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Optimal, Sub-optimal and Adaptive Control Methods for the Design of Temperature Controllers for Intelligent Buildings

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The application of modern control theory to design control systems for buildings are explored. Example problems dealing with HVAC systems and indoor environment control are considered. Linear models of the systems are used to design controllers based on (i) pole-placement technique, (ii) optimal regulator theory and (iii) adaptive control. The responses of the systems subject to disturbances are investigated. The results illustrate the advantage of one method over the other and emphasize the importance of the use of improved methods to design control systems for intelligent buildings.

NOMENCLATURE

- $\mathbf{A}$ system matrix
- A_a air-side area of the cooling coil
- a_h exterior surface heat loss coefficient of the hot water tank
- a_{ij} elements of matrix A
- a_w exterior surface heat loss coefficient of the chilled water tank
- a_r zone heat loss coefficient
- $\mathbf{B}$ input matrix
- b_i, b_{ij} elements of matrix B
- C_c heat capacity of the cooling coil
- C_h heat capacity of the hot water storage tank
- C_i constant coefficients
- C_{pa} specific heat of air
- C_{pw} specific heat of water
- C_w heat capacity of the chilled water storage tank
- C_z heat capacity of the zone air mass
- $\mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3$ disturbance matrices
- d_{11}, d_{12} elements of matrix D_2
- d_{s1}, d_{s2} elements of matrix D_1
- d_{w1}, d_{w2} elements of matrix D_3
- h enthalpy of air
- h_a air-side heat transfer coefficient
- h_{fg} enthalpy of vaporization
- h_s enthalpy of supply air
- h_z enthalpy of zone air
- J performance index
- K gain matrix (Eq. 5)
- $K(n)$ adaptive gain (Eqs. 20, 21)
- L gain vector (Eq. 13)
- M_z mass of zone air
- m_a mass flow rate of air (Eqs. 17, 22, 23)
- m_w rate of moisture addition to the space
- P coefficient of performance
- P_{max} maximum P
- $\mathbf{Q}$ weighting matrix for the states
- q_s rate of sensible heat load
- q_L rate of latent heat load
- $\mathbf{R}$ weighting matrix for the control input
- S variable, matrix Riccati equation
- $\mathbf{T}$ temperature vector
- $\mathbf{T}_a$ actual outdoor temperature
- T_a forecasted outdoor temperature
- T_b boiler temperature
- T_c coil temperature
- T_e boiler room temperature
- T_h hot water temperature
- T_o heat source temperature for the heat pump
- T_s supply air temperature
- T_w domestic hot water temperature
- T_{wi} feed water temperature
- ΔT_{max} maximum temperature differential for the heat pump
- t time
- t_f final time
- t_o initial time
- $\mathbf{U}$ control input vector
- U_1, U_2 control inputs for the discharge air temperature system (Eq. 18)
- U_2, U_3 control inputs for the boiler and the domestic hot water circuits (Eqs. 1, 2)
- U_{1max} maximum value of U_1
- U_{2max} maximum value of U_2
- V_1 control input, mass flow rate of supply air (Eq. 24)
- V_2 control input, mass flow rate of chilled water (Eq. 25)
- V_3 control input, input energy to the chiller (Eq. 26)
- V_{imax} maximum values of V_i ($i = 1, 2, 3$)
- W_s supply air humidity ratio
- W_z zone air humidity ratio

Greek letters

- Δ variations from the operating point
- α tuning parameter (Eq. 20)
- β tuning parameter (Eq. 20)
- ε small positive error (Eq. 20)
- ε effectiveness (Eqs. 25, 26)
- ξ effectiveness

Subscripts

- o operating point

Superscripts

- t transpose
- / transpose
- * optimal value

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1. INTRODUCTION

IN LARGE commercial buildings, Energy Management Control Systems (EMCS) are used to achieve integration, good control and high operational efficiency of building service systems. Along with the control of heating, ventilating and air conditioning (HVAC), the EMC systems also provide other functions such as alarm circuits, audio, visual communications, fire safety, and security [1].

A building in which the EMCS and the office automation systems are integrated is sometimes referred to as an "intelligent building". Although this definition is perhaps debatable, the key characteristics of "intelligent buildings" are integration and automation. Automation is achieved by installing control systems for various building services. In this regard, not only the number of control points but the detail to which each control loop is designed gives an indication of the level of "intelligence" incorporated in building services.

To illustrate the various hierarchical levels of control system design to building service systems, two examples will be considered. These include: (i) control of heating systems (such as a boiler) used for hot water and space heating and (ii) control of indoor environment by controlling the mass flow rate of supply air (as in a variable air volume system) or its temperature (as in constant volume discharge air temperature control system). These problems are typical in HVAC control applications. Before undertaking the control system design, it is instructive to review the work done in this area.

2. LITERATURE REVIEW

As far as the heating systems for hot water and space heating are concerned, boilers and heat pumps are extensively used. Although a number of studies on modelling of hot water boilers [2, 3] exist, most of these models use simple ON-OFF control actions with differential limits. In practical gas-fired systems [1], controls include air-fuel metering and measurement of CO_2/O_2 to improve combustion efficiency. As a safety precaution, the water pump is interlocked with the boiler to ensure water flows before the burner is turned on. On the room side, the control usually means a two-position room thermostat and in some cases modulating gas valves. As far as theoretical work in designing improved control strategies for boilers is concerned, only recently optimal control issues for integrated boilers are addressed [4]. Although in that study an improved concept in control was developed, it needs to be extended to include room air temperature fluctuations.

There have been several studies done on the use of heat pumps for space heating [5]. Also, the issues related to control of heat pumps are addressed in [6–8]. However, these studies are limited to single zone systems and need to be extended to multizone indoor environments.

With regards to the work done in HVAC systems control, it can be classified according to the type of system involved. For example, there are control issues related to either constant or variable air volume systems, single or multizone systems, single or dual-duct systems [9]. Of all types of HVAC systems, the variable air volume (VAV) systems are widely used in large buildings because of their potential for energy savings. However, they have also been found to be difficult to control [10, 11]. The reason is that when the air flow quantities are varied in response to the space loads, the relative humidity in space(s) increases because of low supply air quantities. At the same time, low supply air quantities could cause potential air quality problems. The net effect is that comfort is compromised. For these reasons, designing optimal control strategies for VAV systems is an important issue.

