Air movement in houses : a new approach

by C.H. Saunders

Air movement in houses is now recognised as a major factor both in condensation risk and in energy efficiency, yet conventional test methods have limitations. Using a multi-channel gas analyser and computer operation, the BRE Scottish Laboratory has extended conventional techniques of testing and of measurement to give a more realistic assessment of the processes at work, as explained here. Field studies have verified the importance of blocking gaps in the ceiling, for 20-30 per cent of the air entering a house will leave via the roof space through these gaps, which are critical for moisture transfer but less important for heat transfer.

The measurement and control of air movement into houses and between different parts of a house is becoming increasingly important for the control of condensation and studies of energy conservation.

Air movement carries water vapour produced in the kitchen and bathroom, which are relatively warm, to other colder parts of the house where condensation can occur, particularly in unheated bedrooms and the roof space. The relative benefits and disadvantages of measures such as the use of extract fans, which remove both heat and water vapour from the house, or blocking gaps in the ceiling, which makes the roof space both colder and drier, can be assessed only if the pattern of air movement within the house and roof is known.

Tracer gases have been used to measure ventilation and air movement rates for many years, the basic techniques being outlined by Dick (réf. 1). However in recent years the advent of digital computers which allow the numerical solution of the differential equations governing tracer concentrations has allowed new measurement techniques to be introduced.

The conventional technique for measuring ventilation rates in houses is to introduce a tracer gas and record the concentration during the subsequent exponential decay. All internal doors are left open and fans mix the air in the house continuously during the measuring period so that the house may be regarded as a single volume with a uniform tracer gas concentration throughout. This method has its limitations since, under normal conditions of occupancy, some of the internal doors in a house will usually be shut, thus restricting the air flow round the house and affecting the ventilation rate in the house as a whole. Results obtained with all internal doors open may therefore be The differential equation governing the rate of change

unrepresentative of realistic ventilation rates in houses. Honma (ref. 2) has developed equations governing the concentration of a tracer gas in several interconnecting rooms for known injection rates into each space. In order to obtain the air flows between rooms it is necessary to solve the equations using a process of successive approximations, which implies the use of a computer.

In the specific cases of two interconnecting rooms or of the dwelling space and roof space of a house it is possible to solve explicitly the differential equations governing the tracer gas concentrations. The paper outlines a least-squares technique for determining the air flow between rooms. The paper also explains a technique which allows the measurement of tracer gas concentrations in several rooms simultaneously. These may then be combined to give an overall house ventilation rate, or analysed independently to give information on air flow between different parts of the house.

Equally the conventional method can be impractical in highly ventilated enclosures, such as the roofs of old houses, as it can be difficult to introduce sufficient tracer gas to obtain a reasonable decay curve and obtain reproductible results. However these difficulties may be overcome by continuously injecting tracer gas at a measured rate and observing the rate of rise of concentration in the enclosure. The differential equation governing the tracer gas concentration may then be solved to give an expression for the ventilation rate in terms of the final steady state concentration... However, more satisfactory results may be obtained if the differential equation is expressed in finite difference terms and analysed numerically.

The paper also shows how a multi-channel gas analyser can be used to measure tracer gas concentrations in the house and roof simultaneously, and this permits quantitative values of the air flow through the ceiling to be calculated using an extension of equations formulated by Dick. These values may be combined with measurements of the air pressure difference to obtain estimates of the area of gaps in the ceiling. Heat and moisture balances in the roof space are used to compare the contributions that air movement and conduction/diffusion make to the transfer of heat and water vapour between a house and its roof and to assess the benefits of blocking gaps in the ceiling.

Theory

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which is simply solved to give

 $c_1(t)$

$$= c_1(0)e^{-\frac{1}{v_1}}$$
 (Equation 2)

of the total amount of tracer in an enclosure in contact with a second enclosure is

$$\frac{d(v_1c_1)}{dt} = x_{21}c_2 - x_{12}c_1 \quad (Equation \ 1)$$

where v_1 is the volume of the enclosure, c_1 and c_2 are the tracer concentrations in the two enclosures, x_{21} and x_{12} are the air flow rates between the two enclosures. This equation may be simplified to apply to different measurement techniques.

