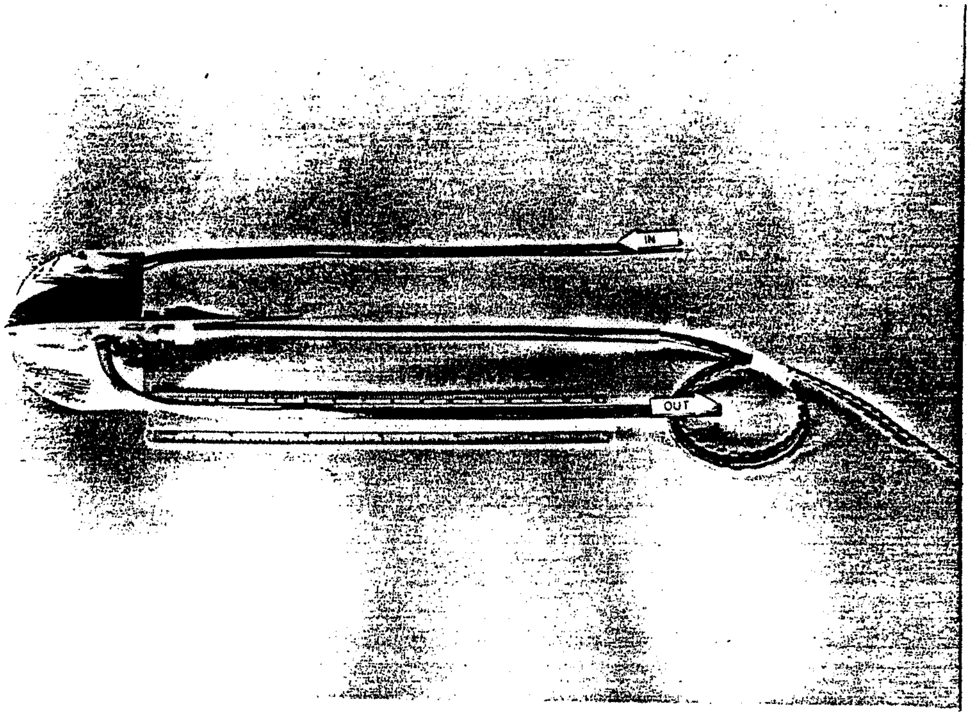


FIGURE 7 LOW MASS CELL BLOCK
FOR ONE ELEMENT

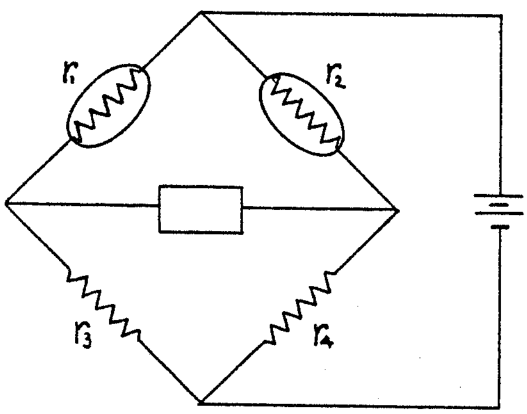


FIGURE 8
WHEATSTONE BRIDGE CIRCUIT

is placed in an adjacent leg of the bridge circuit as shown in Fig. 8, with this arrangement the 3rd and 4th resistors form ballasts for each thermistor, avoiding the possibility of burning the thermistor out if the peak voltage for the thermistor is exceeded. The two thermistors were placed in identical cells in a block of solid metal that has a high thermal conductance. The reason for this block is to conduct heat caused by ambient temperature changes so that it will affect both thermistors the same. With this arrangement there was no drift of the recorder that could be attributed to temperature change. Even with compensation it was thought advisable to try to keep temperature variation to a small value, so further study was done on this. Various arrangements of baths were tried. The most successful as far as temperature variations were concerned consisted of a massive block of copper containing the thermistor elements, see Fig. 9. This was submerged in a glass jar containing 1/2 gallon of water and the whole was surrounded with 4" of insulation, see Fig. 10. This proved to be most satisfactory when the water and the copper block were kept at room temperature with no stirring.

Using a bridge with two thermistors it was then attempted to determine the effect of various ambient temperatures on the signal for various values of current. This was done using the physical set up shown in Fig. 11A with the equipment inside

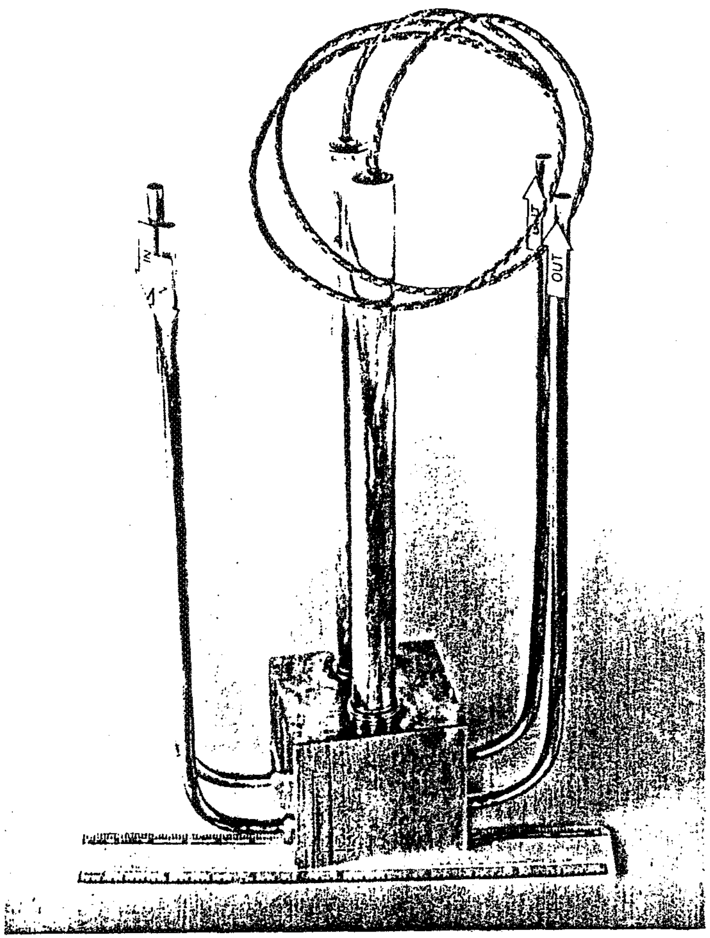


FIGURE 9
HIGH MASS CELL BLOCK
FOR TWO ELEMENTS

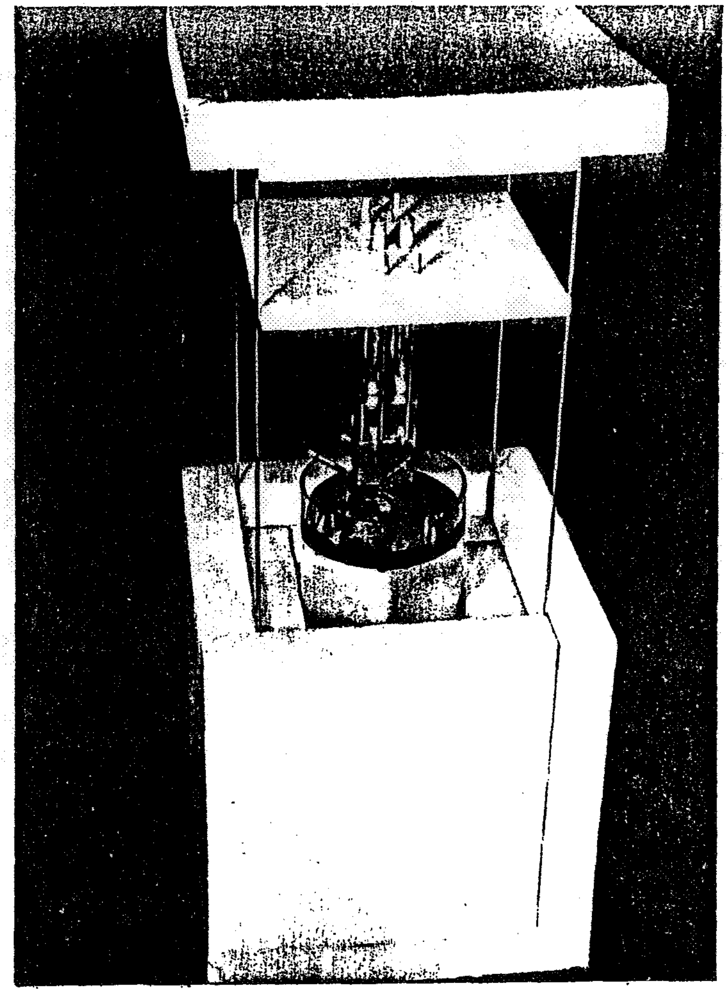
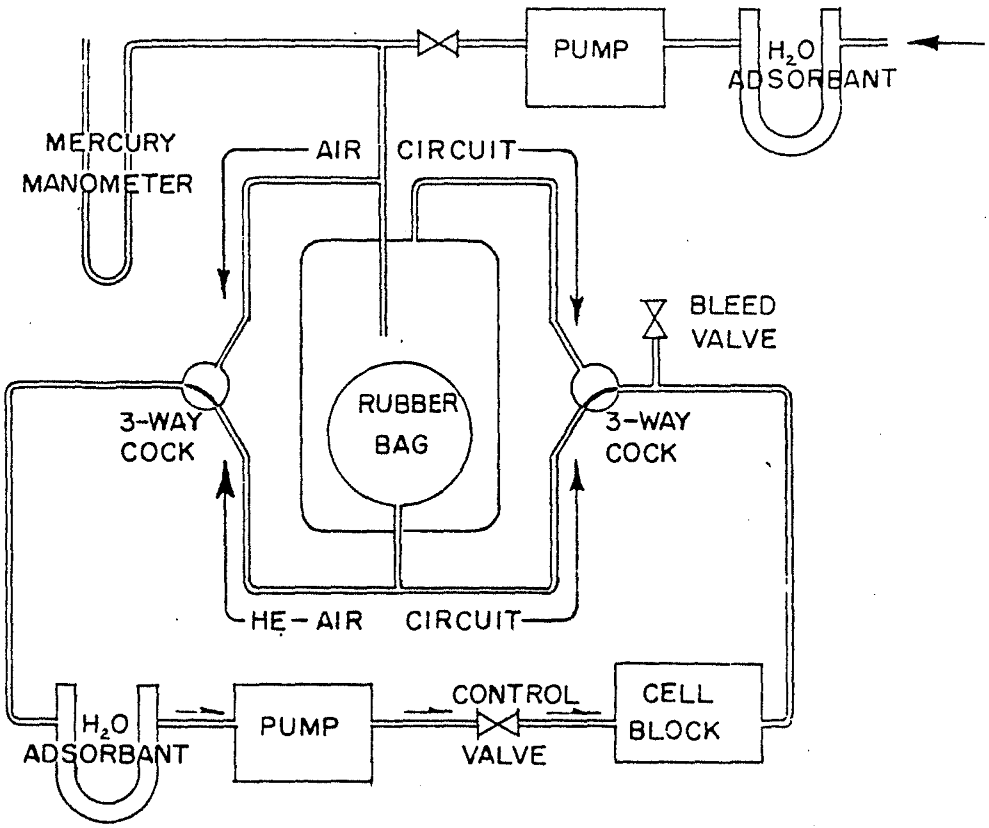
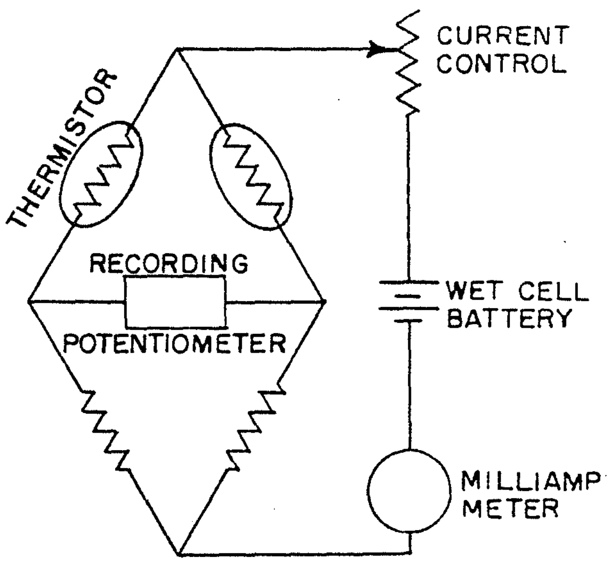


FIGURE 10
INSULATED BATH FOR
HIGH MASS CELL BLOCK



(A) PHYSICAL ARRANGEMENT



(B) THERMISTOR
ELECTRICAL CIRCUIT

FIGURE 11
EXPERIMENTAL ARRANGEMENT (I)

a humidity and temperature controlled box. The temperature in this box was controlled to approximately ± 1/2 F and the humidity to approximately ± 2% relative humidity. In order to make these runs the cells were operated in a pure air atmosphere until a stable base line was being drawn. One cell was then shunted into another circuit containing 1% helium in air. The apparatus was run with this condition until a new steady base was being drawn, and then the cell was shunted back into the original pure air circuit. The signal then was the difference between the initial base line without helium and the new base line with helium. This was plotted in terms of milli-volts against a current base. See Fig. 12.