Even though the energy saving features of multizone HVAC systems are recognized [12, 13], these savings cannot be fully realized without good control strategies for operating them efficiently. To this end, the literature survey indicates that no comprehensive studies have been done on the design of control strategies for HVAC systems. This is due to the fact that even the simplest HVAC system models are nonlinear. However, what has been done by way of design is that industrial controllers, such as proportional and proportional-integral type have been designed and those too for simple processes like temperature or static pressure control, treated as separate processes. For example, the problem of discharge air temperature control is investigated by several researchers. In this system, the air is heated by passing it through a hot water heating coil. The control problem is to regulate the mass flow rate of hot water in order to hold the leaving air temperature through the coil at the desired setpoint. Among the studies done on this subject, the works of Stoecker *et al.* [14], Shavit *et al.* [15] and Mehtha [16] are important. Stoecker *et al.* [14] developed an analytical model for discharge air temperature control and verified the controller design through laboratory experiments. Shavit *et al.* [15] modelled the dynamic performance of a discharge air temperature control with a proportional-integral controller. Mehtha [16] not only designed a proportional-integral controller to maintain the discharge air temperature at a constant value, he also considered comfort factor in his analysis by way of controlling the reset time as a function of temperature fluctuations. Recently, digital controller designs for the discharge air temperature control and methods for automated tuning have been proposed [17, 18].

The problem of controlling relative humidity in spaces is investigated by Howell [19]. He used coil bypass as the control technique. His analysis was based on simulation methods rather than on controller design.

In another class of studies involving the HVAC systems, adaptive control methods [20], optimal and sub-optimal control methods [21–28] have been explored. For example, Farris and McDonald [20] reported an algorithm for direct digital control (based on adaptive optimal control methods) for solar heated buildings. However, they have limited their study to single-zone air space with room air temperature as the output. In another study, Townsend *et al.* [21] designed an optimal control strategy for a general environmental space. They examined optimal control policies for a single-zone environmental space by neglecting the coupling between space dry bulb temperature and relative humidity. The same authors [22] later proposed a digital control implementation for the same single zone environmental control problem. Kaya *et al.* [23] also presented optimal control policies to minimize energy use in HVAC systems. However, their analysis is based on steady state techniques to find an optimal solution which is used as a reference signal to be tracked via a digital implementation scheme.

Optimal and sub-optimal control studies for space heating have been examined by Rink *et al.* [24], Zaheer-uddin [25, 26]. For example, reference [24] examines the optimal control of heat pump systems for space heating when time-of-day energy price incentives for electrical energy (used to run the heat pump) are assumed to be in effect. The advantage of this method is that it considers the interactions between space loads, and the primary plant (heat pump). The limitation is that it is applicable to single-zone systems and uses a simple space loads model to represent the building envelope characteristics. An improvement to this method is demonstrated in [25]. These studies consider more accurate zone models to describe the building envelope. However, they are valid for single-zone systems and only consider the space heating problem where the output variable to be controlled is the indoor air temperature. Nevertheless, the design methods presented in Le *et al.* [27] and Zaheer-uddin [26] have potential applications in the design of control strategies for central HVAC systems. A recent study [28] shows the application of steady state optimal control technique to design control strategies for a VAV system. Thus, it is evident that methods for the design of control strategies for HVAC systems must be explored. It is important to note that, in the absence of such design methods, the practising engineers have developed several different types of control schemes [1] for operating the central HVAC systems. These are often based on trial and error methods, and operating experience. Since HVAC systems fall into the class of multi-input/multi-output systems, they are in fact difficult to tune based on experience alone. The effect of tuning one controller at a time on other controllers and accompanying problems in comfort and energy trade-offs cannot be balanced easily.

Recently Dexter [29] and Kelly [30] have done a comprehensive literature review of the practices in control system simulation and computer control. In [29], digital controller designs and self tuning control algorithms are described. In [30], the state-of-the-art in HVAC control is reviewed. The subject is divided into regulation, supervisory control and optimized building controls.

As far as theoretical developments of design methods are concerned, it is evident that the application of modern control theory to tackle the indoor environment control problems is lacking. One of the primary reasons for this is that even the simplest indoor environment control problems are nonlinear and belong to a class of multivariable systems.

Therefore, rather than depending on classical control methods which are inadequate, it is necessary to explore the application of modern control techniques to design control systems for HVAC. Of course, not all the issues pointed out in the literature could be resolved immediately. However, what might be possible though is that sub-optimal solutions to difficult control problems could be obtained which could be easier to implement. With this as the motivation, this paper illustrates the application of the following design methods to HVAC and indoor environment control. The design methods include: (i) multivariable controller design by pole-placement (for hot water boiler), (ii) optimal regulator design (for a single zone environmental space) and (iii) adaptive control (for discharge air temperature control and a variable air volume (VAV) control system).

3. MULTIVARIABLE CONTROLLER DESIGN

Here, the design of state feedback controllers via pole placement technique [31] is demonstrated by considering a hot water heating system which provides space heating and domestic hot water requirements.

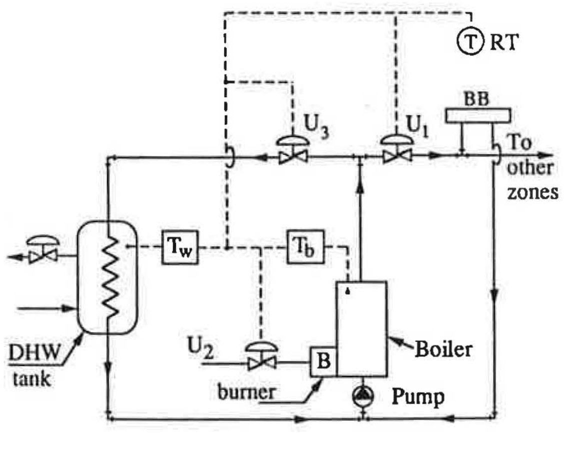
Figure 1 shows the schematic diagram of a hot water boiler supplying space heating and domestic hot water (DHW) needs. Also, shown in the figure (dash lines) is a feedback control system. The space heating is achieved by circulating the hot water in baseboard radiator(s). In response to higher heating load, the room thermostat calls for more heating. To meet this demand either the mass flow rate of hot water through U_1 can be increased or the temperature of hot water can be increased via U_2 which increases the fuel firing rate to the burner. On the other hand, the boiler also supplies domestic hot water needs via a DHW tank shown in the figure. The temperature of hot water T_w can be controlled by varying U_3 or U_2 or both. Although the system has three control inputs, for simplicity only two control inputs will be considered. This means the valve U_1 will be assumed to be fully open as such only U_2 and U_3 act as control inputs.