a. The decay method

In this case the concentration in the room is raised to an initial high value and the outside concentration is assumed to be zero, so that

$$v_1 \frac{\mathrm{d} c_1}{\mathrm{d} t} = - x_{12} c_1$$

Figure 1

The ventilation rate of a pitched roof plotted against wind speed

Taux de ventilation d'un toit en pente en fonction de la vitesse du vent



the term x_{12} / v_1 in the exponent which has the dimensions of time⁻¹ is usually referred to as the ventilation rate in air changes per unit time.

b. Constant injection

In this case pure tracer is injected at a constant rate F m^3/s . To obtain the best possible mixing it is preferable to inject at as many points as possible. Assuming again that c_2 is zero, equation 1 simplifies to

$$\frac{d(\mathbf{v}_1\mathbf{c}_1)}{dt} = \mathbf{F} - \mathbf{x}_{12}\mathbf{c}_1 \qquad (Equation 3)$$

This may be solved in two different ways.

(1)
$$\frac{dc_1}{dt} + \frac{x_{12}c_1}{v_1} = \frac{F}{v_1}$$

gives

so

$$c_{1}(t) = \frac{F/v_{1}}{x_{12}/v_{1}} + ke^{-\frac{N_{12}}{v_{1}}t} = \frac{F}{R_{1}v_{1}} + ke^{-R_{1}}$$

where R_1 is the ventilation rate k is a constant, as

 $t \rightarrow \infty c_1(t) \rightarrow \frac{F}{R_1 v_1}$ $R_1 = \frac{F}{v_1 c_1(\infty)}$ (Equation 4)

In practice there are serious disadvantages to this method. Firstly, the equilibrium concentration will not be reached until several time constants R^{-1} , which means that a long time is required for each measurement. Secondly, in any case it is unusual for the concentration to remain constant due to fluctuations in the wind speed and non-perfect mixing, which means that it is often necessary to determine R_1 from values of $C(\infty)$, which can fluctuate by up to 20 per cent, leading to uncertainty in the ventilation rate calculated from equation 4. These difficulties may be avoided by a second method of analysis.

(2) Rearranging equation 3 gives

$$\frac{\mathbf{x}_{12}}{\mathbf{V}_1} = \mathbf{R}_1 = \frac{1}{\mathbf{C}_1} \left[\frac{\mathbf{F}}{\mathbf{V}_1} - \frac{\mathbf{d}\mathbf{C}_1}{\Delta t} \right]$$

or in finite difference terms

$$R_{1} = \frac{\perp}{C_{1}} \left[\frac{F}{V_{1}} - \frac{\Delta C_{1}}{\Delta t} \right]$$
 (Equation 5)

The tracer concentration is measured at 1 minute intervals and a ventilation rate calculated from (5) for each interval. These are then averaged to remove fluctuations due to the wind and mixing and a reliable ventilation rate found. The availability of a digital computer or programmable calculator simplifies the large amount of calculation that otherwise would be necessary.

Ventilation rates in a roof have been measured by both decay and constant injection techniques and the results are plotted against wind speed in figure 1. It can be seen that there is good agreement between the results from the two methods.

$$c_{r}(t) = c_{h}(0) \frac{x_{hr}}{v_{r}} \frac{1}{R_{r} - R_{n}} \left(c^{-R_{h}t} - c^{-R_{r}t} \right)$$
$$+ c_{r}(0) e^{-R_{r}t} \qquad (Equation 9)$$

where $R_h = \frac{x_{ho} + x_{hr}}{V_h} = \frac{x_{oh}}{V_h}$ is the ventilation rate

derived from decay of tracer in the house.

 $R_r = \frac{x_{ro}}{V_r}$ is the ventilation rate in the roof that would be derived from flooding the roof alone.

Dick assumes $c_r(o) = o$ and derives an expression for

c. Air flow between a house and its roof Measurements of the pressure difference across

ceilings show that under most wind conditions a house is at a higher pressure than the roof, the pressure difference being of the order of 1 N/m^2 . It may therefore be assumed that there is a unidirectional flow of air from the house to the roof with no recirculation downwards. This assumption will be reinforced by the fact that in heated dwellings stack effect will enhance the flow of air from dwelling to roof.

When a tracer gas is injected to a high concentration in a house, it is possible to detect a rise in tracer concentration in the roof. Subsequently the concentration will decay due to ventilation of the roof space. The computation of the flow from the house to the roof is based on equations developed by Dick using the symbolism in the diagram below.