In spite of all the precautions taken to control temperature, humidity, and voltage supply, the equipment was found to be difficult to keep on a stable base line. This is evident from the scatter of points in Fig. 12. The fluctuations observed were larger than those due to possible effects of temperature or humidity and no fluctuations in current were observed that could account for this drift.

A private communication with Division of Building Research, Ottawa, indicated that a manual control as shown in Fig. 13 corrected for the above mentioned drift. This was done by adding two more resistance legs and a galvanometer to the Wheatstone bridge circuit. The current to the circuit was controlled by observing the galvanometer and adjusting the external resistance until zero deflection was obtained. Division of Building Research offered no explanation on the necessity for this control.

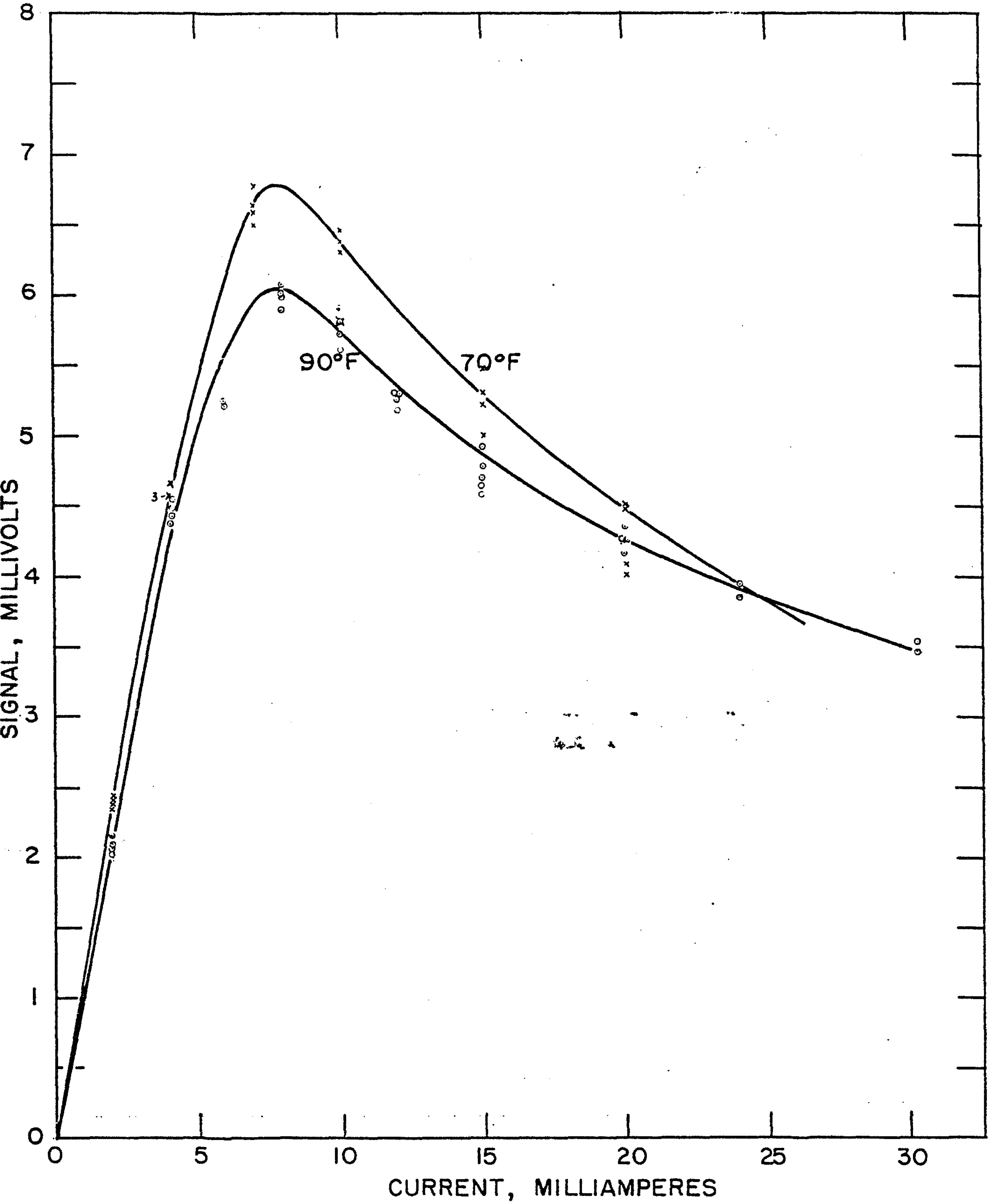


FIGURE 12
STEADY STATE SIGNAL FROM THERMISTOR BRIDGE FOR 1% HELIUM IN AIR FOR TWO AMBIENT CELL TEMPERATURES, PLOTTED AGAINST TOTAL BRIDGE CURRENT

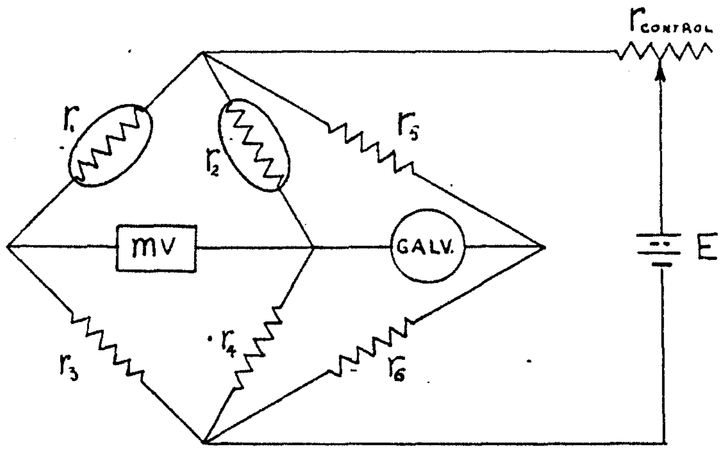


FIGURE 13
MODIFIED WHEATSTONE BRIDGE CIRCUIT

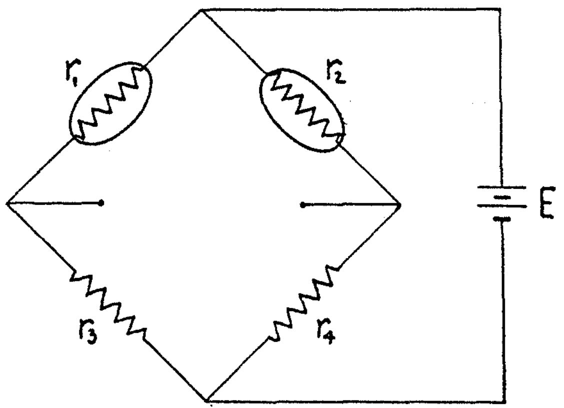


FIGURE 14
WHEATSTONE BRIDGE

VI DEVELOPMENT OF AN AUTOMATIC VOLTAGE CONTROL

Consider the traditional circuit in Fig. 14. It will be noticed that the reference thermistor should have a constant value if there are no external effects. This is not the case in practice. As soon as the active thermistor changes due to tracer gas in the active cell, the reference thermistor will also change in resistance due to the change in voltage of the power supply. The change in the voltage of the power supply is a result of the change in current to the bridge and the internal resistance of the power supply.

Note Section A B in the E I curve of the thermistor, (Fig. 15) is nearly a constant voltage. However, across the adjacent leg B, (Fig. 16) the voltage is proportional to the current. Hence a change in the voltage ratio across the reference thermistor and B will result from changes in the current supply. It is shown in Appendix F that an addition of 1% helium in the active cell will cause a change of 4.5 ohms in the reference thermistor when the resistance of the power supply is 1295 ohms. This resistance change in the reference thermistor is approximately 30% of the resistance change in the active thermistor for 1% helium. This change in the reference thermistor would cause a shift in the zero point of approximately the magnitude that did occur.