By applying the energy balance principle, the following state equations can be written [4]

$$\frac{dT_b}{dt} = C_1U_2 - C_2U_3(T_b - T_w) - C_3(T_l - T_a) - C_4(T_b - T_l) - C_5U_2T_b \quad (1)$$

$$\frac{dT_w}{dt} = C_6U_3(T_b - T_w) - C_7(T_w - T_{wi}) - C_8(T_w - T_l) \quad (2)$$

where T_b is the boiler temperature and T_w is the domestic hot water temperature. The coefficients C_1 through C_8



RT: Room thermostat
BB: Baseboard

Fig. 1. Schematic diagram of a heating system for hot water and space heating.

describe the system parameters. It may be noted that the space heating load was modeled as a static component and its effect on the transient response of the heating system were examined by treating the space heating load as an external disturbance. Since the model equations are nonlinear, the state equations were linearized about an operating point such as

$$\begin{aligned} T_b &= T_{bo}+\Delta T_b, \\ T_w &= T_{wo}+\Delta T_w, \\ U_2 &= U_{2o}+\Delta U_2, \\ U_3 &= U_{3o}+\Delta U_3, \end{aligned} \tag{3}$$

and the linear model (about the operating point) is given by

$$\begin{bmatrix}\Delta \dot{T}_b \\ \Delta \dot{T}_w\end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}\Delta T_b \\ \Delta T_w\end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22}\end{bmatrix}\begin{bmatrix}\Delta U_2 \\ \Delta U_3\end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ 0 & d_{22}\end{bmatrix}\begin{bmatrix}\Delta T_a \\ \Delta T_z\end{bmatrix} + \begin{bmatrix} 0 \\ e_2\end{bmatrix}\Delta T_{wi}. \tag{4}$$

The magnitudes of the matrix elements were computed for a typical system with parameters described in Reference 26.

The control problem may be stated as follows: determine a control law to regulate the inputs ΔU_2 and ΔU_3 such that the hot water temperature and boiler temperature are held at their respective setpoints in the presence of disturbances in ambient air temperature or the variations in the feedwater temperature to the DHW tank. The control law for this system is

$$\Delta \mathbf{U} = -\mathbf{K}\Delta \mathbf{T}, \tag{5}$$

where **K** is the gain matrix which was computed by using the method described in [31].

Figures 2(a) and 2(b) show the open-loop responses of the system. At time zero, the system was assumed to be $-1°C$ from the operating point ($T_{bo} = 131.4°C$ and $T_{wo} = 60.07°C$; $U_{2o} = 0.3$ and $U_{3o} = 0.5$). The time response shown in Figs 2(a) and 2(b) suggest that approximately 15 hours are needed before the system attains steady state conditions. This is of course not acceptable in practice as such a feedback control system is necessary to improve the time response characteristics of the system. The system response with feedback control (Equation (5)) is shown in Figs 2(c)–2(f). Figures 2(c) and 2(d) show the T_b and T_w responses and Figs 2(e) and 2(f) show the magnitude of the control inputs U_2 and U_3. Note that the magnitudes of control inputs U_2 and U_3 shown are the normalized values. That is, they range between 0 and 1. It can be seen that the effect of step change ($-1°C$) is eliminated in about 12 minutes and the system is brought back to the operating point. In the process, the maximum value of U_2 (burner firing rate) was about 0.52 and the mass flow rate value U_3 was 0.9. These values are within normal operating limits of the system.

The results suggest that the constant state feedback control seems to be quite adequate in rejecting impulse disturbances. Of course the transient performance specifications can be further improved by searching for new pole locations, however the need for this is not warranted given the fact that controller performance is good. Tests were also conducted to study the system performance under step disturbances. In this case, the state feedback control although gives good response but shows a small finite error in the final value. This steady state error can be eliminated by adding an error integrator. The important thing to note here is that the performance of the system, in terms of transient response and disturbance rejection, is improved because of the multiple input control. It would be unrealistic to expect similar performance from a system which uses single input (for example only U_2 control) as a control variable.

4. OPTIMAL REGULATOR DESIGN

To illustrate the method, consider the problem of controlling the quantity of supply air to a zone (via damper control) in order to maintain the zone air temperature and humidity ratio within desired limits despite disturbances (such as changing cooling loads) acting on the zone. The energy and mass balance equations on a typical zone (shown in Fig. 3) can be written as

$$M_z \frac{dh_z}{dt} = V_1 V_{1max}(h_s - h_z) + q_s + q_L \tag{6}$$

$$M_z \frac{dW_z}{dt} = V_1 V_{1max}(W_s - W_z) + m_w \tag{7}$$

where the enthalpy h can be expressed as

$$h = C_{pa}T + W_z(h_{fg} + C_{pw}T). \tag{8}$$

In Equations (6) and (7), V_1 is the control variable which regulates the amount of supply air (of enthalpy h_s and humidity ratio W_s) to the zone. The disturbances acting on the zone are (i) sum of all sensible loads q_s, (ii) latent loads q_L, and (iii) the rate of water vapour m_w added to the space. The question is: what is the optimal policy for controlling V_1 so as to reject the disturbances in q_s, q_L and m_w such that the zone air temperature T_z and humidity ratio W_z are maintained within the desired comfort limits? In order to answer this question, Equation (6) will be rewritten in terms of T_z using (8), and the model equations will be linearized about an operating point so that the linear model can be written as

$$\begin{bmatrix}\Delta \dot{T}_z \\ \Delta \dot{W}_z\end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22}\end{bmatrix}\begin{bmatrix}\Delta T_z \\ \Delta W_z\end{bmatrix} + \begin{bmatrix} b_1 \\ b_2\end{bmatrix}\Delta V_1 + \begin{bmatrix} d_{s1} \\ d_{s2}\end{bmatrix}\Delta q_s + \begin{bmatrix} d_{11} \\ d_{12}\end{bmatrix}\Delta q_L + \begin{bmatrix} d_{w1} \\ d_{w2}\end{bmatrix}\Delta m_w, \tag{9}$$

$$\Delta \mathbf{T} = [\Delta T_z \Delta W_z]^t, \tag{10}$$

with system output given by

$$y = [1 \quad 0]\begin{bmatrix}\Delta T_z \\ \Delta W_z\end{bmatrix} \tag{11}$$

implying that we are interested in studying in zone temperature (output) response in the presence of disturbances. This is a standard LQ (linear quadratic) regulator problem. The performance index to be minimized can be written as