Where $v = volume m^3$

c = tracer concentration m^3/m^3 x = flow rate of air m^3h^{-1}

and the subscripts h, r and o refer to the house, roof and outside respectively.

For the house, equation 1 simplifies to

$$v_h \frac{dc}{dt} = -(x_{ho} + x_{hr}) c_h$$
 (Equation 6)

and for the roof

$$v_r \frac{dc_r}{dt} = x_{hr}c_h - x_{ro}c_r$$
 (Equation 7)

These can be solved simultaneously, giving

$$c_h(t) = c_h(0) e^{-\kappa_h t}$$
 (Equation 8)

x_{hr} in terms of a maximum concentration reached in the roof. However, in practice it is not usually possible to start with the concentration in the roof equal to zero. There are two reasons for this. Firstly, it is necessary to allow 5-10 minutes for mixing of the tracer in the house before the run starts, during which time there will be a rise of concentration in the roof. Secondly, even with high ventilation rates it can take 2-3 hours for the roof concentration to fall to zero after the initial rise in concentration. It is necessary to record concentration for only 30 minutes to obtain consistent values for xhr and thus waiting for cr to become zero will be more time-consuming than necessary. Also, estimating the maximum value reached can be as uncertain as estimating the ultimate value when using the constant injection.

To overcome these disadvantages a method of calculating x_{hr} when the initial roof concentration is not zero has been developed.

Equation 9 can be expressed as $c_r(t) - A x_{hr} + B$

where A =
$$\frac{c_{h}(o) \left(e^{-R_{h}t} - e^{-R_{r}t}\right)}{v_{r} (R_{r} - R_{h})}$$
 B = $c_{r}(o) e^{-R_{r}t}$

The value of x_{hr} which gives the best fit to the observed concentration in the roof is found by least squares. The method of least squares is spelled out in Appendix A and x_{hr} is shown to be given by

$$x_{hr} = \frac{\Sigma (c_r(t) A) - \Sigma (AB)}{\Sigma A^2} \qquad (Equation 10)$$

The sums are taken over the run.

Measurement techniques

a. Design and use of a multichannel gas analyser As already noted, it is conventional to measure whole house ventilation rates by opening all internal doors and injecting a tracer gas which is sampled at a central point. In order to obtain good mixing, fans are kept running throughout the period of measurement. The new technique developed at BRE Scottish Laboratory measures the tracer concentration at six points around the house. The concentrations can then be combined to give overall house ventilation rates and used to estimate the exchanges between individual rooms.

An infrared gas analyser measures the concentration of nitrous oxide over one of two ranges, either 0-100 parts per million or 0-400 ppm. Air from 6 mm diameter polythene tube leading to one of the six

points being sampled is pumped through the analyser via a solenoid valve, opened by a signal from a multipoint recorder. After 36 seconds the recorder prints the value of the tracer gas concentration and operates the solenoid valves to begin sampling from the next point.

It is necessary to ensure that the gas analyser has stabilised before the tracer concentration is recorded and the valves switched to the next channel. There are two sources of delay in the system. Firstly, the inherent response-time of the analyser to a stepchange in concentration; this is about 20 seconds and little can be done to reduce it. Secondly, the time taken for the new sample to reach the analyser once the valves have changed. As it is necessary to have a low flow rate through the analyser to minimise the effect on the ventilation in the room under test (0.06 m³h⁻¹ is used, equivalent to an air change rate of 0.002 hr⁻¹ in a room of 30 m³) the air speed in the polythene tubes will be low $(0.6 \text{ m}^{-1} \text{ in this})$ case). Thus a delay of several seconds will be introduced given the tube lengths necessary to cover a typical house and longer in larger buildings.

To reduce this delay, throughout the sampling period a secondary pump draws air through the tubes leading to the other five sampling points, so that when the valves change to the next point only the small volume of air between the valves and analyser has to be flushed out with the new sample. The sampling interval of 36 seconds has been found to be sufficiently long to ensure stability of the analyser when used with the six points, such that each is sampled every 3.6 minutes. This is adequate at the low ventilation rates found in modern housing, but at higher ventilation rates too few values of the tracer gas concentration at each point may be recorded for analysis. For very high ventilation rates, the analyser may be switched to record on one channel continuously.