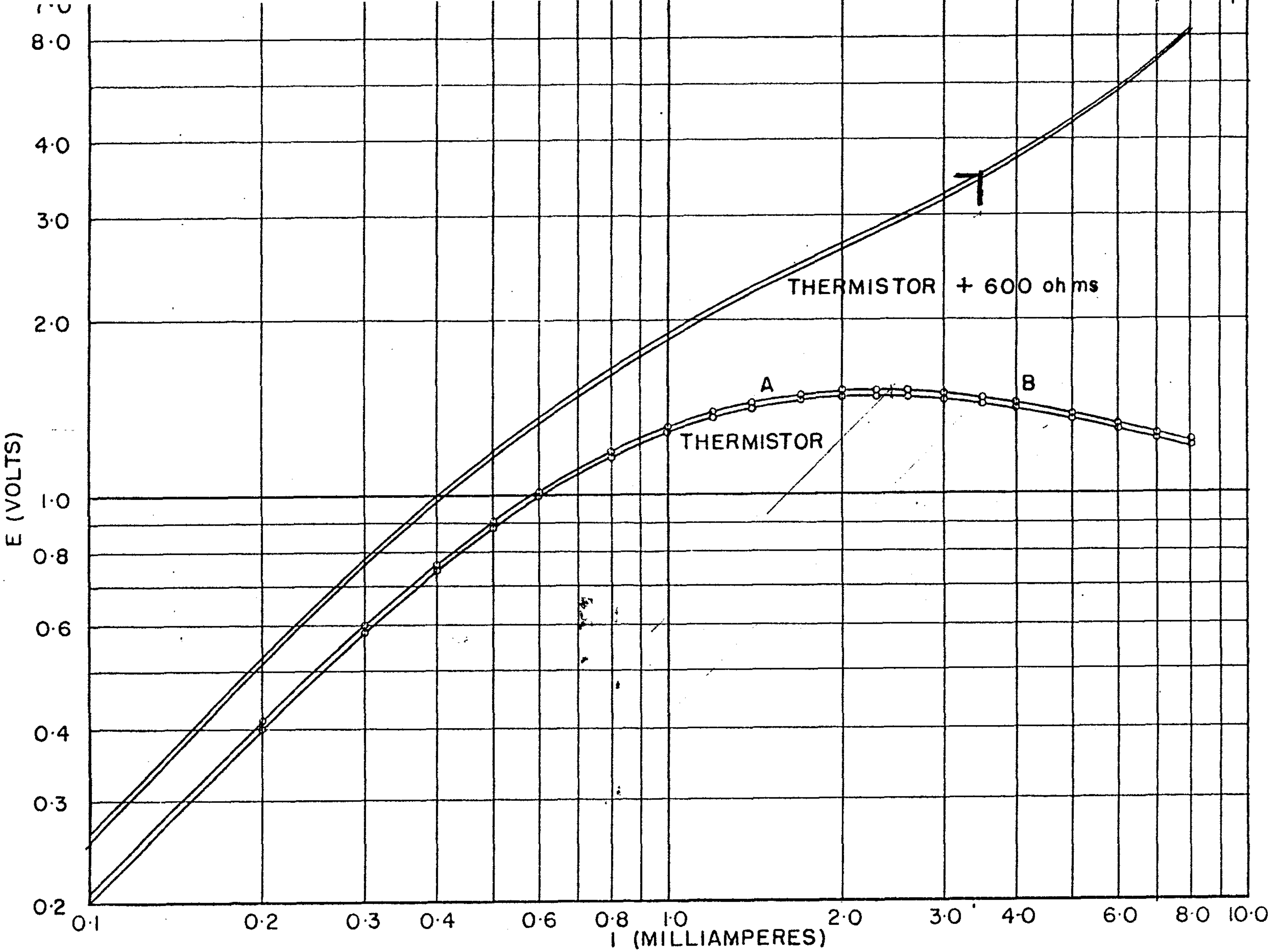


FIGURE 15

E-I CURVE FOR MATCHED PAIR OF THERMISTORS G126 # 372

An improvement was made in the Division of Building Research bridge by automating the voltage control. See Fig. 16. This was done by placing a null amplifier in place of the galvanometer with the output going to a motor used to drive the current control resistance. This amplifier senses the voltage at the junction of B and R_t and drives the current control resistance in such a direction to keep this constant. It is shown in Appendix G that this can keep the voltage supply to the bridge within $\pm$ 8 microvolts. The performance of the automatic control was checked using the apparatus shown in Fig. 17. Air was metered at a known rate into a closed metal tank containing a small amount of helium in air. This was mixed with the contents by the use of inlet jets. A sample was removed continuously and fed to the detector cell and the excess gas was bled to the atmosphere. The plot of signal vs. time is shown in Fig. 18. The zero drift is 0.013mv for five hours. This has been replotted on log concentration vs. time for the instrument (Fig. 19) and an infiltration rate of .790 air changes/hour was determined. The actual infiltration rate was .782 air changes/hour, or an error of 1% .

This improved instrument with temperature, humidity, and voltage control was used to measure the infiltration in three houses and performed satisfactorily.

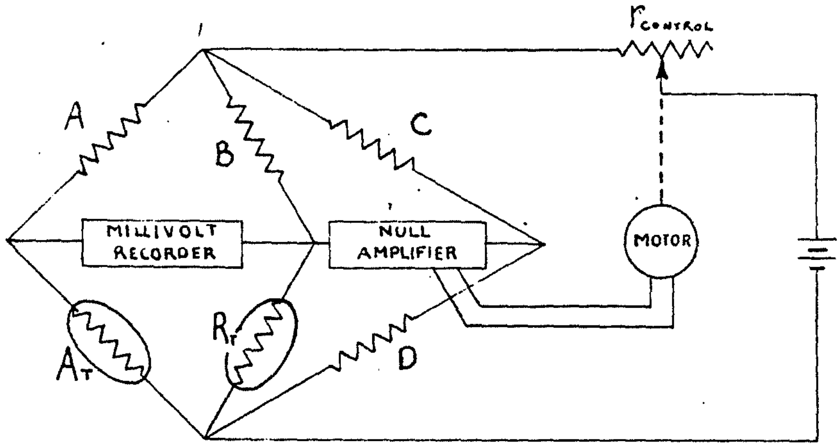


FIGURE 16
AUTOMATED VOLTAGE CONTROL CIRCUIT

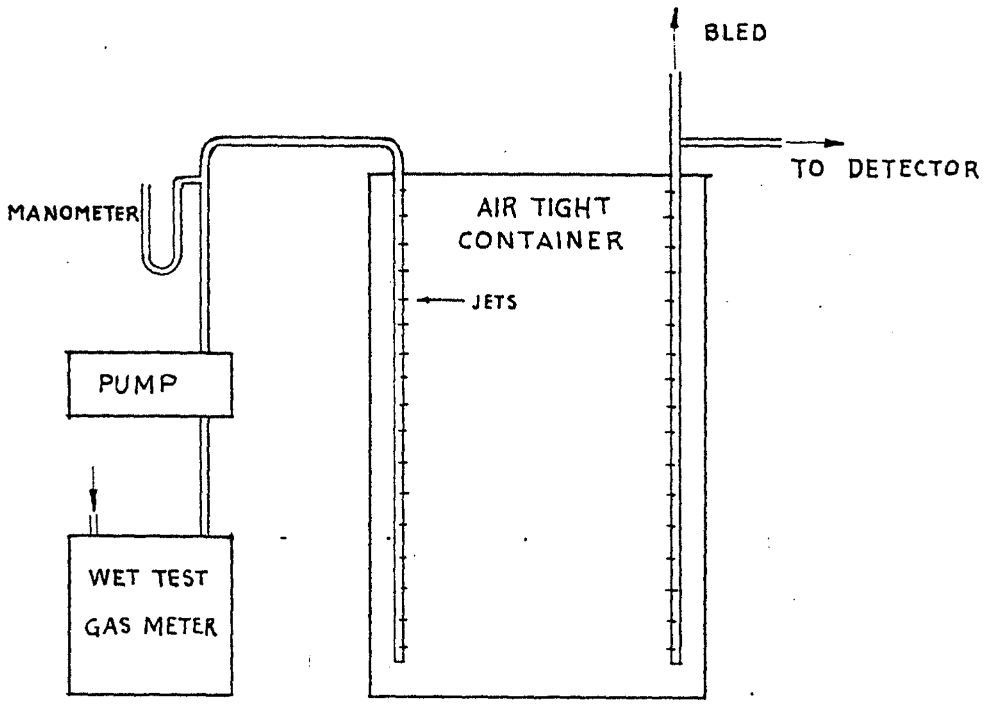


FIGURE 17
EXPERIMENTAL ARRANGEMENT (2)

