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CALCULATION OF VENTILATION REQUIREMENTS IN THE CASE OF INTERMITTENT POLLUTION: APPLICATION TO ENCLOSED PARKING GARAGES

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The ventilation requirements for decontamination are normally determined with a static calculation method. In some cases, the pollutant emission is intermittent, for example in the car park of an office building, where all the cars enter and leave the place nearly at the same time. Generally, in such a case, the volume of the garage is large, consequently the time constant of the system has a high value. So a static approach would no longer stay accurate and a dynamic evaluation is needed. With the help of some assumptions, calculations remain rather simple and results can be plotted on nomographs or computed on a programmable handheld calculator. The amount of energy saved may appear very large in some cases. A sizing optimization will be required but also remains easy to compute. The paper presents the method of calculation for a single ventilation level and the optimization of a two-level ventilation.

Introduction

The classical means of calculating the ventilation requirements for decontamination is a simple static method (ASHRAE, 1977; Baturin, 1972). Using this method, the most difficult part of the problem is to gather information about the source strength as well as the human tolerance levels (ACGIH, 1980). However, in some cases, the pollutant emission cycle does not correspond at all to the steady-state assumed by the classical method. A typical example is the enclosed parking garage of an office building, where all the cars enter and leave at nearly the same time. In such a case, the volume of the garage is generally large; consequently the system has a large time constant, implying that a static approach does not model the situation with sufficient accuracy, and a dynamic evaluation is needed. The purpose of this paper is to present such a dynamic analysis. The fundamental parameters governing the phenomenon will be defined and the results will be compared with the classical method predictions.

It is important to clarify that the problem being treated is to determine the usefulness of a simple dynamic calculation, and not to simulate the response of the structure to general pollution emission problems and ventilation system designs. A simulation model allows for detailed analysis, but it does not provide for a quick determination of the order of magnitude of the ventilation requirements (Lorenz, 1982). The object here is to improve upon the static calculation methods by adding dynamic effects, without significantly increasing the complexity of the analysis.

Equations of the Problem

Hypotheses

In the equations below, X is the instantaneous pollutant concentration; V is the volume of the room (m^3); $\dot{V}$ is the volume of air change (m^3/h); and $\dot{G}$ is the source strength (m^3/h at air temperature). Notice that X is dimensionless, i.e., it corresponds to m^3/m^3. This means that X has generally a very small value. It is sometimes conveniently expressed in ppm (ml/m^3). $\dot{G}$, which is expressed in m^3/h for a question of coherence, is also a very small number.

We consider the outside pollutant concentration X_0 as constant and set this reference value to zero (i.e., X is replaced by $X - X_0$). We also assume that $\dot{V} >> \dot{G}$ and we do not consider effects of pressure and temperature variation. Lastly, we assume a perfect mixing of the air (to be discussed later).

Fundamental equations

As we define the rate of ventilation $n = \dot{V}/V$, let us state the rate of pollution to be $n_p = \dot{G}/V$ (both expressed in h^{-1}). We may write the rate of change of pollutant concentration as

$$\frac{dX}{d\tau} = n_p - nX. \quad (1)$$

The time constant of this system is $\tau_0 = 1/n$. A decay from an initial value X_1 is

$$X = X_1 e^{-\tau n}, \quad (2)$$

and the step response from an initial value X_2 is

$$X = X_2 + (X_{max} - X_2)(1 - e^{-\tau n}), \quad (3)$$

where X_{max} is the asymptotic concentration

$$X_{max} = \frac{n_p}{n}. \quad (4)$$

Choice of units

The equations have been initially derived for gases, as can be seen from the choice of the units. Fortunately, they can also be applied to other kinds of pollutants, provided the units are changed in a coherent way. For instance, the pollutant concentrations (X, X_1, X_2, X_{max}, etc.) can be expressed in g/m³, provided $\dot{G}$ is expressed in g/h, which leads to n_p expressed in g/m³h.

Solution for One Level of Ventilation

System response to a simplified intermittent input

We shall assume a periodic solicitation as shown in Fig. 1. Its period is τ_2 while pollutant is emitted during $\tau_1 < \tau_2$. The ventilation rate n is constant and must be determined in order to limit the peak value of X to a maximum allowed concentration X_{all}.

