Crack flow equations and scale effect

by D. W. Etheridge

1. INTRODUCTION

A major source of ventilation in many dwellings is that arising from air flow through cracks. To date, much use has been made (e.g. see references [1, 2]) of equations of the form

$$\Delta p = \text{Constant} \cdot \nu^n$$

for describing crack flows. In this equation, $\Delta p$ is the pressure drop across the crack, $\nu$ is the volume flow rate through the crack and the exponent has a value of about 1.6. Equations of this type lack generality because they are not dimensionally homogenous. That is, they are in conflict with a fundamental law of fluid mechanics — Reynolds law of similitude.

From an earlier laboratory investigation of crack flows [1], improved semi-empirical equations have been derived. A method of applying them was proposed, but it has been found to have limitations and the present article is an attempt to provide an improved method.

Essentially, crack flow equations are required for two purposes.

(i) For use in a prediction method for investigating the effects of dwelling configuration, mechanical systems and external wind on ventilation rates.

(ii) For estimating the open areas of room components (e.g. doors, windows) when direct measurement is not possible.

The use of the equations in the above two ways and the implications of the equations with regard to the use of model-scale measurements of ventilation rates are discussed and some recent experimental results are presented.

2. DERIVATION OF EMPIRICAL EQUATIONS

Dimensional analysis indicates that for cracks with exact geometric similarity, the flow can be described by the functional relationship (see Appendix 1)

$$C_z = f(R_s)$$

where $C_z$ is the discharge coefficient

$$C_z = \frac{\nu^2}{A \sqrt{2\Delta p}}$$

and the Reynolds number, $R_s$, is

$$R_s = \frac{\bar{u}d_A}{v}$$

In the above equations, $\rho$ and $v$ are the air density and kinematic viscosity respectively and $\bar{u}$ is a mean velocity defined by

$$\bar{u} = \frac{\nu}{A}$$

The symbol $A$ denotes an area of the crack corresponding to some specified cross-section and $d_A$ is a typical dimension defined by

$$d_A = \frac{44}{\text{wetted perimeter}}$$

For most cracks likely to be encountered in ventilation work

$$d_A = 2y$$

where $y$ is the thickness of the crack as illustrated in Figure 1.

In practice, cracks take a variety of shapes and this implies a large number of distinct functions $f$. By introducing a geometric parameter one would hope to reduce this number to a manageable value. By analogy with pipe flows the ratio $z/d_A$ suggests itself as a parameter, where $z$ is the distance through the crack, as illustrated in Figure 1. Hence the crack flow equation is assumed to take the form

$$C_z = f\left(\frac{R_s}{d_A}, \frac{z}{d_A}\right)$$

The feasibility of using such an equation has been investigated [1] by carrying out tests on a large number of simulated cracks for ranges of $R_s$ and $z/d_A$ which are likely to be encountered in practice. The cracks were divided into three basic types — straight-through, L-shaped and double-bend (see Figure 1). For each type of crack it was found that the experimental results could be reasonably correlated by

$$\frac{1}{C_z^2} = B\frac{z}{d_A} \frac{1}{R_s} + C$$

Figures 2 (a—c) show the experimental results and the empirical values of the constants $B$ and $C$ chosen for the three crack types.

Equation (9) can be derived by equating the pressure drop across the crack to the sum of the losses due to skin friction and end effects (see Appendix II). In the above form, the skin friction contribution corresponds to laminar flow. The equivalent equation for turbulent flow is

$$\frac{1}{C_t^2} = B\frac{z}{d_A} \frac{1}{R_s^{0.23}} + C$$
and the results for the double bend cracks are plotted to Figure 3 in the form of a log-log plot. Comparing this with Figure 2 it is clear that equation (8) is more appropriate than equation (10) for correlating the experimental data.

In reference [1], the set of equation (8) in combination with equation (11) is proposed. However, the introduction of (11) is inconsistent and unnecessary, because the flow is completely and properly described by (9) alone and it is therefore now proposed that it should be used alone.

Unfortunately, equation (8) is a simplified representation of a complex flow situation. Bearing in mind the wide variety of crack types which are met in practice, however, it seems that some degree of simplification is unavoidable. Indeed, it is impossible to find a component for which the crack type and the crack dimensions are everywhere constant.

Despite these problems it will be seen from Section 3.2 that equation (8) has been found to be successful both for estimating the open areas of real full-scale components and for describing the flow through them.