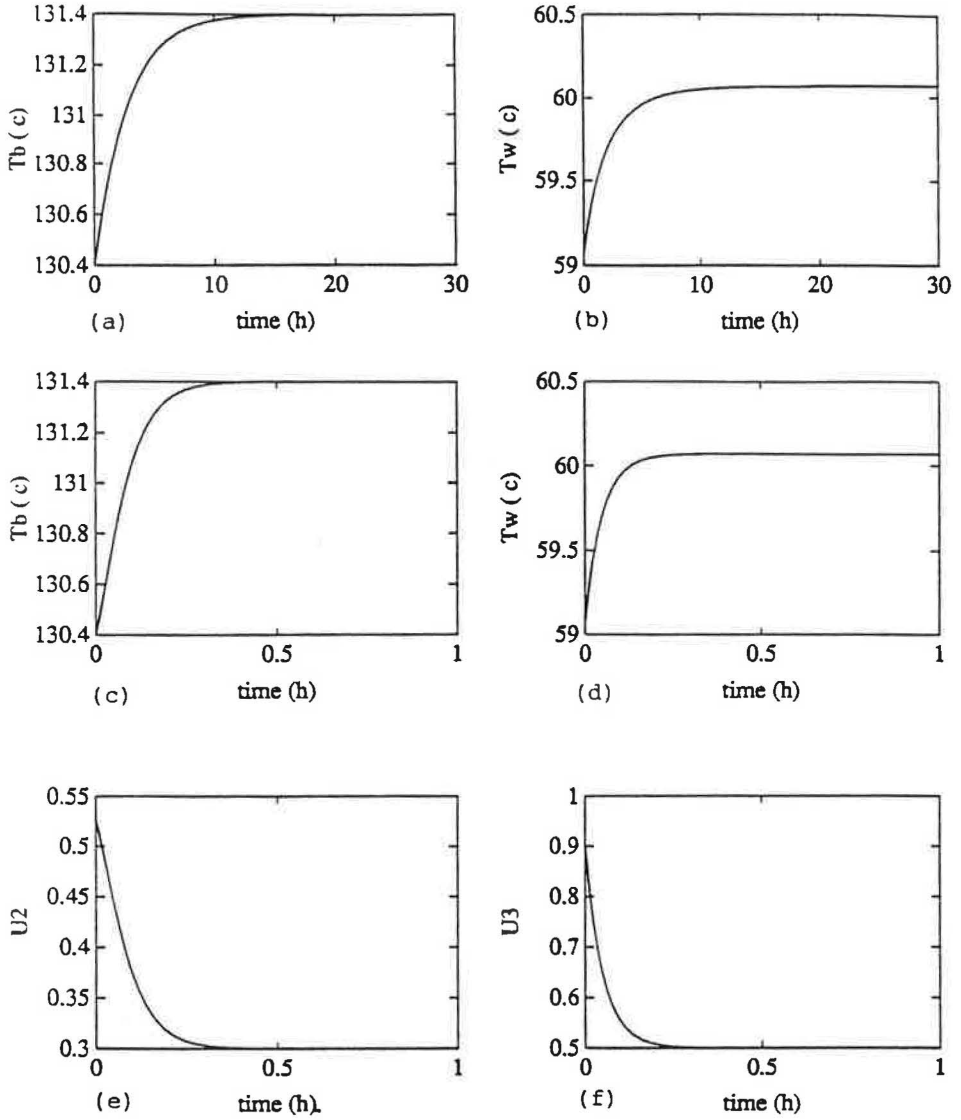


Fig. 2. Temperature and control input responses of the hot water heating system. (a) Open-loop response of T_b; (b) open-loop response of T_w; (c) closed-loop response of T_b; (d) closed-loop response of T_w; (e) U_2 response; (f) U_3 response.

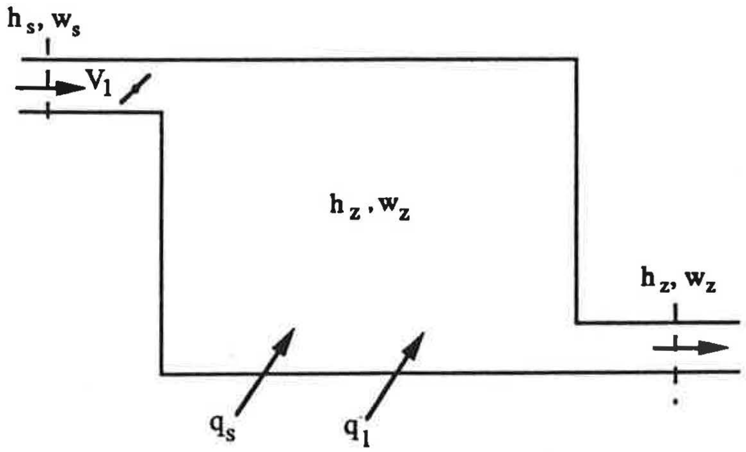


Fig. 3. Energy balance on an environmental zone.

$$J = \frac{1}{2}\int_{t_0}^{t_f} (\Delta \mathbf{T}'\mathbf{Q}\Delta T + \Delta V_1' R \Delta V_1)\,dt \qquad (12)$$

where the terminal cost is neglected. The solution to the regulator problem yields optimal control law [32]

$$\Delta V_1^* = -L\Delta \mathbf{T} \qquad (13)$$

where the gain L is given by

$$L = -R^{-1}B'S(t) \qquad (14)$$

and $S(t)$ is obtained by solving the matrix Riccati equation

$$\dot{S}(t) = -SA - A'S + SBR^{-1}B'S - \mathbf{Q}, \qquad (15)$$

with boundary condition

$$S(t_f) = 0. \qquad (16)$$

The important thing to note is that the optimal rate at which the air is to be supplied to the zone (ΔV_1^*) can be computed via Equation (13). ΔV_1^* is optimal in the sense that it minimizes the performance index J (Equation 12) which penalizes the fluctuations in $\Delta \mathbf{T}$ via the weighting matrix $\mathbf{Q}$ and at the same time ensures that the magnitude of ΔV_1 is varied within acceptable limits via weighting factor $\mathbf{R}$. Thus, we have a mechanism by which $\Delta \mathbf{T}$ and ΔV_1 are computed as per the pre-specified weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ to regulate the transient response of $\Delta \mathbf{T}$ and ΔV_1.

Figures 4(a) and 4(b) illustrate the optimal responses of T_z and V_1 obtained by solving the above LQ regulator problem. Figure 4(a) depicts the zone temperature response when a step change of 3°F (1.667°C) from the operating point (24.08°C) is applied. As shown in the figure the effect of step change is rejected in about 105 s and this was achieved by increasing the supply air flow rate to about 0.6 of the maximum flow rate initially as shown in Fig. 4(b) and gradually reducing the flow rate as T_z approaches the operating point. At the operating point the flow rate was 0.3 to keep the zone temperature constant in the presence of constant external disturbances in q_s and q_L.

The important thing to note here is that the LQ regulator problem allows one to design an optimal control law ΔV_1^* such that transient response characteristics of T_z and the magnitude of V_1 can be influenced as desired. As such this technique is more flexible than the pole-placement design where several trial runs may be necessary to select proper location of poles for an acceptable transient response.

