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The Effect of Edge Insulation on the Steady-State Heat Loss Through a Slab-on-Ground Floor

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This paper examines the effect of introducing edge insulation, either horizontally or vertically, on the heat loss from an otherwise uninsulated slab-on-ground floor. Factors are obtained in terms of the thickness of the insulation, its thermal conductivity, and its width (if horizontal) or depth (if vertical). These factors, when related to the exposed perimeter of the floor, can be combined with the heat loss through the floor without insulation, to give the heat loss from the edge-insulated floor. The predictions compare well with numerical calculations.

1. INTRODUCTION

INSULATION round the edge of a slab-on-ground floor, although less effective than providing the same thermal resistance over the whole floor, can nevertheless give a useful reduction in the total heat transfer.

The CIBSE Guide [1] gives some reduction factors for vertical edge insulation, based on the work of Billington [2], said to apply to edge insulation of thermal resistance 0.25 m^2K/W. Billington's data were based, however, on "definitely preventing lateral heat flow" at the edge of the floor, corresponding to edge insulation of very high thermal resistance. The Guide's factors are therefore somewhat optimistic for adding a small additional resistance.

More recently, theories have been developed involving Fourier series solutions to the heat conduction equation as applied to the whole floor.

Landman and Delsante [3–5] deal with vertical and horizontal edge insulation of uninsulated floors in this way, while Hagentoft [6] presents a method for obtaining the effect of variations in thickness of additional horizontal edge insulation for an already insulated floor. Such solutions require a computer for their evaluation, although heat loss factors for a floor incorporating edge insulation can be presented in graphical or tabular form, as these authors have illustrated in their papers.

A different approach is adopted here. The effect of the edge insulation is treated as an edge correction factor to the heat transfer through an equivalent uninsulated floor. This edge factor, per unit perimeter of the floor, is independent of the size and shape of the floor and so, once obtained, can be applied to any floor incorporating the detail in question at its edges. The edge factor is derived from consideration of how the presence of edge insulation affects the integrated heat flux over the floor, leading to analytic formulae in terms of the parameters of the edge insulation (thickness, thermal conductivity and width or depth). In this way the effect of any combination of the parameters can be readily assessed.

There are two principal ways of providing edge insulation of floors: horizontally near floor level round the perimeter, or vertically below ground providing additional thermal resistance to the foundation wall. These two possibilities require separate treatments.

It is assumed that the edge insulation, however provided, joins or at least overlaps the wall insulation. If this is not so the resulting thermal bridge may largely nullify the effect of the edge insulation.

2. GENERALITY

In an earlier paper [7] it was proposed that the U-value of a floor with a particular edge or foundation detail could be obtained from:

$$U = U_0 + \frac{2}{B'}\Delta\Psi \tag{1}$$

where U_0 = U-value of floor with idealised edge or foundation detail (W/m^2K); B' = (area of floor/$\frac{1}{2}$ exposed perimeter of floor) (m); $\Delta\Psi$ = change in heat loss per unit perimeter due to special edge or foundation detail ($W/m \cdot K$).

The idealized edge or foundation detail to which U_0 refers has uniform thermal properties below ground, with no edge insulation, and assumes an adiabatic boundary at the base of the wall.

$\Delta\Psi$ is positive for factors that increase the total heat flow, for example a thermal bridge at the wall/floor junction, and it is negative for factors such as edge insulation that decrease the total heat flow.

$\Delta\Psi$ can be obtained found by a 2-dimensional analysis. In general an appropriate numerical method, such as finite elements, can be used to assess a foundation detail of almost any degree of complexity, and then applied to

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any size of floor using equation (1) if the uninsulated U-value is known.

The paper examines the case of uniform thermal properties below ground, save for the edge insulation itself. The thickness of the external wall of the building influences the total heat flow, but the thermal properties of the wall above ground are assumed not to affect the heat transfer through the ground.

The following formula for U_0 is given in [7]:

$$U_0 = \frac{2\lambda}{\pi B'} \frac{1}{1+(d_i-d_e)/\pi B'} \ln\left(\frac{\pi B'}{w+d_i+d_e}+1\right) \quad (2)$$

where λ = thermal conductivity of the soil; w = wall thickness; $d_i = R_i\lambda$; R_i = thermal resistance at inside surface of floor; $d_e = R_e\lambda$; R_e = thermal resistance at outside ground surface.