The advantages which equation (8) has relative to equation (11) are several. Firstly, it is dimensionally homogeneous so that the values of the constants do not vary with the particular system of units used. Secondly, it takes into account the effect of Reynolds number i.e. scale effect which is an important effect for small scale models. As a result of this, the estimated values of open area are independent of the flow rate through the open area. This also means that when an open area can in fact be obtained by direct measurement the value can be used directly in the equation. Equation (8) is thus claimed to have wider validity than equation (11).

3. USE OF CRACK FLOW EQUATIONS

3.1 Prediction of ventilation rates

The crack flow equation (8) has been incorporated into a computer program for predicting the flow rates through cracks in multi-unit dwellings. An iterative procedure is used to solve the crack flow and continuity equations.

In practice, the results obtained from this program in the form of a plot of the total air change rate of an actual one-family house against reference wind speed, \( V \). The open areas of this house have been estimated experimentally as described in Section 3.2. For these calculations the pressure distribution over the house was estimated, but for future calculations a scale model has been constructed to enable more accurate distributions to be obtained in a wind tunnel. The predictions will be compared with ventilation measurements made in the house in order that the prediction procedures can be tested.

Also shown in Figure 4 are the corresponding results obtained assuming that the discharge coefficient of the open area is independent of Reynolds number and crack type. It can be seen that this simple square-law approximation gives significantly higher air change rates than the crack flow equations.

3.2 Indirect measurement of open area

For the prediction of ventilation rates for a full-scale dwelling it is necessary to know the open areas of each room component.

In most cases a direct measurement of the full-scale area is not possible and an indirect method is employed. The pressure drop \( \Delta P \) across the component (at some points on the component) is measured, together with the corresponding volume flow rate. It can be shown that the flow equation can be written in the form of a simple cubic

\[
\frac{\Delta P}{\rho V^2} \cdot \frac{A}{S} = \frac{m}{2} \cdot \frac{1}{L^2}
\]

which enables it to be calculated knowing \( \Delta P \), the crack type which determines \( m \) and \( L \), where \( L \) is the length of the crack. Figures 5 and 6 show the solutions of this equation for the two types of cracks. Hence, knowing the above quantities from full-scale tests it is possible to obtain estimates of open areas.

In the following the results of tests on a door in the house which were carried out to verify the above procedure are described. These tests also constitute a check on the validity of equation (8).

The tests were carried out in the small bedroom of the house. Prior to the tests, wind-coupling was laid and the room was extensively sealed with adhesive tape. An air flow plate was used to measure the extraction rate of the air which was exhausted through the small wind-coupling. The static pressure difference across the door was measured with a microanometer.

The dimensions of the door are shown in Figure 8. It should be noted that the quoted dimensions of the door thickness, \( T \), are only approximate since there was some variation of this quantity over the door.

Three separate \( T \), \( \Delta P \) characteristics were recorded. First, the characteristic corresponding to the whole door was obtained. The lower gap of the door was then sealed and the second characteristic recorded. Finally, the door was completely sealed and the base level characteristics were obtained.

The base level characteristic was used to correct the other characteristics. That is, at a given value of \( \Delta P \), denote the base level test with the door unsealed by \( I_1 \), and the corresponding test with the door sealed by \( I_2 \). The flow rate through the door at this value of \( \Delta P \) is then given approximately by

\[
\frac{I_2}{I_1} = \frac{1}{(1 - \frac{T}{L})}
\]

Figure 8 was then used to estimate the open area of the component from each measured point on the \( T \), \( \Delta P \) characteristic. These estimates are given in Figure 8. As also shown here are the estimates of \( T \), which are obtained from the \( T \), \( \Delta P \) characteristic on the assumption that \( L = L_0 \). It is clear that these latter estimates are far from satisfactory. The estimated open area for the two doors configured in this way were

<table>
<thead>
<tr>
<th>Area, ( \Delta P )</th>
<th>From crack flow equation</th>
<th>From direct measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>White door</td>
<td>0.0167</td>
<td>0.0071</td>
</tr>
<tr>
<td>Lower gap sealed</td>
<td>0.0086</td>
<td>0.0070</td>
</tr>
<tr>
<td>Difference</td>
<td>0.0077</td>
<td>0.0076</td>
</tr>
</tbody>
</table>


