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Predicting r.m.s. pressures from computed velocities and mean pressures

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Abstract

There is a need to calculate root mean square (r.m.s.) pressures from the output of steady-state computer programs. We know much less about calculating r.m.s. pressures than about calculating r.m.s. velocities.

R.m.s. pressures can be quickly estimated from calculated mean pressures, mean velocities and r.m.s. velocities using the equations in this paper.

The equations have been used in Wind Engineering but can be applied in any turbulent flow where pressures are required.

1. FORMULAE FOR R.M.S. PRESSURES IN HOMOGENEOUS ISOTROPIC TURBULENCE

The fundamental equations governing fluid flow are the Navier-Stokes equations. The Poisson equation for fluctuating pressures can be derived from the Navier-Stokes equations. From this equation, after many manipulations, Batchelor[1] found the following relationship:

$$p' = \sqrt{0.34}\,\rho u'^2 \tag{1}$$

The r.m.s. pressure p' is calculated directly from the local r.m.s. velocity u' and the density ρ. Hinze[2] followed a similar path and produced the following formula for large Reynolds numbers.

$$p' = \rho u'^2 / \sqrt{2} \tag{2}$$

The r.m.s. pressure predicted by Equation (2) is usually much less than the measured r.m.s. pressure in flows that have mean velocity gradients.

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2. FORMULAE FOR R.M.S. PRESSURES IN GENERAL FLOWS

The first published formula for r.m.s. pressure coefficients that incorporates both mean pressures and r.m.s. velocities is the Model 1 formula:

$$C_P' = 2[k_1 / 3 + 0.816|\overline{C}_P|\overline{U}_0\sqrt{k_0}] / \overline{V}^2 \tag{3}$$

C_P' is the r.m.s. pressure coefficient defined by:

$$p_1' / \rho = C_P'\overline{U}_0^{\,2} / 2 \tag{4}$$

$\overline{C}_P$ is the mean pressure coefficient defined by:

$$(\overline{P}_1 - \overline{P}_0) / \rho = \overline{C}_P(\overline{U}_0^{\,2} - \overline{U}_1^{\,2}) / 2 \tag{5}$$

$\overline{V}$ is a mean reference velocity, $\overline{U}$ is the mean velocity and k is the turbulent kinetic energy. Subscripts 0 and 1 represent upwind and local values respectively. This formula was published by Paterson[3] in 1989 and has since been used by Selvam[4] and Qasim[5].

The derivation of the Model 1 formula contains many assumptions that may not be justified. On these grounds it has been criticised by Selvam[4]. Model 2 is an attempt to put the calculation of r.m.s. pressures on a firmer mathematical base.

At large Reynolds numbers the streamwise component of the Navier-Stokes equations takes the following form.

$$-\frac{\partial U}{\partial t} = \frac{\partial}{\partial s}(P / \rho + U^2 / 2) + F_s \tag{6}$$

P is the instantaneous pressure, U is the instantaneous magnitude of the velocity vector and F_s is the streamwise component of the body force.

A mean pressure coefficient is inserted into the equations to account for cross-streamline total pressure variations. After much manipulation the following equation is found.

$$C_P'^{\,2} = 2u_0'^{\,4} / \overline{U}_0^{\,4} + 4\overline{C}_P^{\,2}(\overline{U}_0^{\,2}u_0'^{\,2} - \overline{U}_1^{\,2}u_1'^{\,2}) / (\overline{U}_0^{\,2} + u_0'^{\,2} - u_1'^{\,2})^2 \tag{7}$$

The relationship between the magnitude of the r.m.s. velocity and the turbulent kinetic energy is:

$$k = u'^{\,2} / 2 \tag{8}$$

This relationship is correct even when the turbulence is not isotropic. Substituting Equation (8) in Equation (7) leads to:

$$C_P'^{\,2} = 8k_0^{\,2} / \overline{U}_0^{\,4} + 8\overline{C}_P^{\,2}(\overline{U}_0^{\,2}k_0 - \overline{U}_1^{\,2}k_1) / (\overline{U}_0^{\,2} + 2k_0 - 2k_1)^2 \tag{9}$$

This reduces to the following equation where the local velocity is zero.

$$C_P'^2 = 8k_0^2 / \overline{U}_0^4 + 8\overline{C}_P^2\overline{U}_0^2 k_0 / (\overline{U}_0^2 + 2k_0 - 2k_1)^2 \tag{10}$$

This is the Model 2 formula. It is a better formula than the Model 1 formula in Equation (3). $k_1 = 0$ at solid walls but this relationship should not be used. It is better to calculate k_1 from the near constant value in the logarithmic boundary layer. This technique is compatible with the use of wall functions in CFD calculations.

3. THE TEXAS TECH EXPERIMENTAL DATA

The full-scale data was collected at the Wind Engineering Field Research Laboratory on Texas Tech University land in Lubbock, Texas. This laboratory is used to obtain reliable wind load data on low-rise structures[6,7].