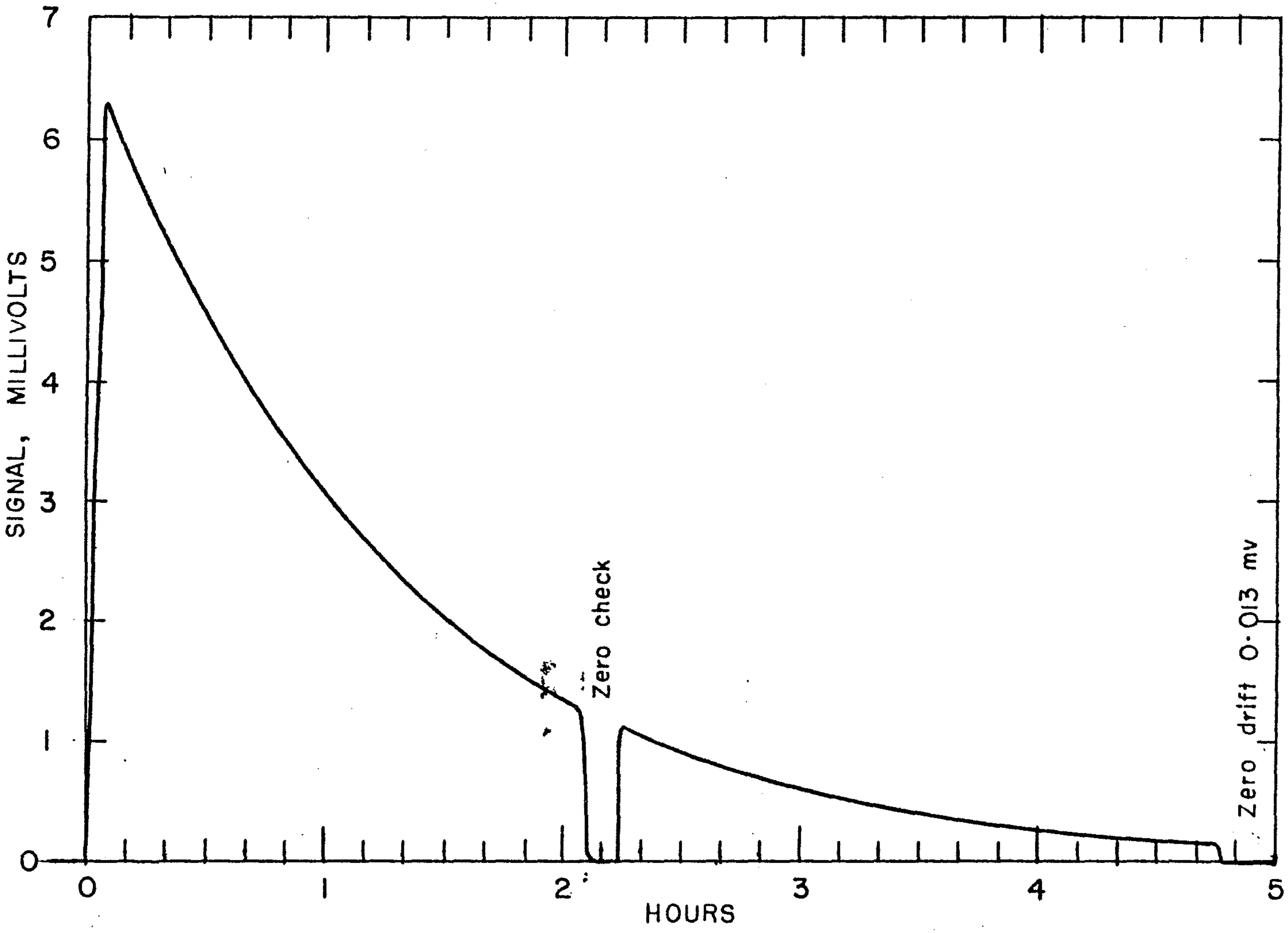


FIGURE 18
HELIUM DECAY CURVE FOR 0·782 AIR CHANGES PER HOUR

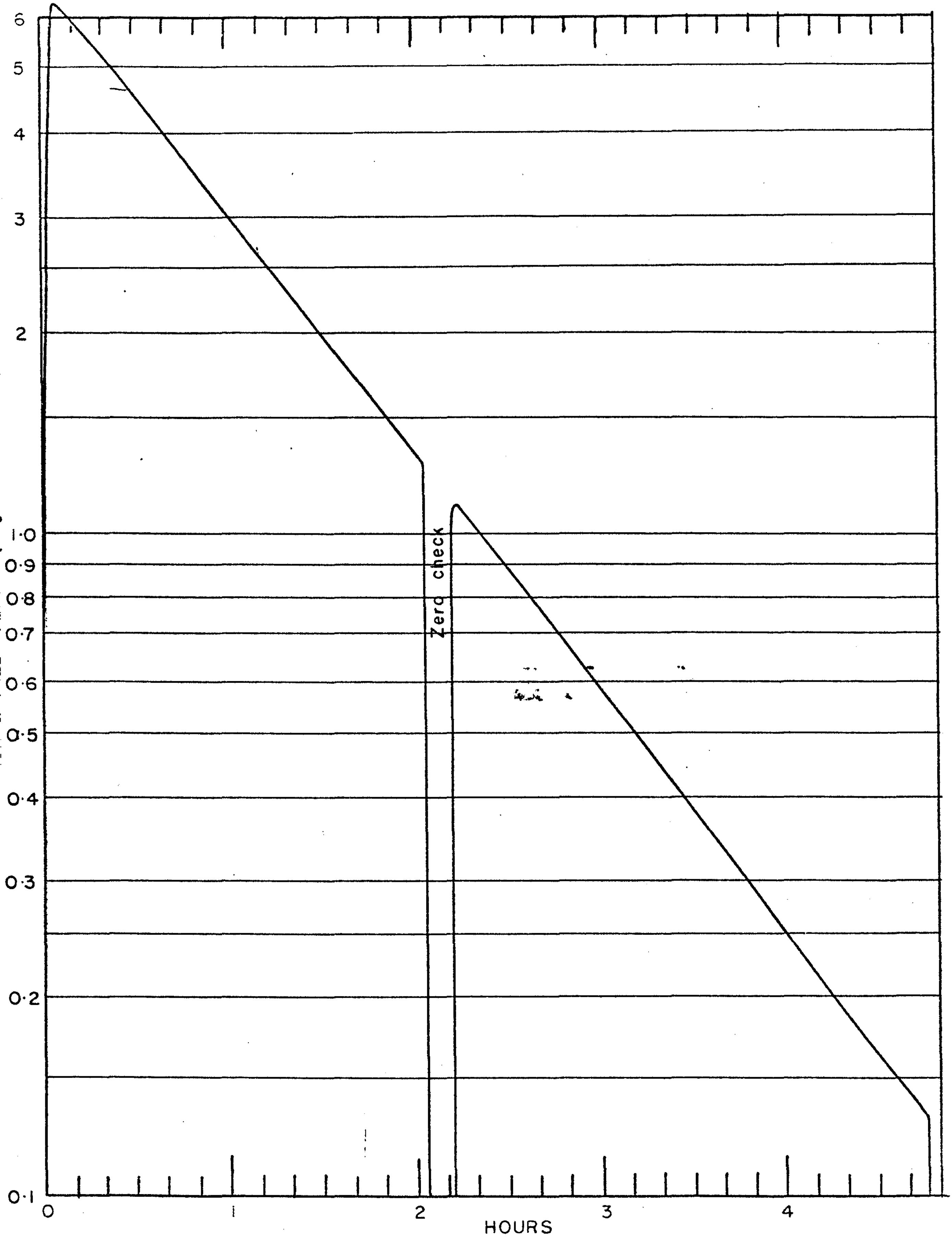


FIGURE 19
HELIUM DECAY CURVE FOR 0·782 AIR CHANGES PER HOUR

VII CONCLUSIONS

As a thermistor is much more sensitive to temperature than hot wire elements it produces a larger signal and is affected more by outside factors. The larger signal will permit use of smaller concentrations of helium in order to obtain infiltration measurements for a specific time. The instrument may be improved by the following suggestions:

(a) As small relative humidity changes would give a signal similar to that of the tracer gas, relative humidity must be controlled. The use of a drying agent has been suggested and was found to be satisfactory.

(b) Changes in the cell block temperature significantly affect the magnitude of the signal, hence temperature changes must be kept small during a test. The insulated box with water bath proved to be satisfactory in this regard, keeping the temperature variation during a run to a small value.

(c) There appear to be two effects of pressure on the cell depending on the speed of application. The signal response to rapid changes is violent and the only suggestion that can be made is that these must be eliminated as far as possible. Slow pressure changes do not appear to have a significant effect as long as the heat transfer can be by conduction. The use of small sized cells should reduce slow pressure changes to a negligible amount.

(d) Without the use of a voltage control the bridge was unstable and had a zero drift, even when all other factors were controlled. The automatic voltage control developed in this investigation appears to have eliminated this drift to a large extent and has given satisfactory results in several full scale infiltration tests.

It is suggested that work needs to be done on the constant concentration method of measuring air infiltration using this instrument. Since this is a continuous method of measuring air infiltration it will be easier to correlate heat loss, air infiltration, and other factors.

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5. H. B. Stout, Basic Electrical Measurements, Prentice Hall, Inc., p. 108.

6. A. L. Lindsay and L. A. Bromley, Industrial and Engineering Chemistry, 42, 8, p. 1509, Aug. 1950.

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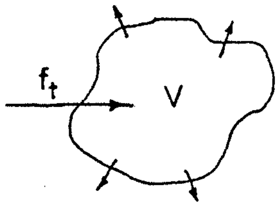
APPENDIX A

OTHER TRACER GAS METHODS

i Constant Flow Rate Method

The procedure for the constant flow rate method would involve the continuous adding of a tracer gas at a constant flow rate, mixing it into the air in the building, and then measuring the concentration of the tracer at some location where an average concentration is present. An analysis of the tracer flow across the boundaries would give a measure of the infiltration rate.



Where:

f_t = flow rate of tracer in space

V = volume of space

n = infiltration rate

c = concentration in space

In this case V and f_t would be held constant.

tracer in = tracer out

$$f_t = Vcn$$

$$n = \frac{f_t}{Vc} \quad (A.1)$$

$$n \propto \frac{1}{c} \quad (A.2)$$

By a proper choice of the flow rate to match the volume of the space, the infiltration rate would have a simple relationship to the reciprocal of the concentration.

The constant flow rate method has the advantages of continuous infiltration recording and relatively simple tracer gas inlet system. It has several disadvantages however.

First, the tracer gas detector must give absolute values of tracer concentration unless relative values of infiltration rates are acceptable. The record of tracer concentration is not linear with the infiltration rate. The concentration of tracer gas in the space will not be constant, hence the steady state condition will not be achieved with resulting problems of adsorption and/or absorption of tracer. The accuracy of detector is variable; that is, at high infiltration rates the sensitivity is lower than at low infiltration rates at the same tracer flow rate.

ii The Constant Concentration Method

The constant concentration method would use a sensing element to control the concentration of the tracer in the space. The flow rate to the space then would be a direct function of the infiltration rate.

$$f_t = Vcn$$

$$n = \frac{f_t}{Vc} \qquad \text{(A.1)}$$

$$n \propto f_t \qquad \text{(A.3)}$$

Where:

f_t = flow rate of tracer

V = Volume of space

n = infiltration rate

c = concentration in space (constant)

Similarily by the proper choice of concentration, the infiltration would have a simple relationship to the flow rate.

Advantages of the constant concentration method are that it is a steady state mechanism and hence has no problems of adsorption and/or absorption. It would give a direct indication of infiltration rate. The tracer detector could be a simple comparison-type instrument not requiring any indication of concentration either relative or absolute. The accuracy is dependent on the rate of flow measurement. The disadvantages of this method are that flow control and its measuring apparatus would be quite complicated.

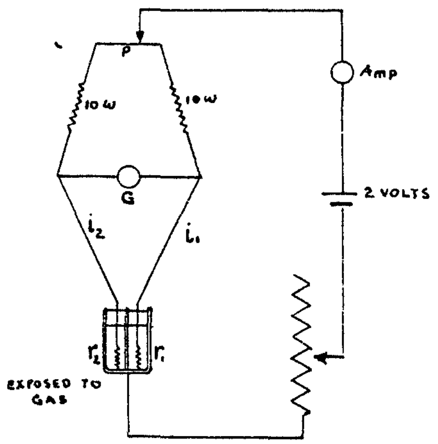
APPENDIX B

THEORY OF THE KATHAROMETER[1]

H. A. Daynes

(Abstract)[2] Gives the theory of the instrument which is of the form of a hydrogen detector based on the increase in heat conduction of a gas or mixture due to the presence of hydrogen. A Wheatstone bridge method is employed for the detection of the change in conductivity.

L. H. Walters, MA



Notation.

θ = temperature of copper block.

i = current in battery circuit.

α = temperature coefficient of resistance of platinum.

In the first katharometer coil:

i_1 = current through coil.

θ_1 = temperature of coil.

r_1 = resistance of coil at temperature θ_1

r_{10} = resistance of coil at temperature 0°C

K_1 = conductivity of gas surrounding coil when the wire is at a temperature θ_1

K_{10} = conductivity of gas at 0°C.

a_1 = constant depending of the shape of wire, etc.,

β_1 = temperature coefficient of conductivity of gas surrounding the coil.

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[1] H. A. Daynes, Proceedings of Royal Society of London, 97, p. 273-286, June, 1920.

[2] Science Abstracts, XXIII, p. 659, Abstract 1653, 1920.

Similar letters with the suffix $_2$ refer to similar quantities in the second coil.

We shall find it convenient to denote the ratio $\frac{a_1 K_{10} r_{10}}{a_2 K_{20} r_{20}}$ by ϕ, a function inversely proportional to K_{20} when the gas is varied. We wish to take account of the three variables i, θ, K_2, when the first cell is used as a standard.

The experimental relations are,

$$i_1^2 r_1 = a_1 K_1 (\theta_1 - \theta) \qquad \text{where} \quad r_1 = r_{10}(1 + \alpha\theta_1)$$

$$\text{and} \quad K_1 = K_{10}\left(1 + \beta_1 \frac{\theta + \theta_1}{2}\right)$$

and $$i_2^2 r_2 = a_2 K_2 (\theta_2 - \theta) \qquad \text{where} \quad r_2 = r_{20}(1 + \alpha\theta_2)$$

$$\text{and} \quad K_2 = K_{20}\left(1 + \beta_2 \frac{\theta + \theta_2}{2}\right)$$

$$\theta_1 - \theta = \frac{i_1^2 r_1}{a_1 K_1} = i^2 \left(\frac{r_2}{r_1 + r_2}\right)^2 \frac{r_1}{a_1 K_1}$$

$$= i^2 \frac{r_{20}^2 (1 + \alpha\theta_2)^2}{\{(r_{10} + r_{20}) + \alpha(r_{10}\theta_1 + r_{20}\theta_2)\}^2} \cdot \frac{r_{10}(1 + \alpha\theta_1)}{a_1 K_{10}\{1 + \beta_1(\theta + \theta_1)/2\}}$$

$$= i^2 b \left[1 + 2\alpha\theta_2 + \alpha\theta_1 - 2\alpha \frac{r_{10}\theta_1 + r_{20}\theta_2}{r_{10} + r_{20}} - \frac{\beta_1}{2}(\theta + \theta_1)\right]$$

to the necessary degree of approximation if $b = \left(\frac{r_{20}}{r_{10} + r_{20}}\right)^2 \frac{r_{10}}{a_1 K_1}$

Similarly

$$\theta_2 - \theta = i^2 b\phi \left[1 + 2\alpha\theta_1 + \alpha\theta_2 - 2\alpha \frac{r_{10}\theta_1 + r_{20}\theta_2}{r_{10} + r_{20}} - \frac{\beta_2}{2}(\theta + \theta_2)\right]$$

since $$\left(\frac{r_{10}}{r_{10} + r_{20}}\right)^2 \frac{r_{20}}{a_2 K_{20}} = b\phi$$

On the right hand side of these equations we may substitute $\theta_1 = \theta + bi^2$ and $\theta_2 = \theta + b\phi i^2$

Then $$\theta_1 - \theta = bi^2\left[1 + (\alpha - \beta_1)\theta + bi^2\left\{\alpha\frac{(2\phi - 1)r_{10} + r_{20}}{r_{10} + r_{20}} - \frac{\beta_1}{2}\right\}\right]$$

and $$\theta_2 - \theta = b\phi i^2\left[1 + (\alpha - \beta_2)\theta + bi^2\left\{\alpha\frac{r_{10}\phi + r_{20}(2-\phi)}{r_{10} + r_{20}} - \frac{\beta_2\phi}{2}\right\}\right]$$

$$\theta_1 - \theta_2 = bi^2\left[(1-\phi)(1+\alpha\theta) - (\beta_1 - \phi\beta_2)\theta + bi^2\left\{\alpha\frac{(1-\phi)^2(r_{20} - r_{10})}{r_{10} + r_{20}} - (\beta_1 - \phi^2\beta_2)\right\}\right]$$

Now $$\frac{r_1}{r_2} = \frac{r_{10}(1+\alpha\theta_1)}{r_{20}(1+\alpha\theta_2)} = \frac{r_{10}}{r_{20}}\{1 + \alpha(\theta_1 - \theta_2) - \alpha^2\theta_2(\theta_1 - \theta_2)\}$$

$$= \frac{r_{10}}{r_{20}}\left\{1 + \alpha bi^2\left[(1-\phi)(1+\alpha\theta) - (\beta_1 - \phi\beta_2)\theta + bi^2\left\{\alpha\frac{(1-\phi)^2(r_{20} - r_{10})}{r_{10} + r_{20}} - \frac{\beta_1 - \phi^2\beta_2}{2}\right\}\right]\right.$$

$$\left. - \alpha^2(\theta + b\phi i^2)bi^2(1-\phi)\right\}$$

$$\frac{r_1}{r_2} = \frac{r_{10}}{r_{20}}\left[1 + \alpha bi^2\left\{(1-\phi) - (\beta_1 - \phi\beta_2)\theta + \alpha bi^2(1-\phi)\left(1 - 2\frac{r_{10} + \phi r_{20}}{r_{10} + r_{20}}\right) - bi^2\frac{(\beta_1 - \phi^2\beta_2)}{2}\right\}\right] \qquad (B.1)$$

; . . . .