Figure 2
Experimental results and empirical lines chosen for the three crack types. (a) straight-through, (b) L-shaped, (c) double-bend

Figure 3
Experimental results for double-bend cracks plotted in accordance with turbulent flow

Figure 4
Calculated effect of wind speed on air change rate of a test house

Figure 5
Solution of crack flow equation for straight-through cracks
All of the doors and windows of the house have had their open area estimated in the above manner and the results obtained are qualitatively similar to those given in Figure 9. This means that the flow through these components can be accurately calculated knowing the pressure difference as well as the components. When using the above technique for obtaining leakage data of houses one is often faced with room components (doors, windows) for which the crack type varies. One therefore has the choice of either dividing each component into smaller parts according to crack type, or treating the component as a whole. If the first choice is adopted, the open area of each part can be measured by the pressure/leakage technique (by ignoring the other parts) and the total open area is then obtained by summing the individual open areas. If it were desired to use all open areas for estimating the results from the test on the door, it would not generally be equal to the sum of the individual parts. However when the component consists of straight through and L-shaped cracks for which the flow equations do not differ greatly, the results from the tests on the door indicate that the effective open area is approximately equal to the total open area.

An example of the use of the flow equations for estimating open areas is given in Figure 10. Here the...
greatly with flow rate, indicates that the crack flow equations for estimating background area were arbitrarily chosen. Nevertheless, the fact that the estimated values of background area do not vary greatly with flow rate, indicates that the crack flow equations can be usefully employed to describe very complex surface phenomena. The relative importance of the effects of Reynolds number and turbulence has been investigated experimentally in a wind tunnel of the type described in reference [6]. A grid of horizontal slits was used to generate the simulated atmospheric boundary layer. Figure 12 shows the resultant profiles of mean velocity, \( U_{rms} \) and turbulence intensity, \( C_{rms} \). These measurements were made with a pitot-static tube which was considered adequate for the present purposes. The intention was to obtain profiles appropriate to an open-country site, and the corresponding power law for the mean velocity is shown for comparison. A single cell model fitted with identical model windows on two opposite faces, as shown in Figure 13, was chosen for the tests. Ventilation rates were obtained from tracer gas decay records measured by a katharometer inserted inside the model. A full report on the tests will be published at a later date, but some relevant results are presented in Figure 14. The measured variation of \( \frac{V}{U_{rms}} \) with \( U_{rms} \) is shown for two wind directions \( \alpha = 0^\circ \) and \( 90^\circ \). At \( \alpha = 0^\circ \) the mean pressure difference across the windows is at a maximum, whereas at \( \alpha = 90^\circ \) the mean pressure difference is negligibly small and the ventilation rate is due to turbulence effects alone. The \( U_{rms} \) axis can be considered as a Reynolds number axis.

4. USE OF SCALE MODELS FOR DETERMINING VENTILATION RATES

Ventilation rates can be determined from model tests in wind tunnels either by measuring the external pressure distribution and using this data for a theoretical prediction or by measurement of ventilation rates in the model. The use of a model for obtaining external pressure distributions for the purposes of design, development and/or research is an established procedure. The problem is whether or not the ventilation rates are best determined theoretically or by measurement at model scale.

A strong argument in favour of model scale measurement is that the established theory does not take account of the effects of wind turbulence and internal air movements. Both of these phenomena are at least present in model tests and there is evidence (e.g. reference [13, 4]) that the former effect can be very significant.

There are however quite strong arguments against the use of model scale measurements. Firstly, it is difficult to model full-scale cracks accurately unless these are large or the model scale is large (1/25 scale, say). Secondly, it is generally not possible to achieve full-scale Reynolds number at model scale because of limitations on tunnel speed. Since the flow through cracks tends to be laminar, Reynolds number effects are likely to be significant. Figure 11 shows the theoretical variation of flow rate, \( V / U_{rms} \), through typical windows with reference wind speed. It can be seen that even with a large crack thickness of 6 mm, Reynolds number effects are significant up to a speed of about 5 m/s. To achieve the same Reynolds number with a 1/25 scale model would require tunnel speeds up to 125 m/s. For the smaller crack sizes, wind tunnel simulation over the required Reynolds number range appears to be even less feasible.
9. CONCLUSIONS

Crack flow equations of the form of equation (1) are not satisfactory because they do not satisfy Reynolds law of similitude. Earlier investigations (reference [1]) have shown that the flow through simulated full-scale cracks can be adequately described by the semi-empirical equation (8). It is now confirmed that this equation should not be used in the manner described in reference [1], but should be used on its own for obtaining the discharge coefficient.