The classical solution consists in limiting the asymptotic concentration $X_{max} = n_p/n$ to the X_{all} value. In that case, it reduces to the classical sizing equation:

$$nX_{all}/n_p = 1.$$

Calling this dimensionless variable N, we obtain $N = 1$.

On the other hand, a dynamic evaluation assumes that the X value follows a perfectly periodic curve, i.e., $X(\tau) = X(\tau + \tau_2)$. We may write (Fig. 1):

$$X_1 = X_2 + (X_{max} - X_2)(1 - e^{-\tau_1 n}),$$

and

$$X_2 = X_1 e^{-(\tau_2 - \tau_1)n}.$$

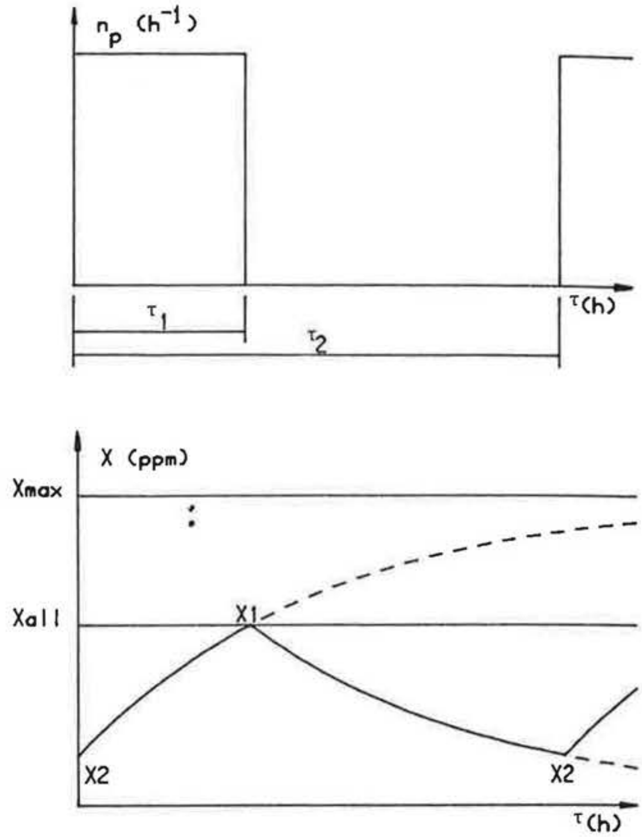

Fig. 1. Shape of the pollutant emission and response of the system for a constant rate of ventilation.

The solution of this system is

$$X_1 = X_{max} \frac{1 - e^{-\tau_1 n}}{1 - e^{-\tau_2 n}}$$

and

$$X_2 = X_{max} \frac{e^{\tau_1 n} - 1}{e^{\tau_2 n} - 1}.$$

Replacing X_1 by X_{all}, and X_{max} by its value, yields

$$X_{all} = \frac{n_p}{n} \frac{1 - e^{-\tau_1 n}}{1 - e^{-\tau_2 n}}, \quad (5)$$

to be solved for n, using an iterative procedure.

Alternatively, this equation can be plotted on a nomograph. Defining the following dimensionless parameters:

$$N = \frac{nX_{all}}{n_p} \quad 0 < N < 1,$$

$$G = \frac{n_p \tau_1}{X_{all}},$$

$$T = \frac{\tau_1}{\tau_2} \quad 0 < T < 1,$$

the equation becomes

$$N = \frac{1 - e^{-GN}}{1 - e^{-GN/T}} \tag{6}$$

A discussion of this last equation is presented in Appendix A.

Discussion about the nomograph

The nomograph is drawn on Fig. 2. First of all, it must be compared to the classical sizing equation $N = 1$. For values of T close to 1, it can be seen that the classical equation provides a quite accurate solution. On the other hand, for small values of T and also small values of G, it oversizes the ventilation system. It is possible to define three different areas in the graph. Area number I is limited by the curve corresponding to $G = 1$ (dashed line on the graph). The theoretical characteristics of this zone have already been explained. In practice it means that we do not try to limit the increase of X from X_2 to X_1 (even with $n = 0$ this increase would be equal to $n_p\tau_1$, i.e., less than X_{all}), but we manage a sufficient purge of the room in order to get an X_2 value low enough to keep X_1 bounded by X_{all}.