R_i includes the inside surface resistance and any insulation of the floor slab.

d_i and d_e are the equivalent thickness at the inside and outside surfaces respectively (the equivalent thickness of a thermal resistance is the thickness of soil having the same thermal resistance).

$\Delta\Psi$ is the heat flow per unit perimeter length and temperature difference for the actual floor, Φ, less that for an equivalent floor with uniform thermal properties in the foundation region, Φ_0, i.e.:

$$\Delta\Psi = \Phi - \Phi_0. \quad (3)$$

For the purposes of this paper, Φ_0 is obtained from equation (2). Equation (2) gives the heat loss per unit area; per unit perimeter it becomes:

$$\Phi_0 = \frac{\lambda}{\pi} \frac{1}{1+(d_i-d_e)/\pi B} \ln\left(\frac{\pi B}{w+d_i+d_e}+1\right) \quad (4)$$

for a floor of width B.

In the case of an uninsulated floor, $d_i \ll B$ and $d_e \ll B$ and an adequate approximation for Φ_0 is:

$$\Phi_0 = \frac{\lambda}{\pi} \ln\left(\frac{\pi B}{w+d}+1\right) \quad (5)$$

where $d = d_i+d_e = (R_{si}+R_{se})\ \lambda$, and R_{si} and R_{se} are respectively the internal and external surface resistances.

Expressions for $(\Phi-\Phi_0)$ are developed in the following sections for horizontal edge insulation and for vertical edge insulation, when the insulation width (if horizontal) or depth (if vertical) is much less than the dimensions of the building. These expressions are then assessed against numerical calculations.

3. HORIZONTAL EDGE INSULATION

Consider edge insulation of width D as illustrated in Fig. 1. Suppose this insulation is of thickness d_{ins} and thermal conductivity λ_{ins}. (When the insulation is placed below ground λ_{ins} should reflect the performance of the material in that situation.)

The *additional* equivalent thickness introduced by the edge insulation, i.e. the equivalent thickness of the insulation less the thickness of the soil it replaces, is:

$$d' = \lambda \cdot d_{ins}/\lambda_{ins} - d_{ins}. \quad (6)$$

First consider infinite insulation placed at the edge ($d' = \infty$), so that there is zero heat flow into the ground in the edge region. This is equivalent to increasing the wall thickness from w to $w+D$ (and so the floor width is reduced from B to $B-2D$); from (5),

$$\Phi_{\infty} = \frac{\lambda}{\pi} \ln\left(\frac{\pi(B-2D)}{w+d+D}+1\right). \quad (7)$$

The reduction in heat loss from infinite edge insulation is then:

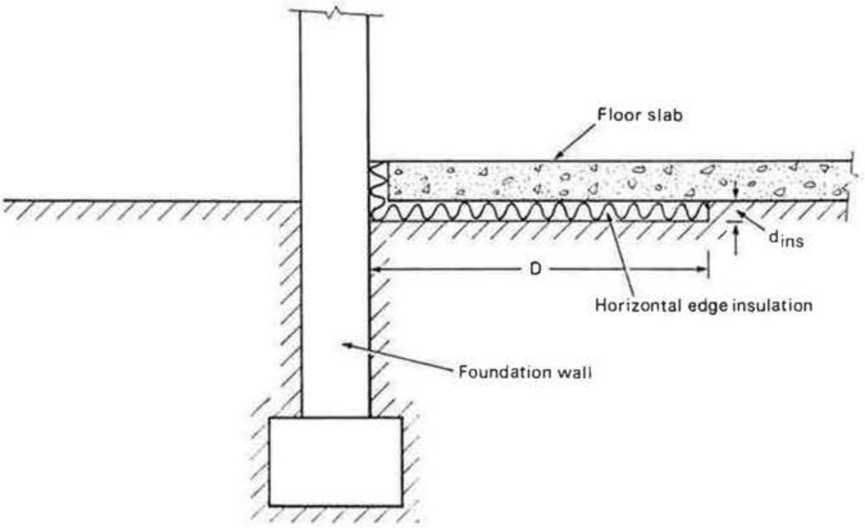


Fig. 1. Horizontal edge insulation placed below the floor slab. It is also returned upwards at the edge of the slab to avoid a thermal bridge. The analysis of this paper applies equally to horizontal edge insulation above the floor slab or incorporated within it.