Special cases

Let $K_{20} = K_{10}(1+\epsilon)$ $\quad \therefore \quad \phi = 1 - \epsilon$

Equation (B.1) then becomes

$$\frac{r_1}{r_2} = \frac{r_{10}}{r_{20}}\left[1 + \alpha bi^2\left\{\epsilon - (\beta_1 - \beta_2 + \beta_2\epsilon)\theta + \alpha bi^2\epsilon\left(\frac{r_{10}\epsilon - r_{10}}{r_{10} + r_{20}}\right) - bi^2\frac{\beta_1 - \beta_2 + 2\epsilon\beta_2}{2}\right\}\right] \qquad (B.2)$$

Suppose the small change ϵ is produced by the admixture of a small percentage of another gas, and suppose over the range considered

$$\beta_2 = \beta_1(1+\delta)$$

Then

$$\frac{r_1}{r_2} = \frac{r_{10}}{r_{20}}\left[1 + \alpha bi^2\left\{\epsilon - \beta_2(\epsilon - \delta)\theta + \alpha bi^2\epsilon\left(\frac{r_{20}\epsilon - r_{10}}{r_{10} + r_{20}}\right) - bi^2\left(\frac{\beta_2(2\epsilon - \delta)}{2}\right)\right\}\right] \qquad (B.3)$$

APPENDIX C

THERMAL CONDUCTIVITY OF GAS MIXTURES*

A. L. Lindsay and L. A. Bromley. University of California.

(Abstract) An equation is developed for the thermal conductivity of gaseous mixtures which requires only a knowledge of the pure component conductivities, heat capacity or viscosity, boiling points, and molecular weights. The equation reproduces 85 mixture conductivities from the literature with an average deviation of 1.9%.

"The final equation for a gas mixture of n components is:

$$K_m = \sum_{i=1}^{n} \frac{K_i}{\frac{1}{X_i} \sum_{j=1}^{n} A_{ij} X_j}$$

here when

$$A_{ij} = A_{12}$$

$$A_{12} = \frac{1}{4}\left\{1 + \left[\frac{\mu_1}{\mu_2}\left(\frac{M_2}{M_1}\right)^{3/4} \frac{\left(1 + \frac{S_1}{T}\right)}{\left(1 + \frac{S_2}{T}\right)}\right]^{1/2}\right\}^2 \frac{\left(1 + \frac{S_{12}}{T}\right)}{\left(1 + \frac{S_1}{T}\right)}$$

when

K thermal conductivity, B.t.u./hr./ft.°F

M molecular weight

T absolute temperature

S Sutherland constant $S_{12} = \sqrt{S_1 S_2}$ except for strongly polar constituents

X mole fraction

U viscosity lb. mass/hr. ft.

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*Industrial and Engineering Chemistry, Vol. 42, No. 8, p.1509, Aug. 1950.

APPENDIX D

PRESSURE EFFECTS ON HOT WIRE ELEMENTS

Hot wire elements were available initially so the first studies were done using these elements. It was decided to study the problem of pressure changes on the cell first. This was done using the set up shown in Fig. D-1. To minimize the effects of water vapor a drying tube containing anhydrous magnesium perchlorate was used. To minimize CO_2 a tube containing potassium hydroxide solution on glass wool was included in the circuit. This was omitted in later experiments since CO_2 concentrations are so small and CO_2 has a thermal conductivity approximately the same as air. The temperature was controlled by placing the cell block in a cabinet in which the temperature was controlled to $\pm$ 0.1 F.

The procedure used in this study was to operate the cell at atmospheric pressure to obtain a steady base line. When this was reached the pressure was rapidly decreased to some specific value by using a diaphram pump. This pressure was maintained in the unit by shutting off a pinch cock and the resulting changes were observed with a recorder.

It was found that consistent results were not obtained and the unit was checked to find possible causes. It was thought that this was possibly due to ambient temperature changes but when temperature measurements were made, the ambient temperature was found to change less than 0.1 F during

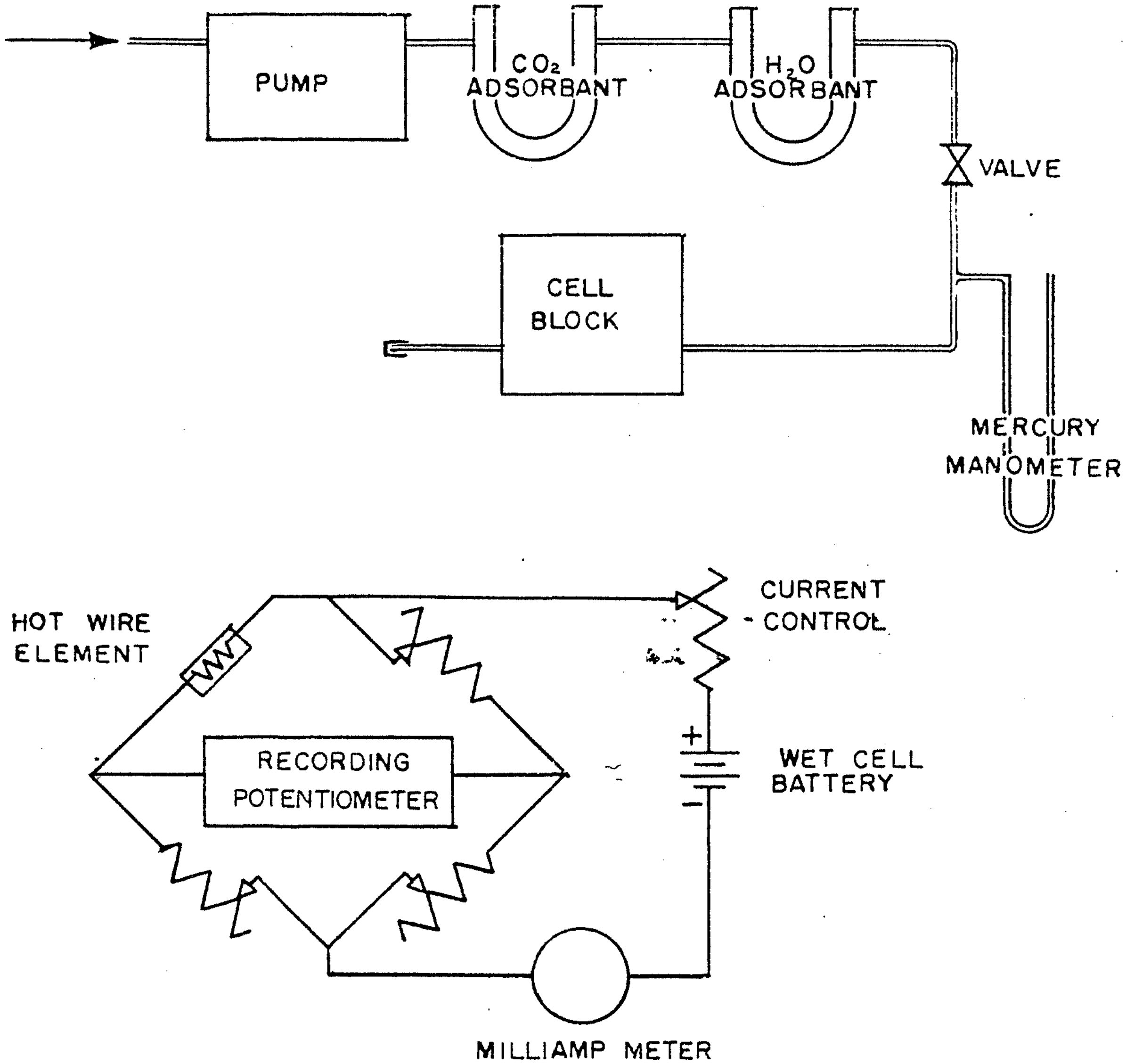

FIGURE D1
EXPERIMENTAL ARRANGEMENT (3)

the period of a run. . Further checks on temperature involved drilling the cell block to accept thermocouples that were soldered flush to the wall of the cell chamber itself. With these thermocouples it was impossible to detect any temperature change using Leeds and Northrup self-balancing potentiometer sensitive to temperatures of 0.1 F.

A study was made of the electrical system to determine reasons for this instability. Current was measured with a milli-amp meter and no fluctuations were observed. A closer check was made using a recording potentiometer to measure the voltage drop across a resistor through which the current passed. Numerous runs were made but no changes in current were observed.

During some of the runs it was observed that the pressure of the system would fall slightly. Attempts were made to tighten the system by replacing parts and sealing joints with a high vacuum grease. It was finally found that there was a small crack in the bottom of the cell block. This was possibly due to a fault in manufacture of the metal.

A new block was made, see Fig. D-2, the unit reassembled and runs were resumed. As before the procedure of making a run was to allow the system to reach a steady base line. The pressure in the system was then rapidly changed to a new value and was held constant until the recorder indicated a new steady base line. The pressure was then released and

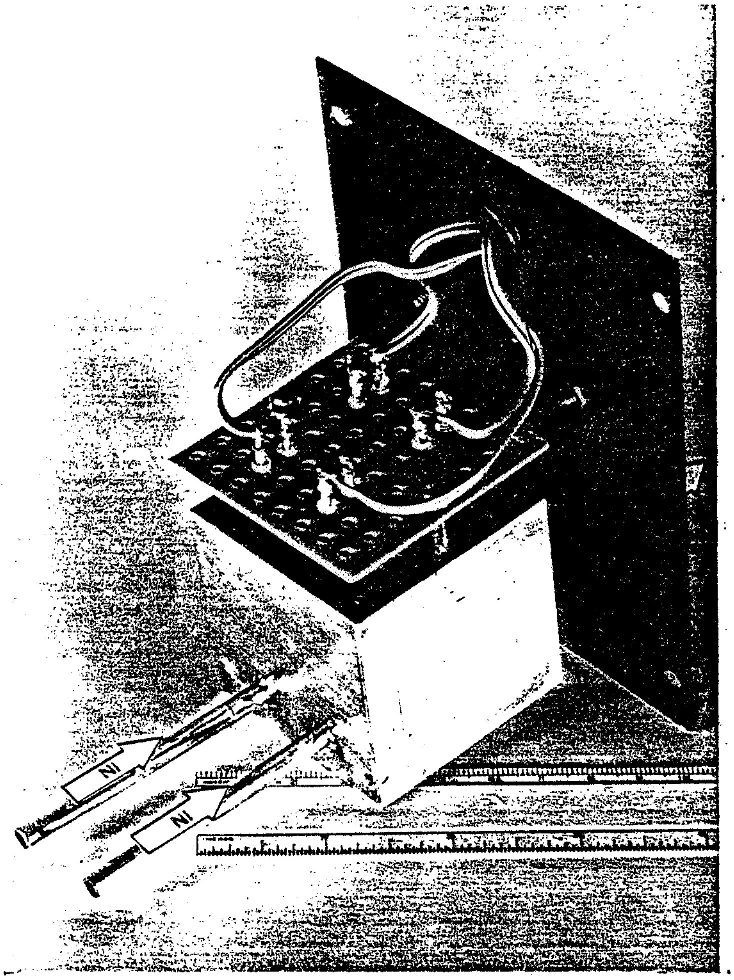

FIGURE D2
HIGH MASS CELL BLOCK
FOR FOUR ELEMENTS (HOT WIRE)

the recorder again allowed to resume its initial base line. A typical chart is shown in Fig. D-3. It will be noted that when the rapid pressure change was made the recorder was deflected off scale. The recorder then reached a peak in approximately 1/4 hour and followed a rough exponential decay curve to a new base line. On the release of pressure, basically the same type of process occurred. Repeated runs did not follow a definite pattern. Runs using different pressures did not appear to have any relationship between the shape of the curve and the pressure used.