In a small house or flat it may be possible to sample from each room, but usually there will be more than six rooms in the house and the volume that each channel samples will not simply be a single room. Care has to be taken in the siting of the sampling tubes and subsequent analysis to ensure optimum coverage of the house. The exact positions adopted will depend on the nature of the information required. For example, condensation studies may concentrate on the rooms near the kitchen or bathroom, or a study of the air flow between a house and its roof will concentrate on the upstairs rooms. Pairs of rooms may be combined by joining their sampling tubes and taking the common tube to the analyser. Large volumes such as roof spaces can be sampled by similary joining tubes from several points.

When using this technique to measure the ventilation rate in a house, a house is first charged with nitrous oxide. Fans, which are positioned to give good mixing throughout the house, are run for about 10 minutes and then switched off, and the decay in concentration of the nitrous oxide at the six points is recorded on the multi-point recorder for approximately 30 minutes. Because of the sequential switching of the solenoid valves, the concentrations are recorded at each point at different times, *ie* at 36 second intervals. These measurements are then linearly interpolated to give coincident concentrations for each point. The concentration for the whole house is then determined by volume weighting.

An example of the decay in concentration in an upstairs flat with the kitchen window open is shown in figure 2. It can be seen that in the kitchen, due to the window being open, the concentration initially falls very rapidly giving a ventilation rate of about 6 air changes per hour. Recirculation from the rest of the flat influences the subsequent concentration. The concentration in all the other rooms decays exponentially and at closely similar rates. The interpolation procedure used to determine the readings in different rooms at the same time has introduced some smoothing into the date. However, it can be seen that, apart from the kitchen with its window open,

Figure 2 The decay of tracer concentration in six rooms of a house

Affaiblissement de la concentration du marqueur gazeux dans les six pièces d'une maison



there are very few fluctuations about the straight lines, indicating that mixing is adequate and that ventilation is quite steady.

It must be emphasised that, owing to the recirculation of tracer gas around the house, the ventilation rates computed from the decay of tracer in each room do not represent the ventilation of each room from the outside air. They do however give a qualitative picture of the air flow around the house and permit the calculation of the ventilation rate for the house as a whole.

b. The measurement of air movement between a house and its roof

When using the infra red gas analyser to measure the air flow from house to roof space, the 6-channel gas analyser described previously is used with 5 channels sampling points around the house on the 0-400 ppm range and 1 channel sampling in the roof space on the 0-100 ppm range, this range being used because the concentrations in the roof space are small. The house is flooded with a nitrous oxide and the concentration in the house and roof recorded for about 30 minutes. Concentrations for the whole house are computed at the same time as the roof values by the procedures of interpolation and volume weighting described previously. The ventilation rate found by this method is used to find the paramater 'A' used in equation 10.

To find the ventilation rate in the roof R_r when the roof alone is flooded with tracer gas, it is necessary to carry out a subsidiary series of tests to establish a relationship between Rr and windspeed and direction for that particular roof. This relationship is then used to find the values of Rr corresponding to the wind speeds and directions recorded at the time of the air movement tests.

Traditionally it has been assumed that the initial tracer gas concentration in the roof must be zero in order to determine x_{hr} . As has been mentioned earlier, this is impossible to achieve in practice. However, equation 10 can be used to determine the value of x_{hr} over the period in which the test is being carried out. An example of the concentrations observed in a house and its roof space is shown in figure 3. Values of the air flow rate from house to roof calculated over different time intervals for this particular test run are given in the table below.

Timer interval	c, (o)	× _{br}
0 - 0.9 hrs	13.7	18.3
0.9 - 1.8 hrs	56.0	19.2
0 - 1.8 hrs	13.7	18.7



Figure 3 The variation of tracer concentration in a house and its roof after the house was flooded

Variation de la concentration du marqueur dans une

maison et son toit après injection maximale

A good agreement between the values of x_{hr} for the different intervals indicates that it is not necessary for the roof concentration to start from zero. Figure 3 also indicates that there is very good agreement between the measured and calculated values of the tracer gas concentration in both the house and the roof space

The pressure which drives air through the ceiling of a house and into its roof is dependent on the wind speed and direction and on the position of any open windows. These interact in complex ways. However, figure 4, shows x_{hr} plotted against wind speed when the wind was blowing from one direction and all windows were shut. As can be seen, xhr is proportional to the wind speed.