$$\Phi_0 - \Phi_\infty = \frac{\lambda}{\pi}\left[\ln\left(\frac{\pi B}{w+d}+1\right) - \ln\left(\frac{\pi(B-2D)}{w+d+D}+1\right)\right]$$

which, on re-arranging the terms, becomes

$$\Phi_0 - \Phi_\infty = \frac{\lambda}{\pi}\left[\ln\left(\frac{D}{w+d}+1\right) - \ln\left(\frac{\pi(B-2D)+w+d+D}{\pi B+w+d}\right)\right]. \quad (8)$$

When the edge insulation has a finite value, a term must be included to represent the heat flow that then occurs through the edge insulation. As discussed in Appendix A, this has a similar form to (8), but with d replaced by $(d+d')$, giving as the net reduction in heat flow:

$$\Phi_0 - \Phi = \frac{\lambda}{\pi}\left[\ln\left(\frac{D}{w+d}+1\right) - \ln\left(\frac{D}{w+d+d'}+1\right)\right] - \frac{\lambda}{\pi}\left[\ln\left(\frac{\pi(B-2D)+w+d+D}{\pi B+w+d}\right) - \ln\left(\frac{\pi(B-2D)+w+d+d'+D}{\pi B+w+d+d'}\right)\right]. \quad (9)$$

Provided that $D \ll B$ and $d \ll B$ the second half of (9) is always very much less than the first half, for any combination of D and d', so that only the first two terms in (9) need be included. This leads to a simple expression for the change in heat loss rate due to the edge insulation:

$$\Delta\Psi = \Phi - \Phi_0 = -\frac{\lambda}{\pi}\left[\ln\left(\frac{D}{w+d}+1\right) - \ln\left(\frac{D}{w+d+d'}+1\right)\right]. \quad (10)$$

This expression does not involve the building dimension, B.

Figure 2 shows the heat loss per unit perimeter from a long floor of width 10 m with edge insulation, calculated as $\Phi_0+\Delta\Psi$ from equations (4) and (10), compared with numerical results by a finite element method.

The other parameters are $\lambda = 2.0$ W/m·K, $w = 0.3$ m, $R_{si} = 0.14$ m^2 K/W and $R_{se} = 0.04$ m^2 K/W. The four values of d' correspond to edge insulation of thermal resistance approximately 0.25, 0.5, 1.0 and 2.0 m^{2}K/W respectively. This comparison shows that equation (10) gives a close approximation to the edge factor $\Delta\Psi$, for variations in both the parameters D and d'. The effect of the edge insulation is slightly underpredicted, but by not more than 2% of the total heat loss in these examples (and this would be even smaller for larger floors), compared with a reduction in heat loss relative to the uninsulated floor of up to 20%.

4. VERTICAL EDGE INSULATION

Vertical edge insulation can be provided with an insulation layer at the edge of the floor slab, Fig. 3(a), or by using low density concrete blocks for the foundations,

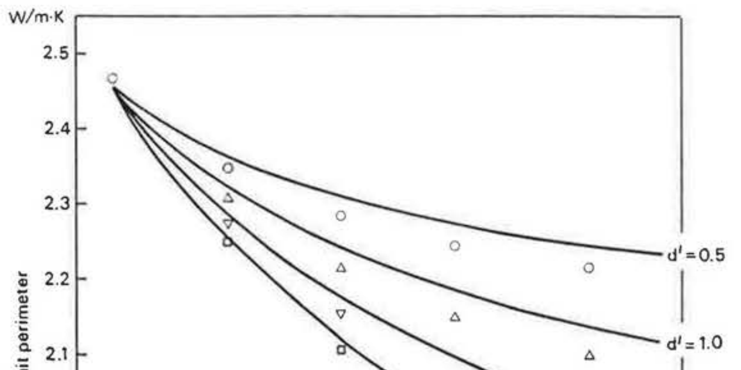
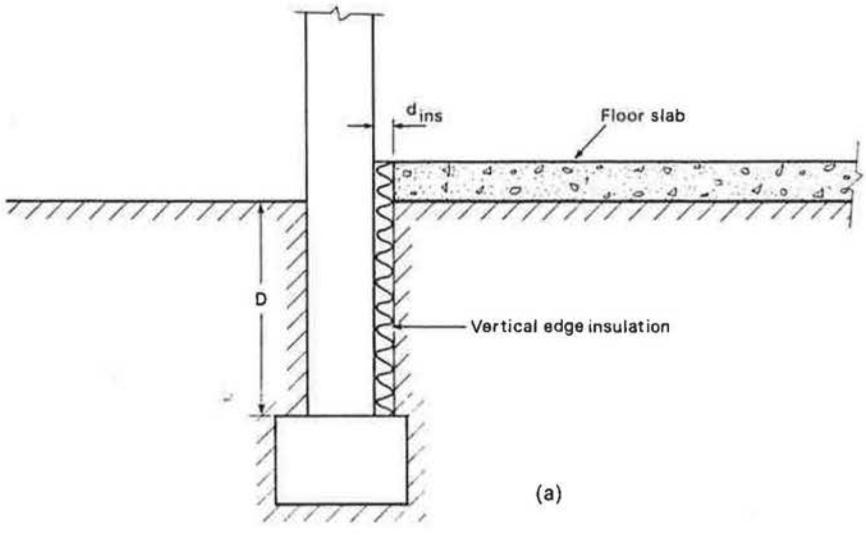