Full-scale tests on a door in a house have been found to support the use of equation (9) for estimating open areas of real full-scale rooms components. It has also been found from tests of all the room components that the equation satisfactorily describes the flow through a wide range of components.

The usefulness of ventilation rates measured at model scale for the design of full-scale dwellings is seen to doubt, because of the large effect of scale implied by the crack flow equations. Calculations indicate that scale effect is apparent even at the Reynolds numbers associated with full-scale dwellings.

Tests carried out on a model in a wind tunnel have illustrated the significance of scale effect when the ventilation rate is due mainly to the existence of a time-mean pressure difference across a crack. When the time-mean pressure difference is zero, and the ventilation rate is due solely to turbulent pressure fluctuations, the scale effect is much smaller.* At the lowest Reynolds numbers tested, the ventilation rate due to turbulence alone is about one half of the ventilation rate which occurs when the time-mean pressure difference is maximised.

The model tests have also illustrated the different flow characteristics of scale-model cracks and circular holes. Circular holes can be used to model the behaviour of real cracks in the limit of high Reynolds number, provided that the open area of the holes are correctly determined.

ACKNOWLEDGEMENTS

The author wishes to thank Mr. P. Phillips who carried out some of the calculations presented here and Mr. J. Nolan who carried out the wind tunnel tests. The permission of British Gas Corporation to publish the work is gratefully acknowledged.

REFERENCES


*See Civil, and Env., 14, 53–64 (1979) for further discussion of this conclusion.
Appendix I

The dimensionless group which describe the flow through geometrically similar pipes can be obtained by dimensional analysis in the manner shown for example in reference [7]. One obtains

\[ \frac{d}{D} = \frac{C}{C} \left( \frac{L}{L} \right) \]

where \( D \) is the pipe diameter and \( C \) is the corresponding dimensionless number. The analysis for flow through geometrically similar cracks is the same, and one obtains

\[ \frac{d}{D} = \frac{C}{C} \left( \frac{L}{L} \right) \]

which, on putting \( C = \frac{V}{V} \), can be expressed in a form involving the discharge coefficient, i.e.

\[ C = \frac{V}{V} \]

or

\[ C = \frac{V}{V} \]

Appendix II

The total pressure drop across the crack can be considered as the sum of pressure drops due to skin friction (all the form encountered in long straight cracks) and due to bends and end effects. This is the procedure which is generally adopted for pipe flow (e.g. see reference [7]). It is assumed that the pressure drops caused by bends and end effects are simply proportional to \( \nu \).

In the following, equation (38) is derived using the above procedure, in a similar way to that given in reference (11).

\[ \text{Fig. 12} \]

For steady laminar flow between parallel walls, the velocity distribution is parabolic (see e.g. reference [8]), i.e.

\[ u(\eta) = \frac{4}{\pi} \left( \frac{1}{2} + \eta^2 \right) \]

In the present notation \( u \) and \( \eta \) denote distance in the crosswise and streamwise directions, respectively, with the origin at the centre line. The crack has breadth \( b \) and length \( l \), \( v(\eta) \) denotes the streamwise velocity at \( \eta \) and \( \nu \) is the volume flow rate per unit width.

On integration one obtains

\[ \frac{1}{C} \int \eta \, \mathrm{d} \eta = \frac{1}{C} \frac{V}{V} \]

and since

\[ C \eta = \frac{1}{C} \frac{V}{V} \]

we can express (38) in the form

\[ \frac{1}{C} = \frac{1}{C} \frac{V}{V} \]

Proceeding in the manner described above, the total pressure drop, \( \Delta P \), is given by

\[ \Delta P = \frac{1}{C} \frac{V}{V} \frac{C}{C} \]

where \( \Delta P \) is the discharge coefficient. Thus the flow through the crack is described by

\[ \frac{1}{C} = \frac{1}{C} \frac{V}{V} \]

which is the required form.

It is interesting to note that the empirical values of \( C \) obtained for the straight through and L-shaped cracks are close to the value 6. This supports the assumption that the pressure drop can be considered as two additive components. The value of \( C \) obtained for the double-bend crack type, indicates that the assumption becomes less reliable as the number of bends is increased. Nevertheless the flow is still adequately described by an equation of this form.