If an extract fan fitted in the bathroom or kitchen is turned on, it removes water vapour from its immediate area but also lowers the air pressure in the house relative to the roof and reduces the flow into the roof. This is shown in figure 4 where the flow from house to roof is also plotted against wind speed when the fan in the downstairs kitchen was turned on.

During the air flow measurements the air pressure difference between the house and roof was measured using an electronic micromanometer. The pressure difference and flow rates may be combined to give an estimate of the area gaps in the ceiling. The

$x_{hr} = 2977 \ A \ \triangle p^{\frac{1}{2}}$

where x_{hr} is in cubic metres per hour, A is the area of the gaps in square metres and Δp is the pressure difference in N/m². This may be expressed as:

$$\log x_{hr} = \log 2977 A + \frac{1}{2} \log \Delta p$$

ventilation déterminé par cette méthode est utilisé pour trouver le paramètre « A » dont il est fait usage dans l'équation 10.

•Pour trouver le taux de ventilation dans le toit R_r lorsque le toit seulement est injecté au maximum du marqueur gazeux, il est nécessaire d'effectuer une série subsidiaire d'essais pour établir une relation entre R_r, la vitesse du vent et sa direction pour ce toit particulier. Puis on utilise cette relation pour trouver les valeurs de R_r correspondant aux vitesses du vent et aux directions enregistrées au moment des essais de mouvement de l'air.

On a toujours pris comme hypothèse de travail que la concentration initiale dans le toit doit être égale à 0 afin de déterminer x_{hr} . Comme il a été dit plus tôt, cela n'est pas réalisable en pratique. Mais on peut, malgré tout, utiliser l'équation 10 pour déterminer la valeur de x_{hr} pendant la période où est effectué l'essai. La figure 3 montre un exemple des concentrations observées dans une maison et son vide sous-toiture.

Le tableau ci-après donne les valeurs du courant d'air de la maison vers le toit calculées pour différents intervalles de temps pour cet essai particulier:

Période	C, (0)	×hr
0/- 0,9 h	13,7	18,3
0,9 - 1,8 h	56,0 /	19,2
0 - 1,8 h	13,7 /	18,7

Si l'on constate qu'il y a accord entre les valeurs de x_{hr} pour les différents intervalles, il n'est pas nécessaire que la concentration dans la toiture parte de zéro. La figure 3 indique également que les valeurs mesurées et calculées de la concentration du gaz traceur concordent très bien à la fois pour la maison et le vide sous toiture.

La pression qui pousse l'air à traverser le plafond d'une maison et à pénétrer dans sa toiture dépend de la vitesse du vent et de sa direction ainsi que de l'emplacement de toute fenêtre ouverte. Ces facteurs sont interdépendants d'une façon complexe. La figure 4 montre, cependant, x_{hp} en fonction de la vitesse du vent lorsque le vent soufflait d'une seule direction et que toutes les fenêtres étaient fermées. Comme on peut le voir, x_{hr} est proportionnel à la vitesse du vent.

Si l'extraction mécanique de la salle de bain ou de la cuisine est branchée, elle retire la vapeur d'eau à proximité immédiate mais abaisse également la pression de l'air dans la maison par rapport au toit et réduit le courant qui entre dans celui-ci. C'est ce que montre la figure 4 où le courant d'air de la maison vers le toit est donné par rapport à la vitesse du vent lorsque le ventilateur de la cuisine (RdC) marchait.

Pendant les mesures du débit d'air, la différence de pression de l'air entre la maison et le toit a été mesurée à l'aide d'un micromanomètre électronique. Il est possible de combiner la différence de pression et les débits pour obtenir la surface estimée des interstices dans le plafond.



Figure 4



Débit d'air entre une maison et sa toiture en fonction de la vitesse du vent

Le Guide CIBS (réf. 2) donne : /

$x_{hr} = 2977 \text{ A} / \Delta p \frac{1}{2}$

où x_{hr} est exprimé en m³/h, A représentant la superficie des interstices en m² et Δp la différence de pression en N/m². Cela peut s'exprimer sous la forme de :

$$\log x_{hr} = \log 2977 A + \frac{1}{2} \log \Delta p$$

La figure 5 montre x_{hr} tracé par rapport à $\triangle p$ sur des échelles logarithmiques pour une maison. La meilleure droite est donné par:

$$\log x_{\rm hr} = 3.37 + 0.46 \log \Delta p$$

Le gradient de 0,46 est suffisamment proche de 0,5 pour justifier l'utilisation de l'équation 10. L'intercept 3,37 = log 2977 A donne A = 0,0098 m² = 98 cm². On peut se faire une idée de cette surperficie en sachant que celle d'une fissure de 1 mm autour des 2,5 m de périmètre d'une trappe est de 25 cm².