It was found in later experiments that if the pressure was not changed rapidly some degree of stability could be observed, but with rapid pressure changes such as those caused by slamming doors, cause fluctuations in the system that take considerable time to stabilize. Rapid pressure changes are characterized by rapid signal fluctuations that are out of proportion to the size of the pressure change. For example squeezing the tube carrying the sample gas can cause the recorder to travel off scale. The reasons for this effect are possibly the forced convective effect and the thermodynamic effect.

The forced convective effect is a result of a jet of air being forced over the element due to the geometry of the unit. In the unit under study the opening to the cell is a

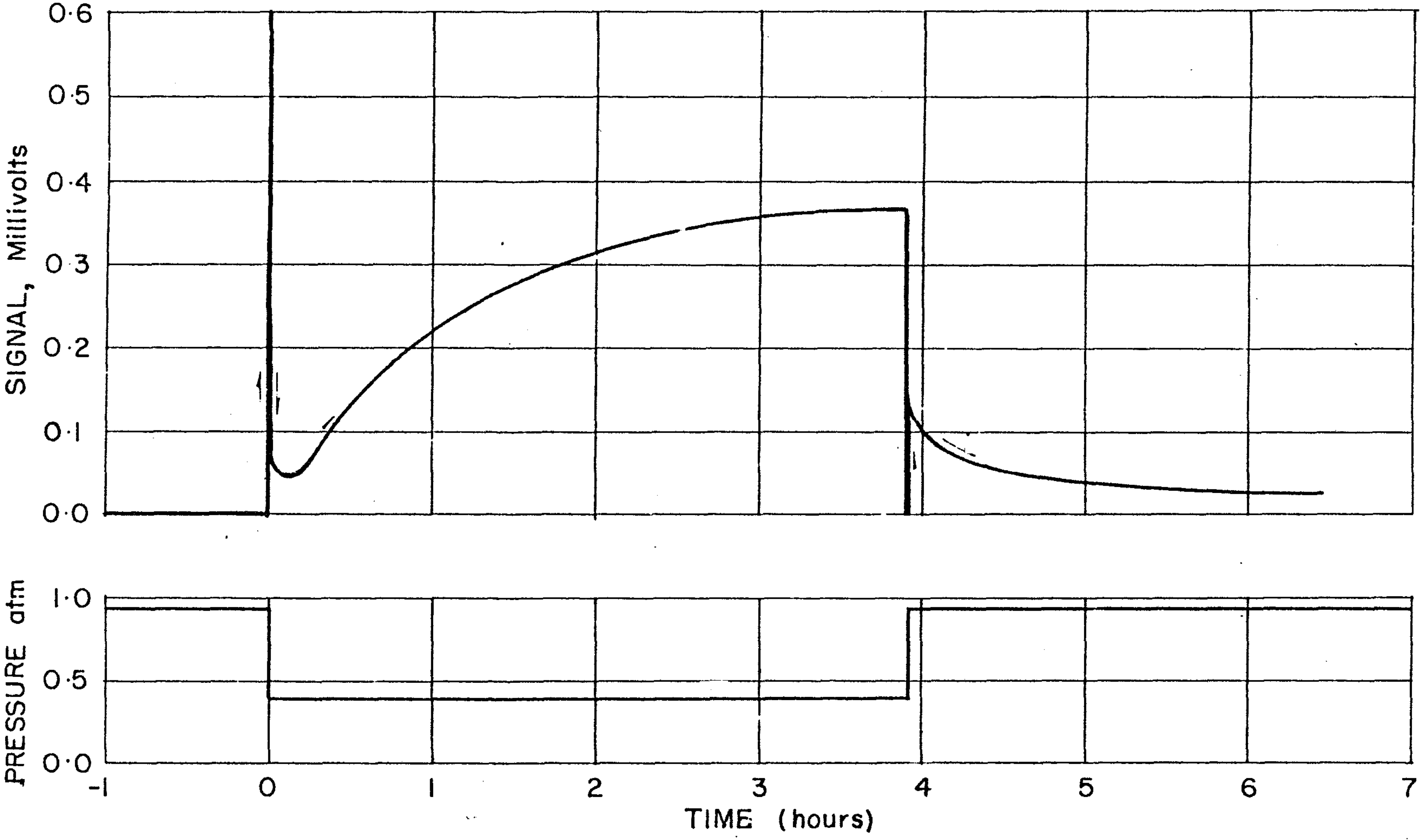

FIGURE D3

TYPICAL SIGNAL FROM HOT WIRE BRIDGE FOR RAPID PRESSURE CHANGES

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small diameter orifice directly in line with the element. Any surge of gas through this orifice would impinge directly on the element causing forced convection and as a result a more rapid method of heat transfer than normal.

This is true only for pressure increases since the flow is from the outside into the cell through the orifice. When there is a sudden drop in pressure the jet action on the thermistor does not take place since the flow is in the outward direction, the jet forming outside the cell.

If the pressure fluctuation is rapid the gas undergoes adiabatic heating or cooling. At first glance this may seem insignificant yet a 1% change in volume will result in adiabatic temperature change of 2.16°F. This is about 1.1 times as great a temperature difference as would exist between the reference and sample thermistor when the sample is exposed to 1% helium in air.

There seems to be two distinct effects due to pressure depending on the speed of application. Consider a slow increase in pressure. In the three methods of heat transfer through the gas in the cell; conduction, convection, and radiation, only convection is a function of the pressure. Conduction and radiation are not affected significantly by pressure changes in the pressure range under consideration, while heat transferred by convection varies as the square root of the pressure.[1]

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1 W. H. Giedt, Principles of Eng. Heat Transfer, p 217

It may be shown that convection heat transfer is a small part of the total heat transfer from the cell. See Appendix F. Therefore, slow pressure changes will affect the signal but may be eliminated by having both active and reference cells at the same pressure.

There appears no way of compensating for rapid pressure fluctuations at present.

APPENDIX E

TEMPERATURE SENSITIVITY OF THERMISTORS

Consider a Wheatstone bridge with a thermistor in one leg only, Fig. E.1.

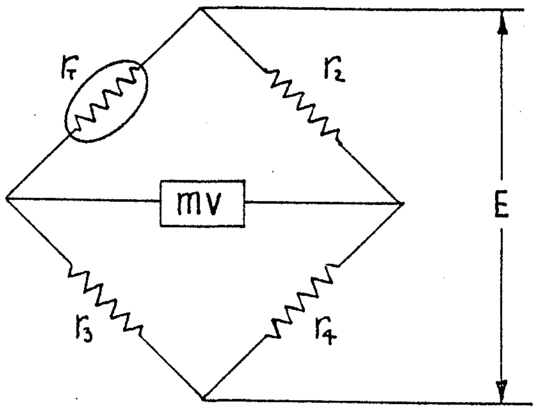


FIGURE E·1
WHEATSTONE BRIDGE CIRCUIT

Notation

$r_2 = 415$ ohms

$r_3 = r_4 = 600$ ohms

$r_T = 415$ ohms

$C = 5691$ temperature units from experimental data

$E = 3.6$ volts

$T =$ °Rankine

For a thermistor

$$\frac{dr_T}{dT} = \frac{-C\,r_T}{T^2}$$ from thermistor calibration data. see Fig. F.5

but $$mv = \frac{E\,\Delta r}{r_2 + r_4} \qquad (3.1)$$

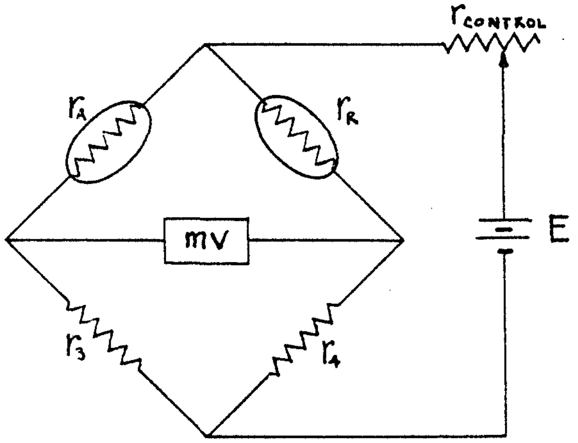
then $$\frac{mv}{\Delta T} = \frac{E}{r_2 + r_4} \cdot \frac{-C\,r_T}{T^2} = 21.3 \text{ millivolts/°F.}$$

or $$\frac{\Delta T}{mv} = .047\text{°F/millivolt.}$$

APPENDIX F

EFFECT OF VOLTAGE SUPPLY CHANGE

Consider what happens when r_A changes in resistance due to tracer gas in the active cell. Consider the circuit shown below.



r_A = active thermistor

r_R = reference thermistor

r_3 & r_4 = fixed resistors

mV= signal volts

E = volts

FIGURE F·1 WHEATSTONE BRIDGE CIRCUIT

The bridge will initially be balanced, $r_A = r_R$. As soon as helium tracer is introduced to the active cell the thermal conductivity will increase causing more heat to be dissipated from the hot thermistor. This will cool the thermistor causing its resistance to increase. This increase in resistance will reduce the current to the bridge causing a reduction in the voltage drop across the control resistance, and hence an increase in voltage drop across the bridge. This increase in voltage across the bridge will cause the reference thermistor to change in value. An attempt will be made to estimate this change in resistance of the reference thermistor. This will be done by attempting to estimate

1. Change in resistance of r_A due to tracer gas.
2. Change in voltage drop across the control resistance due to the change in 1.
3. Effect on r_R

Consider the active thermistor in its cell as shown in Fig. F.2.

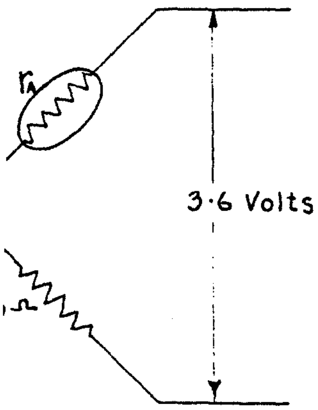


FIGURE F.2
SERIES CIRCUIT

We know that

$$q = I^2 r_A$$

$$r_A = f(T)$$

$$q = f(K, \Delta T)$$

$$\Delta T = f(T)$$

Where

q =	heat flow	K =	thermal conductivity
I =	current	r =	resistance
d =	diameter	T =	temperature °R

Then $r_A = f(K)$

Let us solve this equation.

Consider $q = f(K, \Delta T)$ first.

Heat will be transferred from the thermistor to the cell block by

1. Conduction
2. Convection
3. Radiation

To estimate the heat loss by conduction from the thermistor bead to the cylinder wall, let us assume as a first approximation a sphere within a sphere. This is a fairly good approximation because the ratio of the diameters is approximately 35.

Then $$q_1 = 2\pi K \frac{(T_2 - T_1)}{(d_2 - d_1)} d_1 d_2 \qquad \text{(F.1)}$$

$$2147\ K(T_2 - T_1) \text{ microwatts} \qquad \text{(F.2)}$$

To estimate the heat loss by conduction from the lead wires assume these to be heated at one end and contained within a concentric cylinder, see Fig. F.4.

Then $\Delta q_{in} = \Delta q_c + \Delta q_{out}$

$$K_m A\left(\frac{dT}{dx}\right)_x = \frac{2\pi K_g \Delta x}{\ln\frac{d_2}{d_1}}(T_2 - T_x) + K_m A\left(\frac{dT}{dx}\right)_{x+\Delta x}$$

$$\frac{2\pi K_g (T_2 - T_x)}{\ln\frac{d_2}{d_1} K_m A} = \frac{d^2T}{dx^2}$$

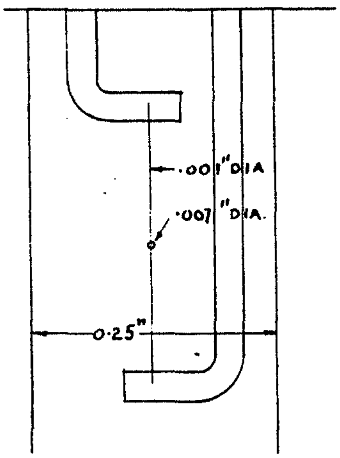


FIGURE F.3
THERMISTOR CELL

The solution of this equation is similar to equation 3.23 in Giedt[1] p. 46.

$$q_2 = K_m A\sqrt{\frac{2\pi K_g}{\ln\frac{d_2}{d_1} K_m A}}\,(T_2 - T_1) \qquad \text{(F.3)}$$

$$q_2 = 293.0\sqrt{K_g}\,(T_2 - T_1) \text{ microwatts} \qquad \text{(F.4)}$$

Where subscripts g and m refer to gas and metal respectively.