Area number III is the zone in which the curves seem horizontal. It is limited by the other dashed line defined by

$$N(G,T) = 1.05N(G,0).$$

Of course, this line is purely arbitrary. In this zone N is nearly independent of T and approximately equals $N(G,0)$, and is the solution of

$$\frac{1 - e^{-GN}}{N} = 1. \tag{7}$$

Notice that the step response from an initial value $X_2 = 0$,

$$X = X_{max}(1 - e^{-\tau n}),$$

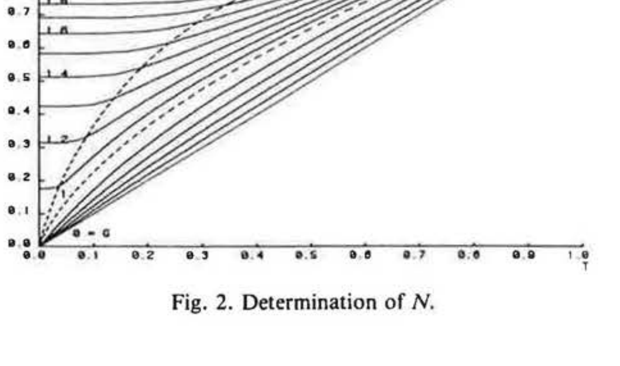

Fig. 2. Determination of N.

provides for $\tau = \tau_1$ and $X = X_{all}$:

$$X_{all} = X_{max}(1 - e^{-\tau_1 n}),$$

or

$$X_{all}\frac{n}{n_p} = N = 1 - e^{-GN},$$

which is exactly the equation of the approximate value of N in area III. We may thus conclude for region number III that $X_2 \cong 0$. Moreover, the system uses a large n value to limit the increase of X while the purge is too important. A two-level ventilation with a great n value during a time τ_1, and a lower n value during the rest of the period, would provide a better solution for a situation described by that area of the graph. Finally, an intermediate zone II appears where both effects are present: purging the room and limiting the increase.

Examples

Three examples, one in each zone, will show clearly the behaviour of the system. For each example, the volume of the room is 50,000 m^3 and $X_{all} = 3$ ppm (μL/L) $= 3 \times 10^{-6}$.

The complete calculation is given in Table 1. The first three columns give the raw data τ_1, τ_2, and $\dot{G}$ from which we compute $T = \tau_1/\tau_2$, $n_p = \dot{G}/V$ and $G = (n_p\ \tau_1)/X_{all}$. We can then compute the classical sizing $n_c = n_p/X_{all}$ (corresponding to $N = 1$), which can be transformed in $\dot{V}_c = n_c V$. We can also use the nomograph of fig. 2 with the values of T and G to fixed N.

Using this value of N, we then compute $n = (N\ n_p)/X_{all}$ and $\dot{V} = nV$ for comparison with the classical solution. Auxiliary results include $X_{max} = n_p/n$, which is the assymptotic value to be compared with the peak value X_{all}, and the number of the nomograph zone in which the examples lie. Figure 3 shows the evolution of X for the three examples. The different behaviours previously described can be clearly observed in these curves. It must be noted that the limits of the zones as they were defined are more theoretical than practical. The three examples were chosen in order to demonstrate the behaviour of the system in each region of the graph, but differences are not obvious for situations lying close to the boundary lines.

Solutions for Two Levels of Ventilation

We now assume a rate of ventilation n_1 during a time τ_1 and a different rate n_2 during a time $(\tau_2 - \tau_1)$. Once again, we will assume that the system is in steady-state conditions, so $X(\tau) = X(\tau + \tau_2)$. The equations are now

$$X_1 = X_2 + (X_{max} - X_2)(1 - e^{-\tau_1 n_1})$$

Table 1. Complete calculation for three cases with additional data: V = 50,000 m^3; X_{all} = 3 ppm (μL/L) : 3×10^{-6}.