La technique qui consiste à injecter la plus grande quantité possible de marqueur, décrite auparavant, ne permet que le calcul du courant au travers d'un AIR MOVEMENT/continued

Figure 5 shows x_{hr} plotted against $\triangle p$ on logarithmic – scales for one house. The best straight line is:

$$\log x_{hr} = 3.37 + 0.46 \log \Delta p$$

The gradient of 0.46 is close enough to 0.5 to confirm the use of equation 10. The intercept $3.37 = \log 2977$ A gives A = 0.0098 m² = 98 cm². This area can be put in perspective by considering that the area of a crack of 1 mm around the 2.5 m perimeter of the hatch cover is 25 cm².

The technique of flooding the house with tracer gas described earlier allows only the flow through a ceiling as a whole to be calculated. It does not take into account the relative contributions of the individual components. Apparatus is being developed to measure the flow around hatch covers and through light drops and holes where pipes pass through the ceiling. These seem to be the most important gaps in the ceiling.

Figure 5

Air flow between a house and its roof against the pressure difference on logarithmic scales

Débit d'air entre une maison et sa toiture par rapport à la différence de pression sur des échelles logarithmiques



Applying the techniques

The flow of air from a house to its roof that has been measured using the technique described earlier has to be taken into account when considering condensation in houses and their roofs and also ventilation heat losses.

Two parameters that are useful in both these fields are:

a. The proportion of air entering a house that leaves via the roof:

$$\frac{\mathbf{x}_{hr}}{\mathbf{x}_{ho}} = \frac{\mathbf{x}_{hr}}{\mathbf{R}_{h}\mathbf{V}_{h}} \qquad (Equation \ 11)$$

b. The proportion of air entering a roof that has come from the house below:

$$\frac{x_{hr}}{x_{hr} + x_{or}} = \frac{x_{hr}}{R_r V_r} \quad (Equation \ 12)$$

where the symbols are as shown in the outline sketch of the house (page 164).

As the ventilation rates of house and roof and the air flow through the ceiling all depend on wind speed, the two ratios above will be expected to vary with wind speed, possibly in a complex manner. In practice both ratios lie between 0.2. and 0.3. at the wind speeds of between 0 and 5 ms⁻¹ which are usually experienced in towns, (*ie* 20 per cent — 30 per cent of the air entering a house leaves via gaps in the ceiling).

Blocking these gaps should therefore reduce the ventilation rates in the house by about this amount, as the air pressure will rise in the house reducing the flow inwards through cracks on the windward side. This reduced ventilation will lead to higher humidities and thus increase the risk of condensation in the house. However, preventing the air flow through the ceiling will greatly reduce the amount of water vapour that is transferred from the house to the roof and therefore greatly reduce the risk of condensation in the roof.

As shown in Appendix B, it is possible to calculate the amounts of heat and water vapour transferred from a house to its roof by air motion, conduction and diffusion. The results are shown in the table below.

Heat	Air motion	Conduction	Total
Transfer	119 W	328 W	447 W
Vapour	Air Motion	Diffusion	87 gmh ⁻¹
Transfer	64 gmh ⁻¹	23 gmh ⁻¹	

It can be seen that whereas 74 per cent of the vapour transfer is by air motion, only 27 per cent of the heat transfert is by air motion.

The effect of blocking the holes in the ceiling on conditions in the roof can be found by calculating the temperatures and vapour pressures in the roof space from heat and water vapour balances, as shown in Appendix B.

	Roof-Void Conditions		
	Temperature	Vapour pressure	Relative humidity
Ceiling gaps open Ceiling	3.6 K	7.4 mbs	94 %
gaps closed	2.8 K	6.2 mbs	83 %

It can be seen that blocking the gaps in the ceiling causes both temperatures and vapour pressure to fall. However, the larger fall in vapour pressure causes the relative humidity to fall, reducing the risk of condensation in the roof.