Fig. 2. The total heat loss per unit perimeter from a long floor of width 10 m with horizontal edge insulation as a function of the insulation width D, for four values of d'. The lines drawn on the figure are from equations (4) and (10) while the symbols show the numerical calculations.



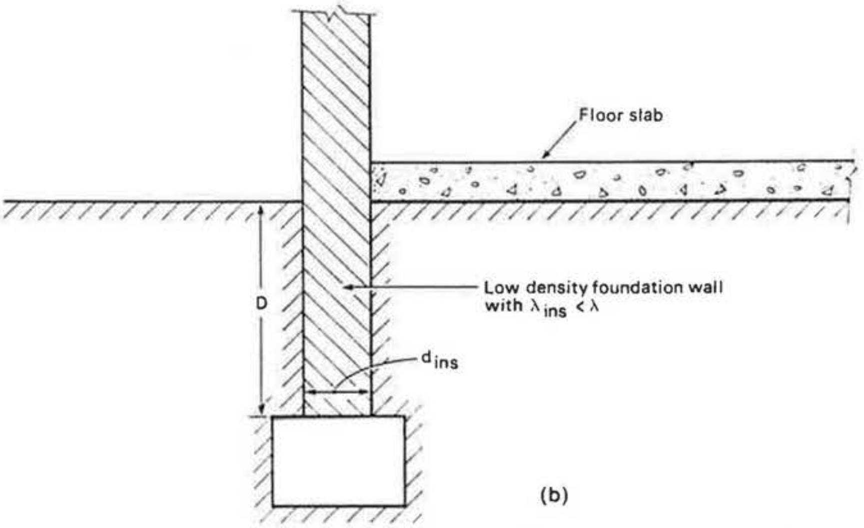


Fig. 3. Vertical edge insulation : (a) by incorporating an additional insulation layer (which could alternatively be placed on the outside of the foundation wall), (b) by using low-density foundation wall.

Fig. 3(b). The parameter D is now the depth below ground to which the insulation extends.

Following a similar approach to that adopted for horizontal edge insulation, consider the situation of infinite thermal resistance from ground level to a depth D. This floor can be considered as one whose original level was at depth D but with a thickness of D of soil added all over both inside and outside. There is no direct thermal connection between these additional blocks of soil since they are separated by an infinite thermal resistance. For that reason let us assume that the heat flow is unidirectional (downwards/upwards) in these additional blocks of soil (likely to be reasonable if D is not too large compared with the floor width, B).

The integrated heat flow is then indicated by (4) with D added to both d_i and d_e, or:

$$\Phi_\infty = \frac{\lambda}{\pi} \ln\left(\frac{\pi B}{w+d+2D}+1\right) \tag{11}$$

and

$$\Phi_0 - \Phi_\infty = \frac{\lambda}{\pi}\left[\ln\left(\frac{2D}{w+d}+1\right) - \ln\left(\frac{\pi B+w+d+2D}{\pi B+w+d}\right)\right] \tag{12}$$

With finite edge insulation there is another term of similar form but with d replaced by $d+d'$, where d' is the additional equivalent thickness of the edge insulation as previously defined (see Appendix B). Dropping the terms involving the building dimension B on the same basis as before:

$$\Delta\Psi = \Phi - \Phi_0 = -\frac{\lambda}{\pi}\left[\ln\left(\frac{2D}{w+d}+1\right) - \ln\left(\frac{2D}{w+d+d'}+1\right)\right]. \tag{13}$$

Figure 4 shows the heat loss per unit perimeter, calculated as $\Phi+\Delta\Psi$ from equations (4) and (13), using the same parameters as for Fig. 2. This agrees very closely with the results of the numerical calculations.