5. ADAPTIVE CONTROL

Adaptive control involves three basic steps [33]: (i) identification of dynamic characteristics of the process, (ii) computation of performance index and comparison with optimal performance and (iii) adjustment of controller parameters to obtain desired control signals. An application of this technique to a solar heated/cooled building is illustrated in [20]. Another interesting application of adaptive control is described in [27]. In this method the reference signal is pre-computed based on forecasted loads and a self tuning scheme is used to update the controller gain. In the following, the application of this technique to control the discharge air temperature from a heating coil will be described.

5.1. *Discharge air temperature control*

Figure 5 depicts a discharge air temperature control system. Air is heated in a heating coil in which hot water is circulated. The hot water is heated by means of a heat pump. Note that a boiler can also be used in place of the heat pump. The temperature of air leaving the coil T_s can be controlled either by (i) controlling the mass flow rate of hot water $U_1U_{1\max}$ in the coil or (ii) the temperature of hot water. Here, the latter approach is considered. As shown in the figure, measurements of ambient temperature T_a, zone temperature T_z, hot water temperature T_h, discharge air temperature T_s and a forecast of ambient temperature $\bar{T}_a$ are required to control the input energy to heat pump U_2 such that the hot water temperature and consequently T_s are maintained at their desired reference values. The advantage of this technique comes from the fact that the reference signal can be pre-computed based on forecasted ambient temperature $\bar{T}_a$ by using steady state optimal control techniques. To illustrate the method, a simple model describing the energy balances on T_z and T_h is written as follows:

$$C_z\frac{dT_z}{dt} = -a_z(T_z - T_a) + m_a C_{pa}(T_s - T_z), \qquad (17)$$

$$C_h\frac{dT_h}{dt} = U_2U_{2\max}P - U_{1\max}\xi(T_h - T_s) - a_h(T_h - T_e), \qquad (18)$$

$$P = 1 + (P_{\max} - 1)\left(1 - \frac{T_h - T_o}{\Delta T_{\max}}\right). \qquad (19)$$

In Equation (17), the rate of heat stored is equated to heat losses from the zone to the ambient and the rate of heat supplied. Similarly, in Equation (18), the rate of heat stored in hot water is equated to the rate of heat supplied by the heat pump, the rate of heat delivered to the coil and the rate of heat loss from the hot water storage to surroundings at temperature T_e. P in Equation (19) is the coefficient of performance of the heat pump which is expressed as a function of T_h and T_o (source temperature).

The control problem may be stated as follows: determine a strategy to adaptively control the heat pump input U_2 such that the discharge air temperature is made to track an optimal reference temperature T_s^* under the presence of disturbances (changes in T_a) acting on the zone. By using the method described in [27], the steady state optimal solution to (17–19) can be found which can be implemented by the following control algorithm

$$U_2(n+1) = \alpha U_2(n) + \beta K(n)[\varepsilon + T_h^* - T_h] \qquad (20)$$

where α and β are constants (with $\alpha + \beta = 1.0$), T_h^* is the optimal hot water temperature and ε is finite error. A small value of ε is necessary to make the system more responsive to tracking errors. $K(n)$ in (20) is the adaptive gain given by

$$K(n) = \frac{a_z(T_z - T_a) + a_h(T_h - T_e)}{U_{2\max}P}, \qquad (21)$$

T_h^* in (20) is computed from

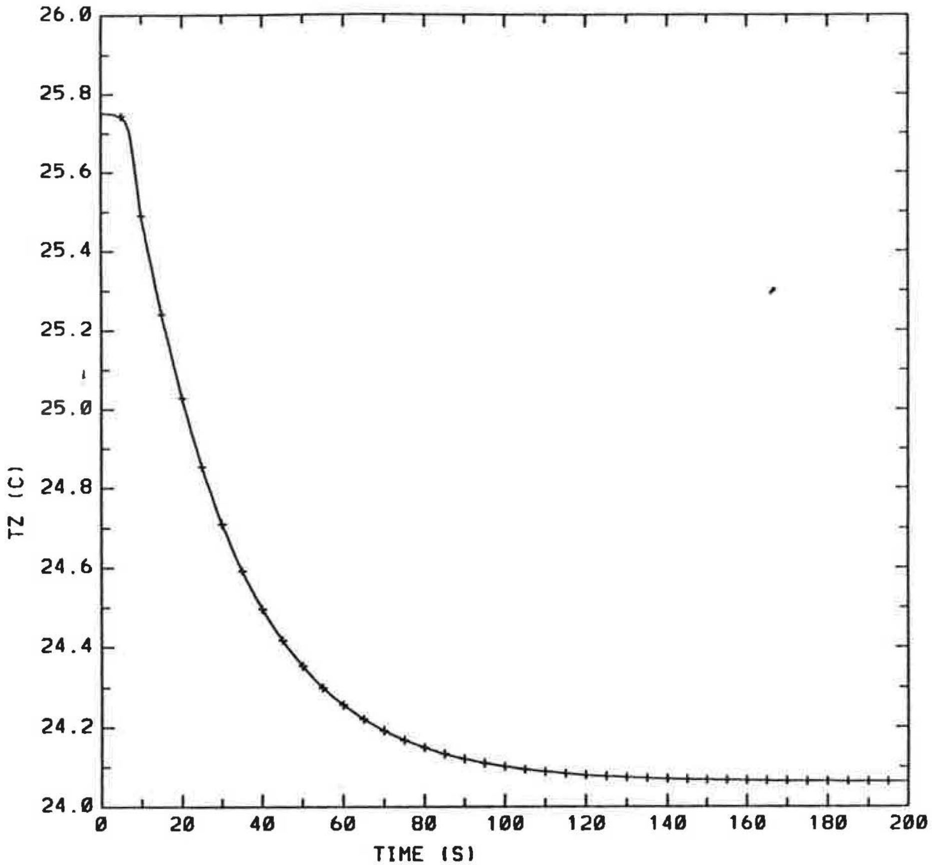


Fig. 4(a). Optimal response of zone air temperature to a step change in T_L.