Data						Results						
Raw Data			Calculated from Raw Data			First Method: Classical Solution		Second Method: Use of the Nomograph			Additional Considerations	
τ_1 (h)	τ_2 (h)	$\dot{G}$ (m^3/h)	T	n_p (h^{-1})	G	n_c (h^{-1})	$\dot{V}_c$ (m^3/h)	N	n (h^{-1})	$\dot{V}$ (m^3/h)	X_{max} (ppm)	Zone
2	20	0.03	0.1	6×10^{-7}	0.4	0.2	10,000	0.125	0.025	1,250	24	I
6	10	0.045	0.6	9×10^{-7}	1.8	0.3	15,000	0.85	0.255	12,750	3.5	II
2	10	0.15	0.2	3×10^{-6}	2.0	1.0	50,000	0.80	0.80	40,000	3.75	III

This table is derived from Fig. 2 and the definition of the parameters. Notice that X_{max} is not the peak value but the theoretical asymptotic value.

and

$$X_2 = X_1\, e^{-(\tau_2-\tau_1)n_2},$$

with

$$X_{max} = \frac{n_p}{n_1}.$$

The solutions are as follows:

$$X_1 = X_{max}\frac{1 - e^{-\tau_1 n_1}}{1 - e^{-(\tau_2-\tau_1)n_2-\tau_1 n_1}}$$

and

$$X_2 = X_{max}\left(1 - \frac{1 - e^{-(\tau_2-\tau_1)n_2}}{1 - e^{-(\tau_2-\tau_1)n_2-\tau_1 n_1}}\right)$$

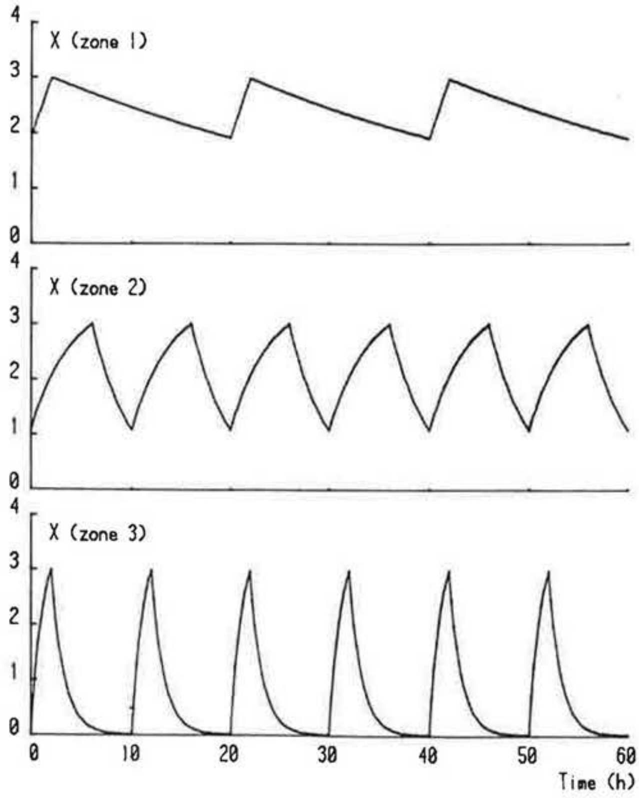

Fig. 3. Typical example for each zone.

It is easy to verify that for $n_2 = n_1$, the equations reduce to the previous ones.

Let us now suppose that we selected a value of n_1 by any means. It is possible to compute the value of n_2 which solves $X_1 = X_{all}$. Indeed,

$$n_2 = -\frac{1}{(\tau_2-\tau_1)}\ln A, \tag{8}$$

with

$$A = \frac{N1 + e^{-GN1} - 1}{N1\, e^{-GN1}}, \tag{9}$$

where we introduce the dimensionless parameter:

$$N_1 = \frac{n_1 X_{all}}{n_p} \qquad 0 < N1 \leq 1.$$

This equation is discussed in Appendix B.