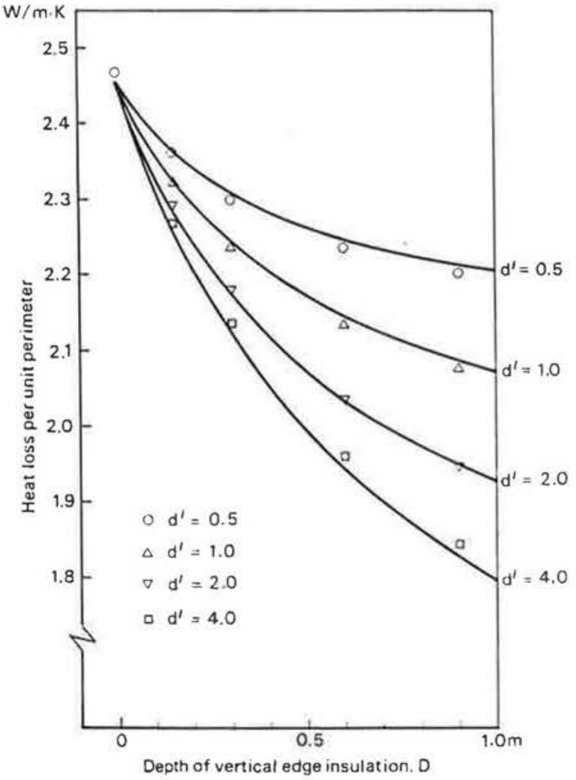


Fig. 4. The total heat loss per unit perimeter from a long floor of width 10 m with vertical edge insulation as a function of the insulation depth D, for four values of d'. The lines drawn on the figure are from equations (4) and (13) while the symbols show the numerical calculations.

5. DISCUSSION

As indicated by Figs 2 and 4, the formulae given in this paper give a good representation of the effect of edge insulation of limited width or depth applied to an otherwise uninsulated floor. They give results that are similar to those of Landman and Delsante [4, 5], who also considered edge insulation of uninsulated floors.

It has been assumed that the width of horizontal insulation or the depth of vertical insulation, denoted by D in both cases, is small compared with the width of the building: $D \ll B$. The formulae do not indicate the correct heat flux in the limit $D \to \frac{1}{2}B$.

It was noted earlier that equation (10) for horizontal edge insulation slightly underpredicted the effect of the edge insulation on an uninsulated floor. This discrepancy becomes larger when considering the effect of adding extra edge insulation to an already insulated floor. Compared with the values given by Hagentoft [6] (the latter estimated by Hagentoft to be accurate to better than 0.1%) and confirmed by the present author using finite element techniques, the prediction of $\Delta\Psi$ can be in error by a factor of up to about 2 for well-insulated floors. There are two principal reasons for this.

Firstly, the analysis in Appendix A is based on an assumption of a constant temperature at the ground surface except in the edge region. This is a good approximation for an uninsulated floor but becomes progressively poorer as the all-over insulation is increased.

Secondly, when d becomes comparable with B it is no longer valid to drop two terms in equation (9), as was done with only a very small error for uninsulated floors to obtain equation (10).

The consequence is that, strictly, $\Delta\Psi$ cannot be given independently of the overall building dimensions, in the case of adding extra edge insulation to insulated floors. In such cases, however, $\Delta\Psi$ is small and is unlikely to affect the total heat loss by more than 10%, so that even if $\Delta\Psi$ is in error by a factor of 2, the total heat loss will be obtained to within 5%, which is adequate for most purposes considering the uncertainty in soil properties. The error is such that the estimation is on the "safe" side: the effect of the edge insulation will be at least that predicted.

In practice, the effect on heat loss is most significant in the consideration of edge insulation of uninsulated floors (additional edge insulation is added to insulated floors primarily for reasons of frost protection of foundations in cold climates). The formulae proposed in this paper give a very satisfactory quantification of the effect on the heat loss, allowing the costs and effectiveness of edge insulation to be properly balanced.