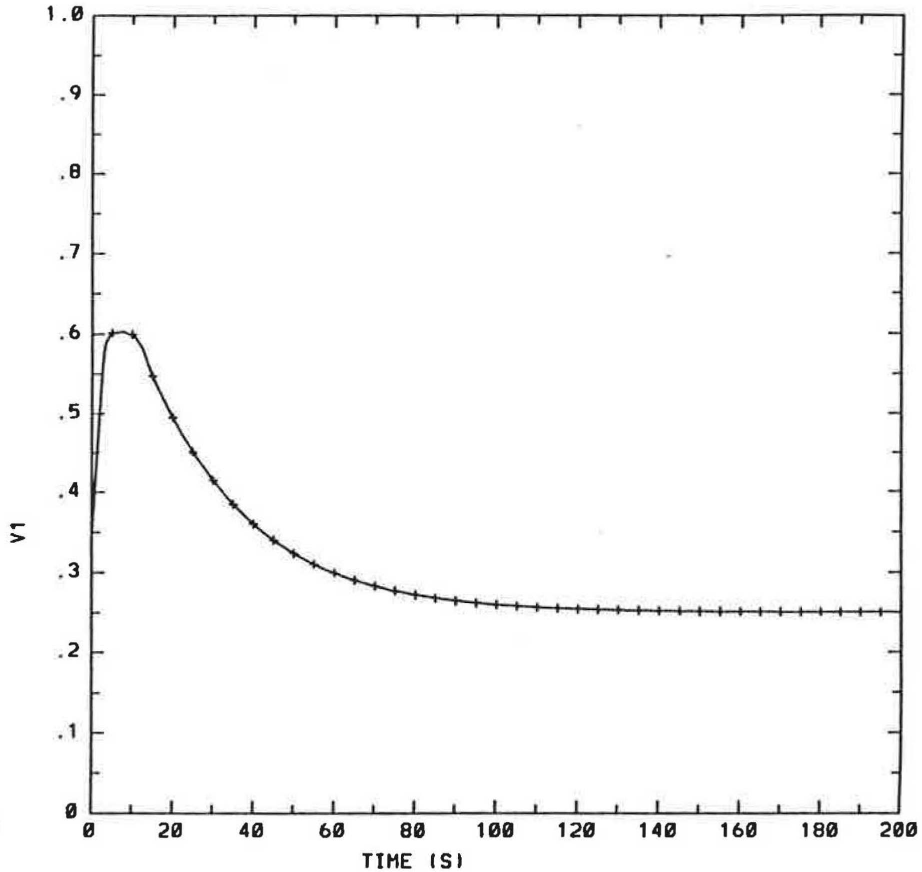


Fig. 4(b). Optimal control input (V_1) response.

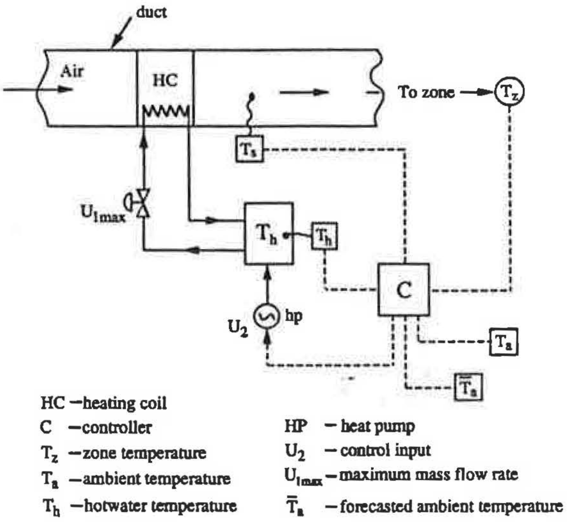


Fig. 5. Discharge air temperature control system.

$$T_h^* = T_s^* + \frac{m_a C_{pa}(T_s^* - T_z)}{U_{1max}\xi}, \tag{22}$$

where

$$T_s^* = T_z + \frac{a_z(T_z - \bar{T}_a)}{m_a C_{pa}}. \tag{23}$$

The new control signal $U_2(n+1)$ is updated based on previous value $U_2(n)$ and the error between T_h^* and T_h. It may be noted that the gain is updated according to (21) and T_h^* is computed based on forecasted $\bar{T}_a$ via (22–23). The control algorithm in a sense steers T_h to track T_h^* which ensures T_s to follow T_s^*.

Figures 6(a), (b) and (c) illustrate the time response of control input U_2, water temperature T_h and zone temperature T_z respectively due to a step decrease in ambient air temperature T_a. The input energy to the heat pump is increased initially (Fig. 6(a)) so that the water temperature (Fig. 6(b)) also increases to an optimal value T_h^*, consequently the effect of disturbance on T_z is rejected. This is shown in Fig. 6(c) in which T_z reaches the desired setpoint value 20°C. The advantage of this control algorithm comes from the fact that following a sudden increase in heating load, the question which often arises is to what temperature level T_h be raised? Over charging of the storage is wasteful of energy (through standby losses) and under charging would result in improper control of T_z. The control algorithm addresses this issue by tracking T_h towards a pre-computed optimal T_h^*. In the process, the energy consumption is minimized and zone temperature is maintained close to the setpoint.

5.2. *Model Reference Adaptive Control*

A class of adaptive systems, known as model-following or Model Reference Adaptive Control (MRAC), could also be used for indoor environment control. In MRAC, the desired performance of the system can be expressed in terms of a reference model [34].

As shown in Fig. 7, the system utilizes a simple feedback loop and an adaptation loop. The controller parameters are adjusted based on the error between the outputs from the reference model and the actual plant which could be a HVAC system with an environmental zone. Thus, the method gives an approach for adjusting the controller parameters so that the closed loop response of the actual system will be close to the reference model [34].

In order to illustrate the method, we consider a variable air volume system shown in Fig. 8. In this system the recirculated air is mixed with fresh outdoor air. The mixed air is cooled in a cooling coil in which chilled water is circulated from the chiller and storage tank arrangement shown in the figure. The chilled water temperature is controlled by increasing or decreasing the input energy to the chiller via controller C_3 and the supply air temperature is controlled by modulating the mass flow rate of chilled water via controller C_2. The mass flow rate of supply air to the zone is controlled via the damper control C_1. The control problem considered is as follows: assuming that the VAV system is at a certain operating point (in this example it was assumed that the chiller is operating at 50% capacity so that $V_3 = 0.5$ and $V_2 = 1.0$) it is required to control the supply air flow rate to the zone via the damper control C_1 such that the zone temperature is maintained close to the setpoint in spite of the variations in cooling loads acting on the zone. The model equations for the VAV system are

$$C_z(\mathrm{d}T_z/\mathrm{d}t) = -V_1 V_{1max} C_{pa}(T_z - T_s) + a_z(T_a - T_z) + q_s(t), \tag{24}$$

$$C_c(\mathrm{d}T_c/\mathrm{d}t) = V_2 V_{2max} C_{pw}\varepsilon(T_w - T_c) - A_a h_a\left(T_c - \frac{T_s + T_c}{2}\right), \tag{25}$$

$$C_w(\mathrm{d}T_w/\mathrm{d}t) = -V_3 V_{3max} P - a_w(T_w - T_a) + V_2 V_{2max} C_{pw}\varepsilon(T_c - T_w), \tag{26}$$

$$T_s = \left(\frac{V_1 V_{1max} C_{pa} T_z + A_a h_a T_c - 0.5 A_a h_a}{V_1 V_{1max} C_{pa} + 0.5 A_a h_a}\right), \tag{27}$$

$$P = (P_{max} - 1)\left(1 - \frac{T_o - T_w}{\Delta T_{max}}\right). \tag{28}$$

In order to design an adaptive controller for the VAV system by using the MRAC approach, the output responses of the system have to be defined by specifying a reference model rather than overshoot and settling time as is the case in classical control. Therefore, the selection of a reference model requires some understanding of the process in order to decide as to how the process output ideally should respond to the command signal. For the VAV system we could use a second-order model. With this choice of reference model, the mechanism for adjusting the parameters for the controller must be determined such that the overall system is stable and which makes the output error between the reference model and the process go to zero.