Determination of n_1

In order to select n_1, we need another constraint which we shall choose as the minimisation of the energy consumption. Thus we will need the derivative of n_2 with respect to n_1:

$$\frac{\delta n_2}{\delta n_1} = \frac{-\tau_1}{(\tau_2-\tau_1)}B, \tag{10}$$

where

$$B = \frac{1 - e^{-GN1} - GN1(1 - N1)}{(N1 + e^{-GN1} - 1)\,GN1}. \tag{11}$$

It can be shown that this derivative is always negative. Consequently, an increase of n_1 will logically produce a decrease of n_2.

The power demand needed to heat the air is

$$W_h = \frac{nV\varrho c\Delta t}{3600\eta_h}, \tag{12}$$

where n = ventilation rate (h^{-1});
V = volume of the room (m^3);
ϱ = specific mass of the air (kg/m^3);
c = specific heat of the air ($J/kg\,°K$);
Δt = temperature difference (°K);
η_h = efficiency of the heating system.

The power required by the fan is

$$W_f = \frac{\Delta_p}{\eta_f}\frac{nV}{3600}, \tag{13}$$

where Δ_p is the pressure drop in the ventilation network (Pa), and η_f is the efficiency of the fan;

$$\Delta_p \cong L\left(\frac{nV}{3600}\right)^2. \tag{14}$$

L ($Pa\,sec^2/m^6$) can be easily obtained from a point of the network characteristic diagram.

We can now express the total power demand in the form

$$W = K(K_2^2 n^3 + K_0 n) = W_h + W_f, \tag{15}$$

with W in watts, K in Wh, K_2 in h, and K_0 dimensionless for n in h^{-1}. So

$$KK_2^2 = \frac{LV^3}{\eta_f(3600)^3}$$

and

$$KK_0 = \frac{V\varrho c\Delta t}{3600\eta_h}.$$

The term $(K_2^2 n^3 + K_0 n)$ is expressed in h^{-1}, i.e., it represents a kind of equivalent ventilation rate. Now we can arbitrarily choose one of the three constants K, K_0, and K_2. One convenient choice is $K_2 = 1$ h. In this case

$$K = \frac{LV^3}{\eta_f(3600)^3(1)^2}, \tag{16}$$

and

$$K_0 = \frac{\varrho c\Delta t\eta_f(3600)^2(1)^2}{LV^2\eta_h}. \tag{17}$$

The advantage of this form is that W is proportional to an equivalent ventilation rate whose variations will be studied to determine the optimum. The constant K will therefore not be used. With the choice of K_2, only K_0 will appear in the equations. Finally, if no heating of the air is required, K_0 itself will disappear from the equations.

K_2 was chosen equal to 1 h; since a fan will always be required, this power demand will never be zero. On the other hand, if no heating is necessary, K_0 is zero. For practical purpose adequate mean values of η_f and η_h will be necessary as we will integrate the equations over an entire year. It is also valuable to estimate the mean yearly Δt as $Dd/365$ where Dd is the number of yearly degree-days.

The expression of K and K_0 are adequate for a system in which both the energy consumed by the fan and by the heating device is electricity. It remains usable if to obtain an expression of primary energy. In this case η_f and η_h must include the efficiency of the electricity production. Notice that K_0 will remain unchanged and so will the optimum.

If electricity is not used for heating, the calculation is once again possible in primary energy by including η_f in the efficiency of the electricity production. Finally, in order to achieve a cost optimization (not including investments), K_0 must be multiplied by the ratio of the price of 1 kWh of the energy source used for heating by the price of 1 kWh of electricity. So the consumption of the heating device is transformed in an equivalent electrical consumption from a cost point of view.