6. CONCLUSION

Straightforward expressions have been obtained for the effect of adding horizontal or vertical edge insulation to slab-on-ground floors.

The predictions of these expressions agree closely with numerical calculations in the case of otherwise uninsulated floors. Using the methodology described, a unique factor is obtained for a given edge configuration. This factor can then be applied to the evaluation of the heat loss from a floor of any size or shape incorporating the detail at its edge. The factor can also be used with insulated floors although the effect of the edge insulation is underestimated.

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APPENDIX A
HORIZONTAL EDGE INSULATION

Consider an isolated edge (i.e. the opposite edge of the floor does not affect the heat flow in the region under consideration, with insulation of thermal resistance R' at the edge region only, which extends over a distance D. The internal temperature is T_i and the external temperature is T_e. In the $x-y$ plane $y = 0$ is the floor surface, and $x = 0$ is at the middle of the insulated region (see Fig. 5). Within the ground ($y < 0$):

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0 \tag{A1}$$

and the boundary conditions along the ground surface, $y = 0$, are:

$$T = T_e, \quad x < -\tfrac{1}{2}D \tag{A2}$$

$$\frac{T_i - T}{R'} = -\lambda\frac{\partial T}{\partial y}, \quad -\tfrac{1}{2}D < x < \tfrac{1}{2}D \tag{A3}$$

$$T = T_i, \quad x > \tfrac{1}{2}D. \tag{A4}$$

(A2) and (A4) correspond to constant surface temperature away from the insulated region, while (A3) corresponds to unidirectional heat flow through the insulation.

Let

$$\left.\begin{aligned} d' &= R'\lambda \\ z &= x+iy \end{aligned}\right\} \tag{A5}$$

and transform the ground into the semi-infinite strip illustrated in Fig. 6 by the transformation

$$\left.\begin{aligned} d' &= \tfrac{1}{2}D\delta \\ z &= \tfrac{1}{2}D\cos(iw) \end{aligned}\right\} \tag{A6}$$

where

$$w = \xi + i\eta \tag{A7}$$

i.e.

$$\left.\begin{aligned} x &= \tfrac{1}{2}D\cos(\eta)\cosh(\xi) \\ y &= \tfrac{1}{2}D\sin(\eta)\sinh(\xi) \end{aligned}\right\}. \tag{A8}$$

For convenience take unit temperature difference ($T_i = 1$, $T_e = 0$) so that, noticing that z and w are conjugate functions, (A1) to (A4) become:

$$\frac{\partial^2 T}{\partial \xi^2}+\frac{\partial^2 T}{\partial \eta^2}=0, \quad \xi > 0, 0 < \eta < \pi \tag{A9}$$

$$T = 0, \quad \eta = 0, \xi > 0 \tag{A10}$$

$$T = 1 + \frac{\delta}{\sin(\eta)}\frac{\partial T}{\partial \xi}, \quad 0 < \eta < \pi, \xi = 0 \tag{A11}$$

$$T = 1, \quad \eta = \pi, \xi > 0. \tag{A12}$$

For infinite thermal resistance at the edge region ($d' = \infty$, so

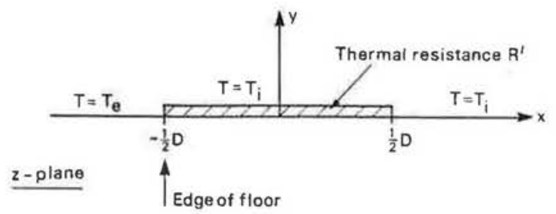

Fig. 5. Boundary conditions at the ground surface for horizontal edge insulation.

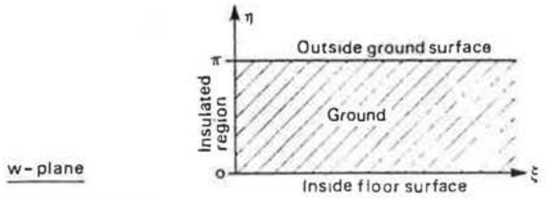

Fig. 6. The ground in the transformed co-ordinates.