By following the approach described in [34], the equations for updating the regulator parameters were obtained. These equations are described in terms of a command signal, the error, the reference model and an adaptation rate. The results obtained by using this approach are shown in Figs 9(a)–(d).

The solid lines in Figs 9(a),(c) represent the output

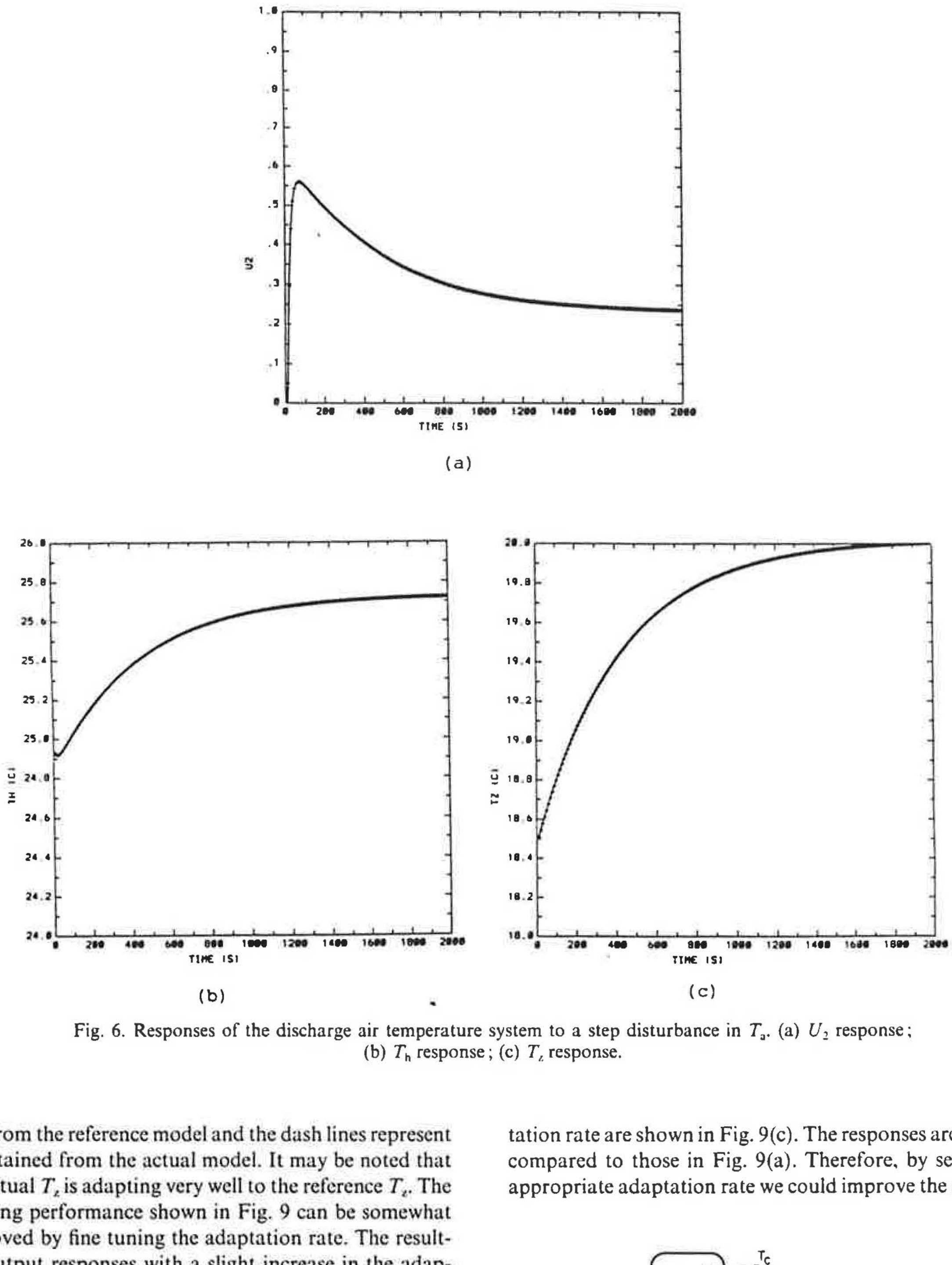


Fig. 6. Responses of the discharge air temperature system to a step disturbance in T_a. (a) U_2 response; (b) T_h response; (c) T_z response.

(T_z) from the reference model and the dash lines represent T_z obtained from the actual model. It may be noted that the actual T_z is adapting very well to the reference T_z. The tracking performance shown in Fig. 9 can be somewhat improved by fine tuning the adaptation rate. The resulting output responses with a slight increase in the adap-

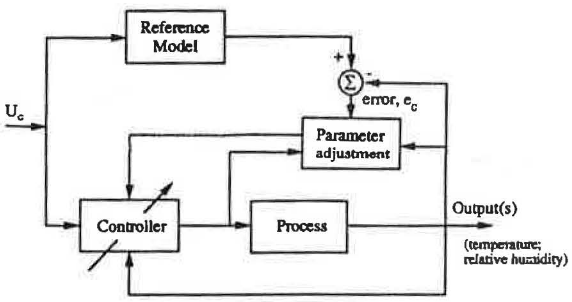


Fig. 7. Model reference adaptive control system.

tation rate are shown in Fig. 9(c). The responses are faster compared to those in Fig. 9(a). Therefore, by selecting appropriate adaptation rate we could improve the system

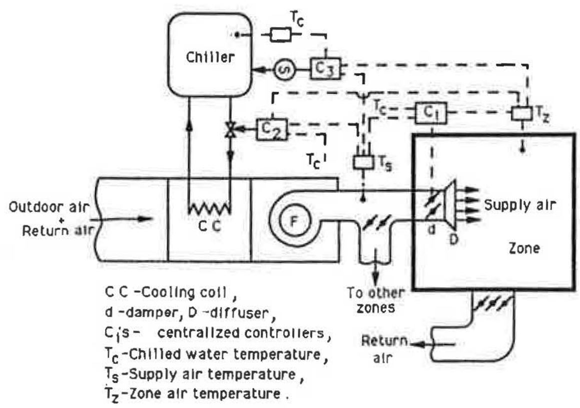


Fig. 8. Schematic diagram of a VAV system.