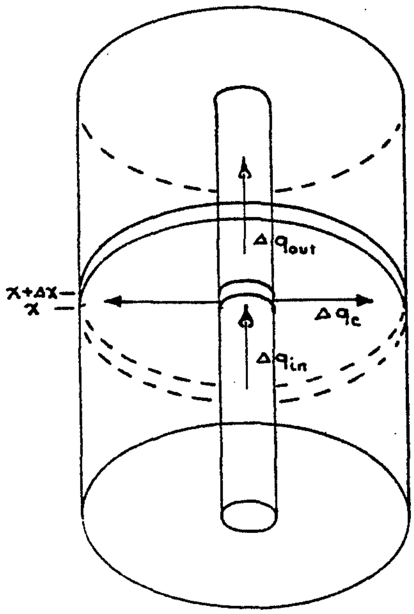


FIGURE F.4
CONDUCTION OF HEAT IN CONCENTRIC CYLINDERS

To estimate the heat loss by convection from the sphere

$$q_3 = h_c A (T_2 - T_1)$$

but $h_c = .24\left(\frac{\theta}{d}\right)^{.25}$ Giedt[1] p. 218 table 10.1

$$q_3 = 1.60 (T_2 - T_1) \text{ microwatts} \qquad \text{(F.5)}$$

[1] W. H. Giedt, Principles of Engineering Heat Transfer, D. Van Nostrand Co. Inc.

Consider the heat loss by radiation from the sphere.

Assume $e_s = \cdot 80$

$$h_r = \cdot 1714\, e_s \left[\frac{\left(\frac{T_s}{100}\right)^4 - \left(\frac{T_R}{100}\right)^4}{T_s - T_R} \right]$$

$$q_4 = h_r A (T_2 - T_1) = 0 \cdot 32 (T_2 - T_1) \qquad (F.6)$$

Since q_3 and q_4 are small, we will combine these making the assumption that heat loss from the wire by convection and radiation is proportional to the heat loss by conduction in q_1 and q_2. That is,

$$\frac{q_{5+6}}{q_{3+4}} = \frac{q_2}{q_1} \qquad q_{3+4} = 1 \cdot 92 (T_2 - T_1) \qquad (F.7)$$

$$q_{5+6} \doteq 2 \cdot 16 (T_2 - T_1) \qquad (F.8)$$

Consider the case with pure air where K = .0148, then

$$q_1 = 31 \cdot 8 \; (T_2 - T_1) \qquad (F.2)$$

$$q_2 = 35 \cdot 6 \; (T_2 - T_1) \qquad (F.4)$$

$$q_{3+4} = 1 \cdot 92 (T_2 - T_1) \qquad (F.7)$$

$$q_{5+6} = 2 \cdot 16 \; (T_2 - T_1) \qquad (F.8)$$

$$q_{TOTAL} \doteq 71 \cdot 5 \quad (T_2 - T_1) \text{ microwatts} \qquad (F.9)$$

From Fig. 15 the actual input would be 5.21 milliwatts and for these conditions ΔT will be approximately 96°F since the wall temperature will be 70°F and the thermistor temperature is given from the resistance temperature curve, Fig. F.5

watts

$$q_{TOTAL} = 71.5\,(T_2 - T_1) = \frac{71.5\,(96)}{1000} = 6.86 \text{ millivolts}$$

Obviously since this is greater than the actual input, 5.21 millivolts, our analysis as to the geometry of the heat flow was incorrect. Suppose we assume the heat flow from the thermistor bead is to part of a sphere instead of a whole sphere. It will be noticed that a large part of the heat loss from the thermistor is by conduction from the lead wires. In our first approximation for q_1 no allowance was made for other conducting surfaces in the sphere considered. In the actual problem the lead wires are hot and do affect the heat loss from the ends of the bead from which they project. We also made the assumption that the heat loss from the wires was not affected by other heat sources. This is not the case. A prominent heat source at the end of the wire will have less effect than is the case with concentric spheres.

watts

For simplicity we will arbitrarily reduce the heat loss from the wire by 10% and assume the remainder of the heat loss change is from the sphere.

Then equation (F.9) becomes

$$q_T = \left[(33.7)\,x + 0.9\,(37.9)\right](T_2 - T_1) = 5.210 \text{ microwatts}$$

Where $x = 0.598$ is the reducing factor for q_1 and q_{3+4}, for making q_T equal to the measured value.

Substituting the corrected values in q_1 through q_6

$$q_T = \left[1284\,K + 263.9\sqrt{K} + 3.09\right](T_2 - T_1) \qquad \text{(F.10)}$$

But $q_T = i_A^2\, r_A$

Consider the case where E_B is constant

$$i_A = \frac{E_B}{(r_A + 600)}$$

but $$q = i_A^2 r_A$$

then $$q = \frac{E_B^2}{(r_A + 600)^2} r_A \qquad (F.11)$$

but $r_A = f(T)$; this is known from Fig. F.5

Then q can be calculated for various values of T and hence ΔT if T_1 is fixed.

For various values of ΔT, q has been calculated using equation F.11,

ΔT	q	$q/\Delta T$
90	5295.4	58.837
91	5284.2	58.068
92	5272.6	57.311
93	5260.4	56.564
94	5247.8	55.828
95	5234.8	55.103
96	5221.2	54.388
97	5207.2	53.683

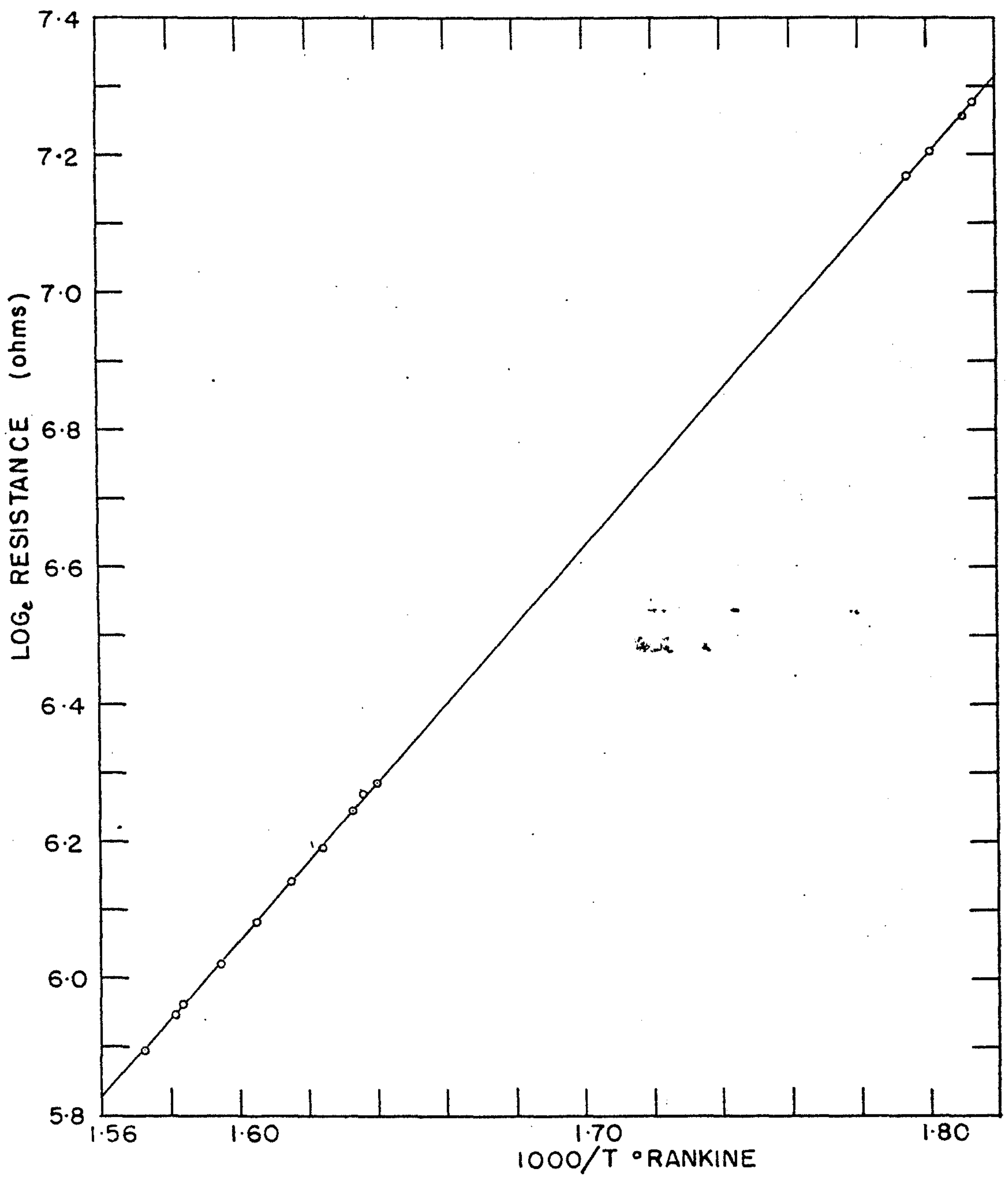


FIGURE F·5
LOG_e RESISTANCE VERSUS RECIPROCAL TEMPERATURE FOR THERMISTOR G126 #372-322

The use of values of $\frac{q}{\Delta T}$ from the above table were used in equation F.10 with results shown below:

ΔT	K
90	.016791
91	.016458
92	.016132
93	.015812
94	.015497
95	.015189
96	.014886
97	.014590

These have been plotted in Fig. F.6

Now

$$i_1 = \frac{E_B}{r_B} = 7.10372$$

$$i_2 = \frac{E}{r_{control} + r_{B_2}} = \frac{12.8}{1295 + r_{B_2}} \qquad \text{(F.12)}$$

To find the resistance of the bridge with 1% helium in the active cell,

$K_2 =$	.01555	from Fig. 6B
$\Delta T =$	93.824	from Fig. F.6
$r_{A_2} =$	428.617	from Fig. F.5