$\delta = \infty$) the solution of (A9) to (A12) is:

$$T = \eta/\pi. \tag{A13}$$

The *difference* due to finite thermal resistance at the edge region compared with infinite thermal resistance is therefore defined by the solution for the temperature field θ where

$$\theta = T - \eta/\pi \tag{A14}$$

i.e.

$$\left.\begin{aligned} &\frac{\partial^2\theta}{\partial\xi^2}+\frac{\partial^2\theta}{\partial\pi^2}=0, \quad \xi > 0, 0 < \eta < \pi \\ &\theta = 0, \quad \eta = 0, \xi > 0 \\ &\theta = 1 - \frac{\eta}{\pi} + \frac{\delta}{\sin(\eta)}\frac{\partial\theta}{\partial\xi}, \quad 0 < \eta < \pi, \xi = 0 \\ &\theta = 0, \quad \eta = \pi, \xi > 0. \end{aligned}\right\} \tag{A15}$$

(A15) does not admit a general analytic solution but a close approximation may be obtained as follows. Considering initially the case $\delta = 0$, a Fourier series solution of (A15) is given in [9]:

$$\theta = \frac{2}{\pi}\int_0^{\pi}\left(1-\frac{\eta'}{\pi}\right)\sum_{m=1}^{\infty} e^{-m\xi}\sin(m\eta)\sin(m\eta')\,d\eta'. \tag{A16}$$

The integral is readily evaluated, and denoting the imaginary part of an expression by Im{ },

$$\begin{aligned} \theta &= \frac{2}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}e^{-m\xi}\sinh(m\eta) \\ &= \frac{2}{\pi}\mathrm{Im}\{e^{-\xi}e^{i\eta}+\tfrac{1}{2}e^{-2\xi}e^{2i\eta}+\tfrac{1}{3}e^{-3\xi}e^{3i\eta}+\cdots\} \\ &= \frac{2}{\pi}\mathrm{Im}\{-\ln(1-e^{-\xi+i\eta})\} && \text{(A17)} \\ &= \frac{2}{\pi}\tan^{-1}\left(\frac{\sin(\eta)}{e^{\xi}-\cos(\eta)}\right). && \text{(A18)} \end{aligned}$$

The heat flux density along $\xi = 0$ is then given by

$$\begin{aligned} q &= -\lambda\left.\frac{\partial\theta}{\partial\xi}\right|_{\xi=0} \\ &= \frac{\lambda}{\pi}\frac{\sin(\eta)}{1-\cos(\eta)}. \end{aligned} \tag{A19}$$

(A19) is valid for zero applied thermal resistance and corresponds to an effective ground resistance $R_g(\eta)$ of

$$R_g(\eta) = \frac{\pi}{\lambda}\frac{1-\cos(\eta)}{\sin(\eta)}. \tag{A20}$$

If a thermal resistance R' is now applied at $y = 0$, $-\tfrac{1}{2}D \leqslant x \leqslant \tfrac{1}{2}D$ in the real coordinates, this becomes

$$R'\left.\frac{\partial\xi}{\partial y}\right|_{\xi=0} \text{ which is equal to } \frac{\delta}{\lambda\sin(\eta)}$$

in the transformed coordinates. We now assume that, at least for small values of the applied resistance, this can be added to the effective ground resistance $R_g(\eta)$. The heat flux density then becomes

$$q = \cfrac{1}{\cfrac{\pi}{\lambda}\cfrac{1-\cos(\eta)}{\sin(\eta)} + \cfrac{\delta}{\lambda\sin(\eta)}}$$

$$= \frac{\lambda}{\pi}\frac{\sin(\eta)}{1+\delta/\eta-\cos(\eta)}. \qquad \text{(A21)}$$

The net additional heat flow from inside to outside the building is the integral of the heat flux density along $\eta = 0$ from $\xi = 0$ to ∞. This is equal to the integral to the heat flux density along $\xi = 0$ from $\eta = 0$ to the point at which the line of heat flux entering at that point runs parallel to the ξ-axis at $\xi = \infty$. The lines of heat flux, being normal to the isothermals θ = constant, are given by the conjugate function of (A17), i.e. the real part:

$$\frac{2}{\pi}\,\mathrm{Re}\{-\ln(1-e^{-\xi+i\eta})\} = \text{constant}$$

or

$$1-2e^{-\xi}\cos(\eta)+e^{-2\xi} = \text{constant}. \qquad \text{(A22)}$$

Putting $\xi = \infty$ in (A22) gives the constant as unity, so the only flux line remaining in the system at $\xi = \infty$ is that for which