In the average house about 30 per cent of the heat loss is by ventilation. Consequently blocking the gaps in the ceiling can be expected to reduce the total heat loss by about 6-9 per cent. This benefit would be relatively greater in highly insulated houses.

Conclusions

This paper has shown how to extend the existing techniques for measuring ventilation rates and air flows within houses using tracer gases. By way of illustration it has shown the importance of the transfer of air from a house to its roof space.

Results from field studies in a number of houses have shown that typically 20-30 per cent of the air that enters a house leaves via the roof space through gaps in the ceiling. It has been shown, by means of heat and moisture balance in the roof space, that the air flow through these gaps is the major mode of transport of water vapour from the house to the roof, but the air movement is relatively less important for the transport of heat. It has been shown that blocking gaps in the ceiling will reduce the relative humidity in roof spaces significantly.

Appendix A - Derivation of equation 10

Assume there are N observations of the tracer gas concentration in the roof, $c_r(t_i)$ observed at times t_i . For any value of the air flow from the house to roof x_{hr} , values of the roof concentration $c'_r(t_i)$ to correspond to the observed values $c_r(t_i)$ can be calculated, using equation 9.

$$c_{r}^{\prime}(t_{i}) = \frac{c_{h}(o) x_{hr}}{V_{r}} \cdot \frac{1}{R_{r} - R_{h}} \left(c^{-R_{h}t_{i}} - R_{r}t_{i} \right) + c_{r}(o)^{-R_{r}t_{i}} = A(t_{i}) x_{hr} + B(t_{i})$$

where

$$\begin{split} A(t_i) \ &= \ \frac{c_h(o) \ \left(e^{- \ R_h t_i} - e^{- \ R_r t_j} \right)}{V_r \ (R_r - R_h)} \ . \\ B(t_i) \ &= \ c_r(o) \ e^{- \ R_r t_i} \end{split}$$

The best value of x_{hr} is that which minimises the sum of squares of the differences between the observed and calculated values of the room concentration.

$$S = \sum_{i=1}^{N} (c_{r}(t_{i}) - c_{r}'(t_{i}))^{2}$$
$$= \sum_{i=1}^{N} (c_{r}(t_{i}) - A(t_{i}) x_{hr} - B(t_{i})^{2}$$

ie the value of x_{hr} which makes

$$\frac{\partial S}{\partial x_{hr}} = \frac{1}{\sum_{i=1}^{N} (-2 c_r(t_i) A(t_i) + 2 A^2(t_i) x_{hr} + 2 A(t_i) B(t_i)) = 0}$$

giving
$$x_{hr} = \frac{\sum_{i=1}^{N} c_r(t_i) A(t_i) - \sum_{i=1}^{N} A(t_i) B(t_i)}{\sum_{i=1}^{N} A^2(t_i)}$$

Appendix B - Heat and moisture balance in a roof

References/Bibliographie

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a. Heat balance

During the night when there is no solar radiation, the heat entering a roof is made up of

1. Conduction from the house below = $U_c A_c (T_h - T_r)$

AIR MOVEMENT/continued

2. Air movement from the house below = $x_{hr} \rho_c (T_h - T_r)$

The heat leaving the roof is made up of:

1. Conduction through the roof surface = $U_r A_r (T_r - T_o)$

2. Ventilation from the outside = $x_{or} \rho_c (T_r - T_o)$

where :

 $T_r = air temperature in roof space$

 T_h = air temperature in house = 15 °C

- $T_o =$ air temperature outside $= 0 \ ^oC$
- U_c = the U value of the ceiling = 0.64 Wm⁻²K (10 mm plasterboard + 50 mm insulation)

 U_r = the U value of roof surface = 1.9 Wm⁻²K (12,5 mm sarking boards 10 mm tiles)

- $A_c =$ the area of the ceiling = 45 m²
- A_r = the area of the roof surface = 55 m²
- x_{hr} = the air flow from house to roof = 30 m³h⁻¹ = 0.0083 m³ s⁻¹
- x_{or} = the air flow from outside into the roof = $60 \text{ m}^3 \text{h}^{-1} = 0.0166 \text{ m}^3 \text{ s}^{-1}$

for V_r roof volume = 30 m³

and R_r roof ventilation rate = 2 ach⁻¹

 ρ c = density x specific heat of air = 1250 Jm⁻³ °C⁻¹.