The wind-tunnel data comes from the Fluid Dynamics and Diffusion Laboratory at Colorado State University[8]. The model scale was 1:100. 72 approach wind directions were used. The mean and turbulent flow properties in the approaching flow were set to match those measured in the field.

The numerical data comes from a finite-volume simulation of flow over the Texas Tech Building[3]. Calculations were done with approach flow angles at intervals of 10°. The flow was calculated on a 26 x 29 x 14 irregular Cartesian grid. This grid is sufficiently fine for errors due to turbulence modelling to exceed those due to grid resolution. The mean pressure coefficient is in close agreement with both wind-tunnel and full-scale data. A k-ε turbulence model was used. Recirculation was forced at the front corners by setting the adjacent longitudinal velocity to zero. The Reynolds number of the calculation was set to match that of the full-scale experiments.

4. RELATIONSHIPS BETWEEN C_P' AND $\overline{C}_P$

The locations of pressure tappings on the roof and walls of the Texas Tech experimental building are shown in Figure 1. The locations of the tappings in wind-tunnel tests are the same as in full-scale tests.

R.m.s. pressure coefficients are plotted against mean pressure coefficients for roof tappings from full-scale results in Figure 2a. The results are a composite from the nine roof tappings 50101, 50205, 50209, 50501, 50505, 50509, 50901, 50905 and 50909. Approach flows from all angles are included.

Figure 2b is the equivalent relationship derived from wind-tunnel results. The results are a composite from all 17 roof tappings at all 72 approach angles. The equivalent relationships predicted by Models 1 and 2 are shown in Figures 2c and 2d. Results are a composite from 247 points on the roof surface at 10 different approach flow angles.

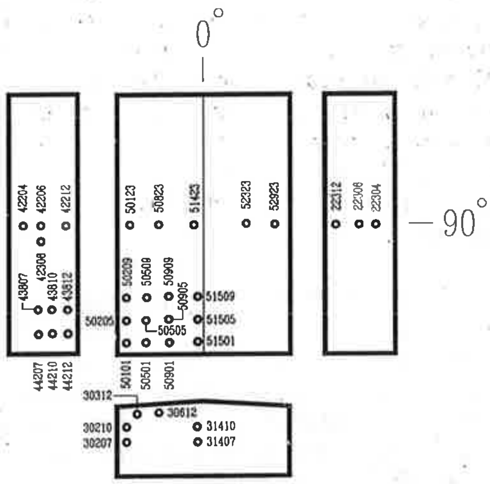

Figure 1. Pressure Tap Locations on the Texas Tech Experimental Building.

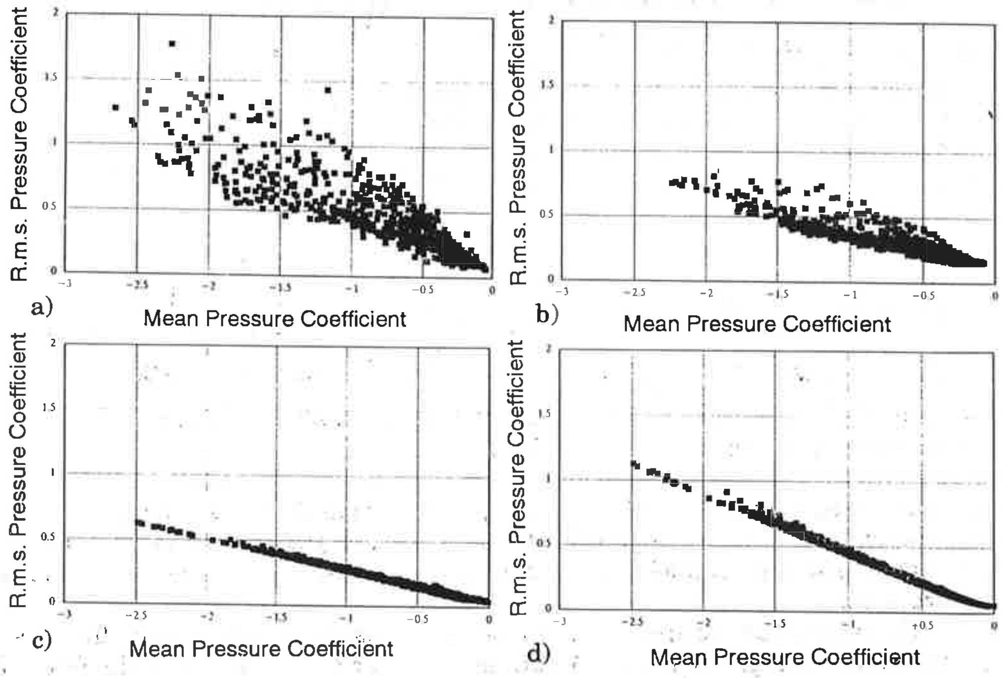

Figure 2. R.m.s. vs Mean Pressures from Roof Tappings: a)Full-Scale, b)Wind-Tunnel, c)Computer Model 1, d)Computer Model 2.