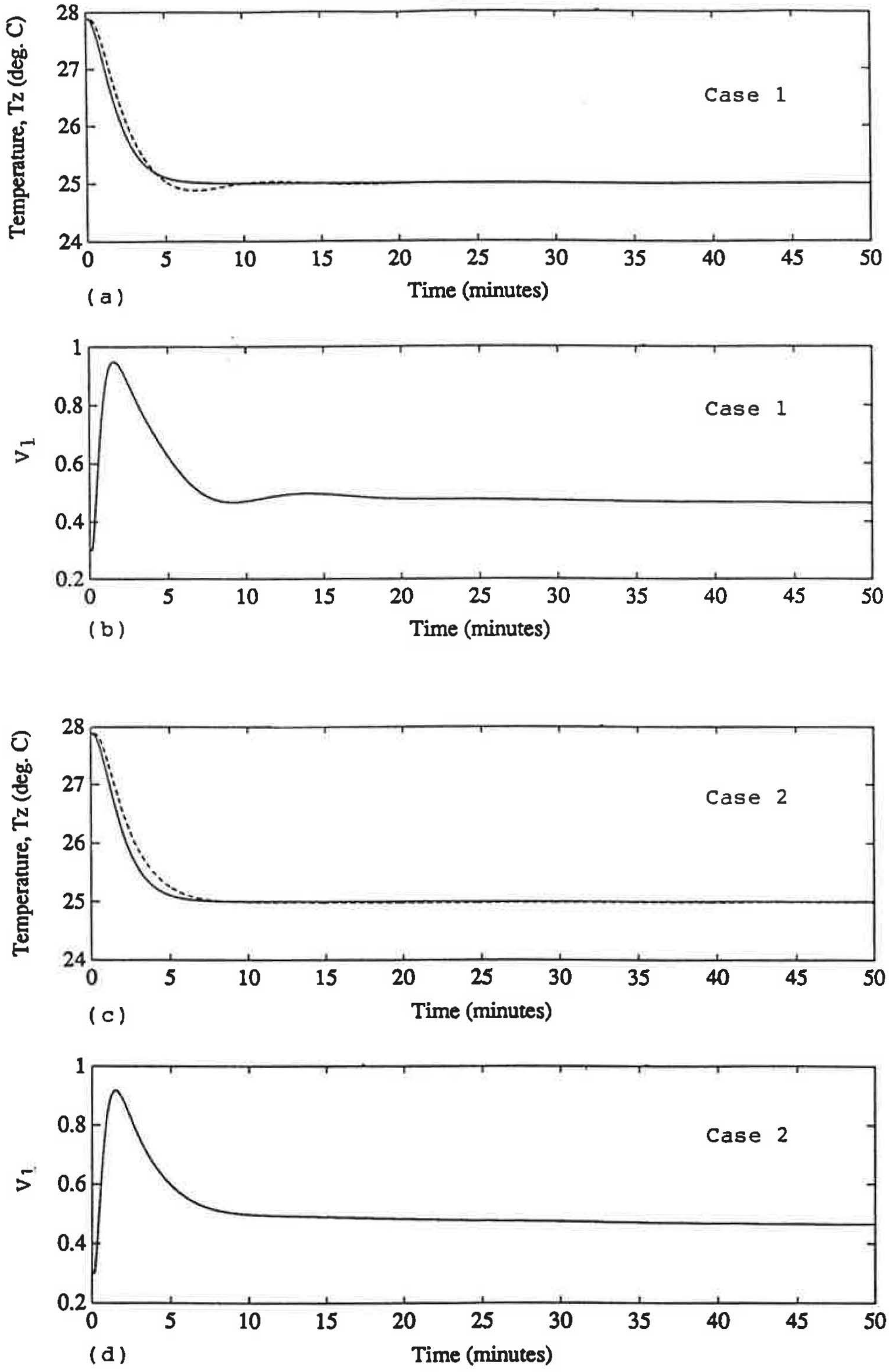


Fig. 9. Output responses from the model reference adaptive control system with two different adaptation rates. (Case 1—lower adaptation rate, Case 2—higher adaptation rate.) Case 1 : (a) T_z response ; (b) control input response. Case 2 : (c) T_z response ; (d) control input response.

performance. On the other hand, higher adaptation rates could also cause instability. Therefore, there is a trade-off between higher adaptation rates and the system response.

Figures 9(b)–(d) show how the mass flow rate of air to the zone (normalized with respect to the maximum flow rate) was controlled to maintain the zone temperature constant at 25°C. It may be noted that following a step increase in the cooling load on the zone, the mass flow rate of the supply air is initially increased followed by a gradual decrease until it reaches steady state.

6. CONTROL SYSTEMS FOR LARGE BUILDINGS

In the majority of modern commercial buildings, direct digital control (DDC) systems are used to control the HVAC systems. In spite of the superior control capabilities of DDC systems, several problems remain in terms of achieving good indoor environment control. These problems are largely due to the fact that the controller design and tuning in HVAC is still based on classical control methods in which each control loop is tuned independent of the other loops. This is not likely to be satisfactory because the HVAC system and the building enclosure form an integral part of the overall system which is coupled and nonlinear. The load patterns on the zones change frequently and associated with it are the dynamics of the building enclosure. Any combination of these factors could affect the control system performance if no provisions are made for these factors at the design stage. One approach for improving the control system performance is to utilize the past history of the load patterns and the future forecast of the weather. We are presently working on implementing these concepts in order to design good tracking controllers for the multizone VAV systems.

7. SUMMARY AND CONCLUSIONS

The applications of optimal, sub-optimal and adaptive control methods to indoor environment control problems have been illustrated with examples. Each method has some advantages over the other. By far the most effective are the state feedback controllers since small offsets which occur as a result of their use do not significantly affect the building comfort parameters. Optimal control methods offer greater flexibility in the design process in that both energy and comfort trade-offs can be examined. We have also shown that good temperature control can be achieved by a sub-optimal controller which regulates the system based on forecasted and actual loads. The application of model reference adaptive control to a VAV system has been illustrated. It has been shown that by proper choice of the adaptation rate, the output response of the VAV system can be improved. Applications of these methods for multizone indoor environment control are presently being studied.

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