In equilibrium the heat entering the roof equals that leaving, so:

$$U_{c}A_{c} (T_{h} - T_{r}) + x_{hr} \rho c (T_{h} - T_{r})$$

= $U_{r}A_{r} (T_{r} - T_{o}) + x_{or} \rho c (T_{r} - T_{o})$

giving

$$T_{r} = \frac{(A_{c}U_{c} + x_{hr} \rho c) T_{h} + (A_{r}U_{r} + x_{or} \rho c) T_{o}}{A_{c}U_{c} + x_{hr} \rho c + A_{r}U_{r} + x_{or} \rho c}$$

Inserting the numerical values quoted gives $T_r = 3.6^{\circ}$ which changes to $T_r = 2.8 \text{ °C}$ if $x_{hr} = 0$.

Using the value of $T_r = 3.6$ °C one may calculate the four heat flows outlined above.

- 1. Conduction from house $U_c A_c (T_h T_r) = 328 \text{ W}$
- 2. Air flow from house to roof $x_{hr} \rho_c (T_h T_r) = 119 W$
- 3. Conduction through roof surface $U_r A_r (T_r - T_o) = 376 W$
- 4. Ventilation to outside $x_{or} \rho_c (T_r - T_o) = 75 W$

b. Moisture balance

Water vapour enters a roof by

1. Diffusion through the ceiling = A_cD_c ($P_h - P_r$)

2. Air motion through gaps in the ceiling

$$\frac{hr}{k} \frac{\rho}{\rho} (p_h - p_r)$$

and leaves by

X

1. Diffusion through the roof surface A_rD_r ($P_r - P_o$)

2. Ventilation to the outside $\frac{x_{or}}{k} \rho (P_r - P_o)$

where the terms are as defined for the heat balance, except for

- p_r = the vapour pressure in the roof
- p_h = the vapour pressure in the house = 10.2 mb = 1 020 Nm⁻²

(gives 60 % RH at
$$T_h = 15$$
 °C)

- $p_o =$ the vapour pressure outside = 5.5 mb = 550 Nm⁻² (gives 90 % RH at $T_o = 0$ °C)
- D_c = the diffusivity of the ceiling = 0.5 GNs/kg
- $D_r =$ the diffusivity of the roof surface

$$= 0.02 \text{ GNs/kg}$$

$$p = is$$
 the density of air = 1,2 kg m³

K = is a factor relating concentration of water vapour pressure = $1.57 \times 10^5 \text{ Nm}^{-2}$

In equilibrum the water vapour entering a roof equals that leaving, so

$$A_{c}D_{c} (P_{h} - P_{r}) + \frac{x_{hr} \rho}{k} (P_{h} - P_{r})$$
$$= A_{r}D_{r} (P_{r} - P_{o}) + \frac{x_{or} \rho}{k} (P_{r} - P_{o})$$

giving

$$P_{r} = \frac{(A_{c}D_{c} + x_{hr} \rho/k) P_{h} + (A_{r}D_{r} + x_{or} \rho/k) P_{o}}{A_{c}D_{c} + x_{hr} \rho/k + A_{r}D_{r} + x_{or} \rho/k}$$

inserting the numerical values quoted gives $P_r = 7.4$ mb falling to $P_r = 6.2$ mb if $x_{hr} = 0$.

Using $P_r = 7.4$ mb the four mass flows may be calculated.

- 1. Diffusion through ceiling A_cD_c ($P_h P_r$) = 6.3 × 10⁻⁶ kg s⁻¹ = 22.7 gmh⁻¹
- 2. Air motion through ceiling = $\frac{x_{hr}}{k} \rho (P_h P_r)$ = 1.8 × 10⁻⁵ kg s⁻¹ = 64 gmh⁻¹
- 3. Diffusion through roof surface = $A_r D_r (P_r P_o)$ = 2.1 × 10⁻⁻⁷ kg s⁻¹ = 0.7 gmh⁻¹
- 4. Ventilation to the outside = $\frac{x_{hr}}{k} \rho (P_r P_o)$ = 2.41 × 10⁻⁵ kg s⁻¹ = 87 gmh⁻¹