$$1-2e^{-\xi}\cos(\eta)+e^{-2\xi} = 1 \qquad \text{(A23)}$$

and at $\xi = 0$ this flux line is located at

$$\cos(\eta) = \tfrac{1}{2}. \qquad \text{(A24)}$$

The integral of (A21) now gives the additional heat flow per unit perimeter of floor, for finite thermal resistance in the edge region compared with infinite thermal resistance:

$$\Phi-\Phi_\infty = \frac{\lambda}{\pi}\int_0^{\cos^{-1}(1/2)} \frac{\sin(\eta)\,d\eta}{1+\delta/\pi-\cos(\eta)} \qquad \text{(A25)}$$

$$= \frac{\lambda}{\pi}\ln\left(\frac{\frac{1}{2}\pi}{\delta}+1\right)$$

$$= \frac{\lambda}{\pi}\ln\left(\frac{\frac{1}{4}\pi D}{d'}+1\right) \qquad \text{(A26)}$$

(A26) establishes the logarithmic form in terms of the ratio D/d', of the effect of horizontal edge insulation.

The factor $\frac{1}{4}\pi$, however, is approximate because the upper limit of the integral in (A25) was obtained from consideration of the temperature field in the absence of the edge insulation. The latter alters the whole temperature field, whence the lines of heat flux. Replacing $\frac{1}{4}\pi$ by unity ensures that in equation (10) the effect of the edge insulation reduces to zero as $d' \to 0$ and as $D \to 0$, as is essential, and the validity of this adjustment is confirmed by the comparisons with numerical calculations.

The analysis has considered the case of constant floor surface temperature except in the edge region. It is assumed that the effect of surface resistance is small, save to add to d' in the edge region. It is also assumed that the effect of wall thickness adds linearly to that of the insulation, as has been established in the case of all-over insulation [7].

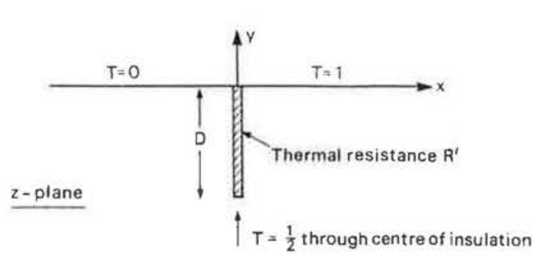

Fig. 7. Boundary conditions for vertical edge insulation.

APPENDIX B

VERTICAL EDGE INSULATION

The case of vertical edge insulation is illustrated in Fig. 7. For the situation of zero thermal resistance at the inside floor surface,

$$T = 1, \quad x > 0,\ y = 0 \qquad \text{(B1)}$$

$$T = 0, \quad x < 0,\ y = 0. \qquad \text{(B2)}$$

Symmetry considerations also require that $T = \frac{1}{2}$ at the middle of the insulation, i.e.

$$T = \tfrac{1}{2}, \quad x = 0,\ y < 0. \qquad \text{(B3)}$$

The vertical edge insulation is idealized to a strip of zero thickness but finite thermal resistance R'.

Let

$$z = x+iy \qquad \text{(B4)}$$

and transform the actual shape into a straight line in the z'-plane by the transformation (see Fig. 8):

$$z' = \sqrt{z^2+D^2}. \qquad \text{(B5)}$$

The analysis can then proceed as in Appendix A, except that in the region $-D < x' < D$ the applied thermal resistance is $\frac{1}{2}R'/\sqrt{D^2/x'^2-1}$ and the boundary temperature is $\frac{1}{2}$, leading to

$$\Phi-\Phi_\infty = \frac{\lambda}{\pi}\ln\left(\frac{\pi D}{d'}+1\right). \qquad \text{(B6)}$$

For the same reasons as are discussed in Appendix A, $\pi D/d'$ is adjusted to $2D/d'$.

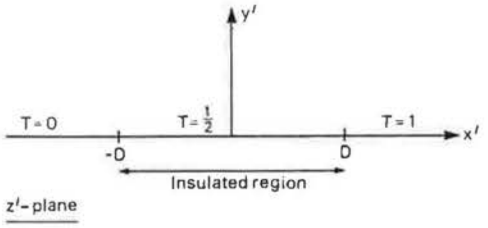

Fig. 8. Initial transformation of vertical edge insulation to situation analogous to Fig. 5.