There is a near linear relationship between C_P' and $\overline{C}_P$ in Figures 2a, 2b, 2c and 2d. The slope of the line of best fit is about 0.6 in Figure 2a, 0.3 in Figure 2b, 0.12 in Figure 2c and 0.45 in Figure 2d. The scatter of results is largest in Figure 2a. In Figures 2c and 2d the scatter comes from the variation of the turbulent kinetic energy near the roof surface. The results from Model 2 agree much better with both wind-tunnel and full-scale data than those from Model 1.

R.m.s. pressure coefficients are plotted against mean pressure coefficients for two wall tappings from full-scale data in Figure 3a, from wind-tunnel data in Figure 3b, from Model 1 data in Figure 3c and from Model 2 data in Figure 3d. The wall tappings are 22306 and 42206. Each tapping lies in the centre of the long face of the building.

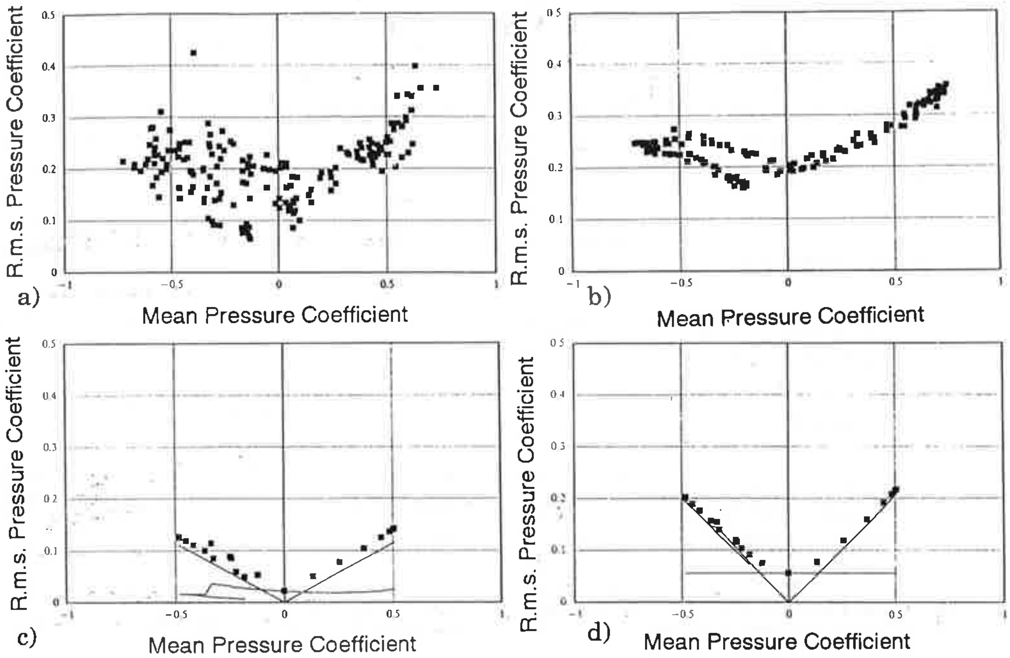

Figure 3. Pressures from Tappings 22306 & 42206: a)Full-Scale, b)Wind-Tunnel, c)Computer Model 1, d)Computer Model 2.

The mean pressure coefficients are positive when the wind is blowing onto the face (at approach flow angles of 20 to 160°), and negative at other approach flow angles. The wind-tunnel tests overpredict the r.m.s. pressures at near zero mean pressures.

R.m.s. pressure coefficients are plotted against mean pressure coefficients for two wall tappings from Model 1 data in Figure 3c and Model 2 data in Figure 3d. The values predicted by both model formulae are too small over the whole range.

Both model formulae correctly predict an increase in r.m.s. pressure with an increase in the magnitude of the mean pressure. The values predicted by Model 2 are in better agreement with full-scale and wind-tunnel results than those predicted by Model 1.

These formulae have been compared with the formula developed by Selvam[4]. Selvam's formula agrees with the Model 2 formula for large values of $|\overline{C}_P|$ but gives the unrealistic result $C'_P = 0$ when $\overline{C}_P = 0$.

5. CONCLUSIONS

Two numerical models are presented compared with wind-tunnel and full-scale results from flow over the Texas Tech experimental building. There are significant differences between the full-scale and wind-tunnel results. The r.m.s. pressures calculated from Model 2 are in much better agreement with wind-tunnel and full-scale results than those from Model 1. The predictions of r.m.s. pressures by both formulae at near zero mean pressure coefficients are poor.

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