then bridge resistance is

$$r_B = \frac{(r_A + 600)(r_R + 600)}{(r_A + 600) + (r_R + 600)} = 510.388$$

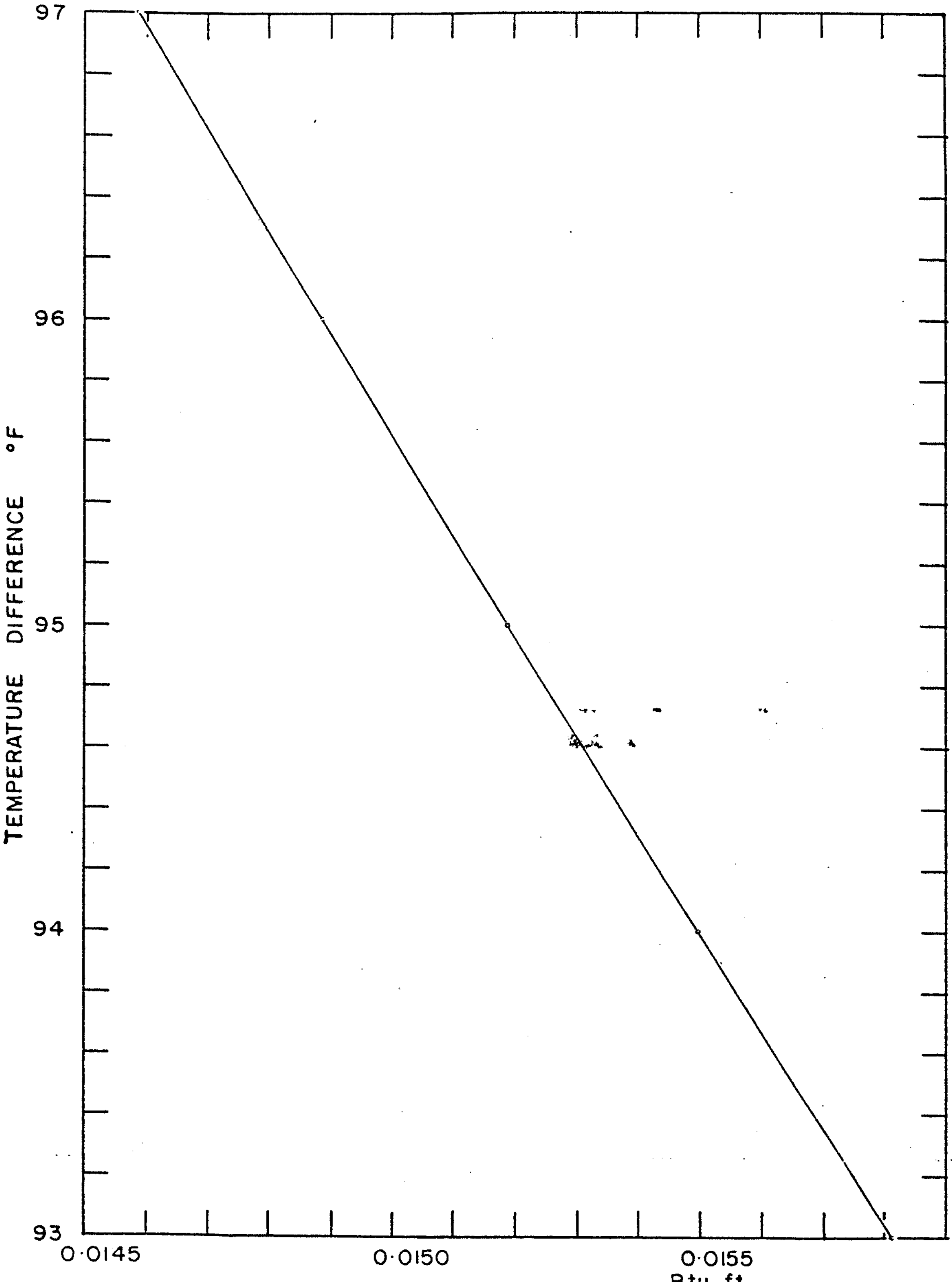


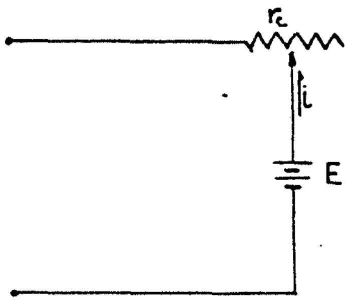
FIGURE F·6

TEMPERATURE DIFFERENCE VERSUS THERMAL CONDUCTIVITY OF GAS IN CELL

Then $i_2 = 7.08955$

$\Delta i = -.01417$

Consider the external circuit in Fig. F.7.



$$E = i\, r_{control}$$

$$dE_B = i\,\cancel{\partial r_c}^{\,0} + r_c\,\partial i$$

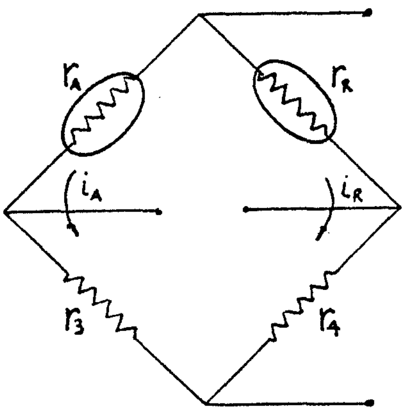
$$dE_B = r_c\, di$$

$$\Delta E_B = \frac{1295\,(-0.01417)}{1000}$$

$$= -0.1835 \text{ volts} \qquad (F.13)$$

FIGURE F.7
EXTERNAL CIRCUIT

Consider the internal circuit in Fig. F.8.



For r_R and r_4

$$E_B = i_R r = 600\, i_R + 1.675 - 60\, i_R$$

$$= 540\, i_R + 1.675$$

$$dE_B = 540\, di_R \qquad (F.14)$$

For the thermistor

$$r_R = \frac{E_T}{i_A} = \frac{1.675 - 60\, i_R}{i_R}$$

$$dr_R = \frac{-1.675}{i_R^2}\, di_R$$

FIGURE F.8
INTERNAL CIRCUIT

$$dr_R = \frac{-1.675}{i_R^2}\,\frac{dE_B}{540} \qquad (F.15)$$

$$\Delta r_R = \frac{-1.675\,(-1)(-.1835)}{(.0035518)^2\; 540} = -4.5 \text{ ohms}$$

APPENDIX G

SENSITIVITY OF VOLTAGE CONTROL

Consider the E I curve (Fig. G.3) of the thermistor. At its point of operation the curve may be approximated by a straight line since this is an inflection point on the curve. The equation of a straight line that closely fits the curve is $E_T = 1{\cdot}675 - 60i$ between 3.5 and 4.0 milliamperes.

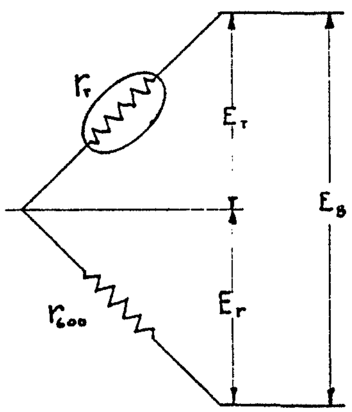


FIGURE G·1
SERIES CIRCUIT

Consider the thermistor in series with ballast resistor, Fig. G.1.

$$E_B = E_T + E_R$$

$$= 1{\cdot}675 - 60i + 600i = 1{\cdot}675 + 540i$$

$$r = \frac{E}{i} = \frac{E_B(540)}{(E_B - 1{\cdot}675)}$$

$$\frac{dr}{dE} = \frac{-540(1{\cdot}675)}{(E_B - 1{\cdot}675)^2}$$

But since $d\,r_{600} = 0$ all this dr is due to change in r_T of the thermistor, that is

$$\frac{d\,r_T}{d\,E_B} = \frac{-904{\cdot}5}{(E_B - 1{\cdot}675)^2} \qquad (G.1)$$

Consider the bridge circuit, Fig. G.2.

$$mV = E_B\left[\frac{\Delta r}{r_2 + r_4}\right] \qquad (3.2)$$

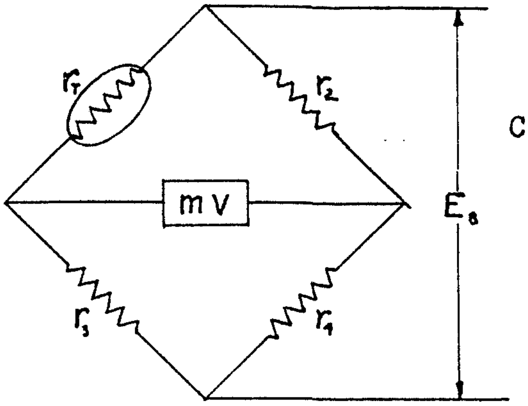


FIGURE G·2
WHEATSTONE BRIDGE CIRCUIT

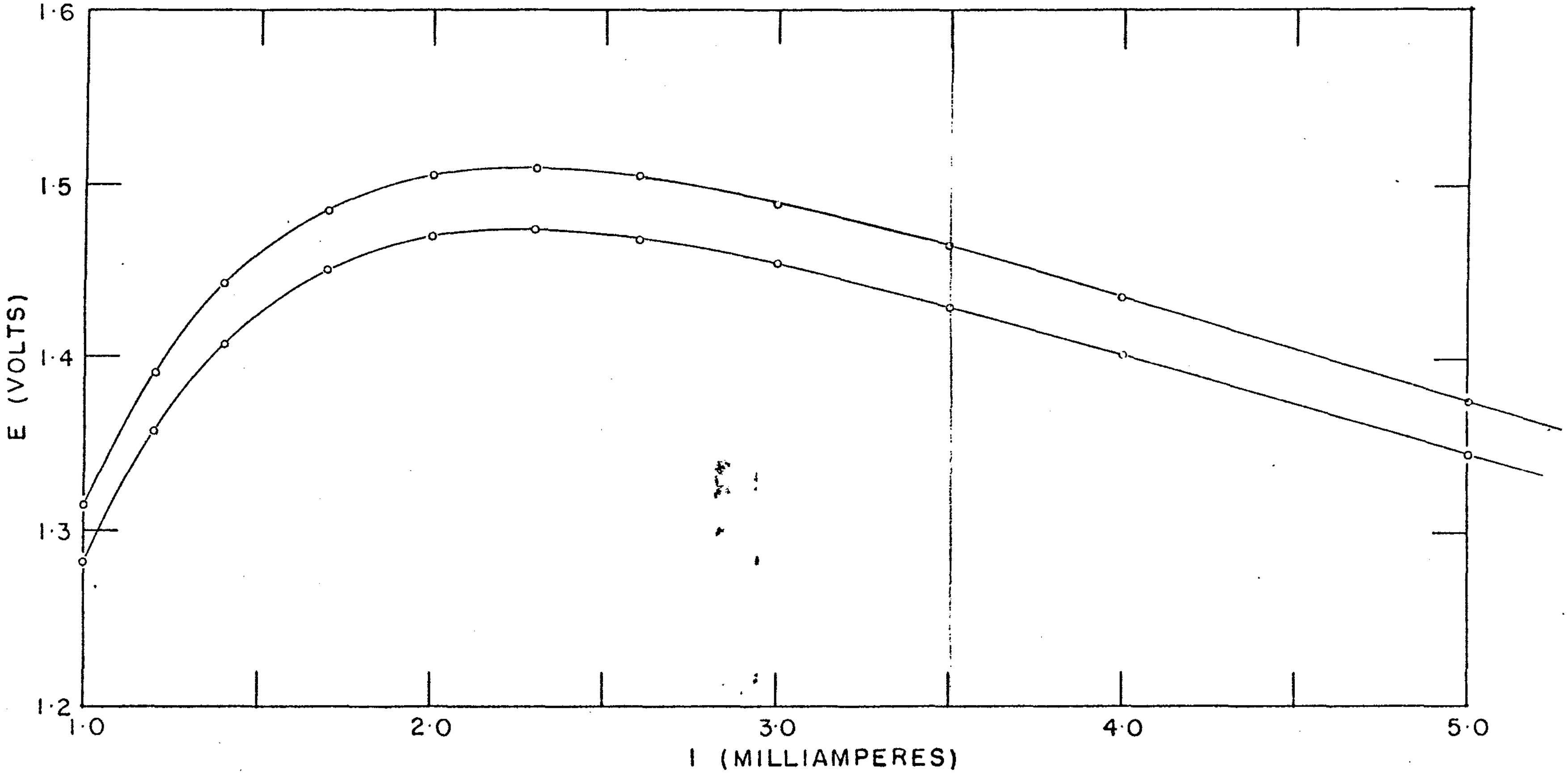


FIGURE G·3

E-I CURVE FOR MATCHED PAIR OF THERMISTORS G126 #372

The output is the input to an amplifier whose sensitivity is at least $\pm$ 7 microvolts. That is, it will drive a balancing motor when the input signal is less than 7 microvolts.

$$mv = E_8\left[\frac{\Delta r}{r_2 + r_4}\right] = \pm 7 \times 10^{-6} \text{ volts}$$

but $$\Delta r = \frac{-904.5 \;\Delta E_8}{(E_8 - 1.675)^2}$$

$$\Delta E_8 = \frac{(E_8 - 1.675)^2\,(r_2 + r_4)}{-904.5 \quad E_8} \text{ mv} \tag{G.2}$$

$\Delta E_8 = \mp 8.08$ microvolts when

$E_8 = 3.6$ volts $\quad r_2 = 415$ ohms $\quad r_4 = 600$ ohms

That is; voltage control is $\pm$2.25 microvolts/volt.