The energy consumption over a period τ_2 is

$$Q = \tau_1 W_1 + (\tau_2 - \tau_1) W_2,$$

where W_1 and W_2 are calculated with $n = n_1$ and n_2, respectively. In another form,

$$\frac{Q}{K} = (K_2^2 n_1^3 + K_0 n_1)\tau_1 + (K_2^2 n_2^3 + K_0 n_2)(\tau_2 - \tau_1), \tag{18}$$

where n_2 is a function of n_1. Solving: $(\partial Q/K)/\partial n_1 = 0$ will give the optimum n_1:

$$\frac{\partial Q/K}{\partial n_1} = 3K_2^2 n_1^2 \tau_1 + K_0 \tau_1$$

$$- 3K_2^2 \frac{\tau_1}{(\tau_2 - \tau_1)^2}(\ln^2 A)B - K_0\tau_1 B = 0,$$

so

$$n_1^2 - \frac{\ln^2 A}{(\tau_2 - \tau_1)^2}B = \frac{K_0}{3K_2^2}(B - 1). \tag{19}$$

This equation can be transformed to use dimensionless parameters:

$$\left(N1\,G\frac{1 - T}{T}\right)^2 - (\ln^2 A)B = P(B - 1), \tag{20}$$

where

$$P = \frac{K_0(\tau_2 - \tau_1)^2}{3K_2^2} \quad \frac{\varrho c\Delta t\eta_f(3600)^2(\tau_2 - \tau_1)^2}{3LV^2\eta_h}$$

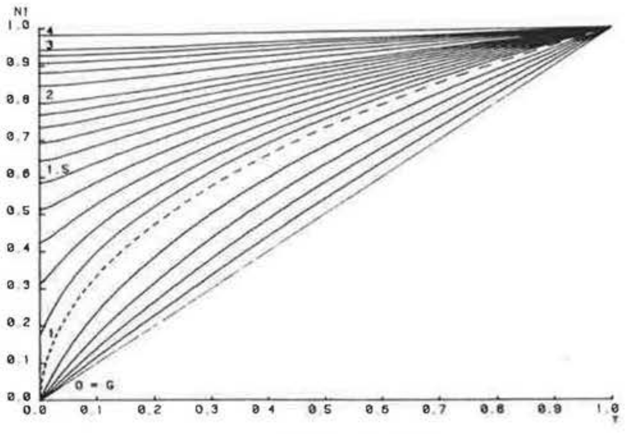

Fig. 4. Determination of $N1$.

is a new dimensionless parameter.

It must be remembered that A and B are also functions of G and $N1$.

Nomograph

As we have now four dimensionless parameters, it will only be possible to draw a nomograph if one of these parameters is fixed. Let us consider $P = 0$ (no heating required). The nomograph is shown in Fig. 4 and is similar to the one corresponding to a single level of ventilation. The values of n_1 are higher than the corresponding values found for a single level of ventilation, while the values of n_2 are lower. From the point of view of separable regimes, the two-level ventilation nomograph has only two distinct zones. The nomograph in Fig. 4 does not have a regime with horizontal curves corresponding to zone III in Fig. 2. All future references to zones correspond to the zones in Fig. 2.

Examples

Let us take the three examples we have already studied. We will consider $P = 0$ and compare the classical sizing solution n_c, the one-level ventilation solution n, and the two-level ventilation solution n_1 and n_2. n and n_c come directly from Table 1. n_1 is calculated in exactly the same way n was, except that the nomograph of Fig. 4 is used. n_2 is then calculated from Eq. (8). We will also consider the benefit due to this two-level solution from the one-level solution (Table 2).

For the example previously chosen in zone I, n_1 and n_2 are very close to n and the benefit is low. The investment of a two-speed fan would generally be prohibitive. On the other hand, a two-speed fan with air flow rates so close together is impossible to find. In zone II, the benefit is much more interesting and the solution using a two-speed fan can be seriously studied. In zone III, the benefit is very large. Unfortunately, it can be shown that when n_1 is rather close to n_c (which is often the case in zone III) a small change in n_1 produces a large change in the required n_2. So a major risk is to be taken on n_2 which reduces the benefit very much. In this zone, the best solution is often found by choosing $n_1 = n_c$ and $n_2 = 0$. Anyway, for rather low values of G (for instance $1 < G < 2$), the two-speed fan solution can also be seriously studied.

Influence of P

It can be shown that an increase of P will result in an increase of n_1 and a decrease of n_2. Moreover, it is possible to find a limit value for which $n_1 = n_c$ and $n_2 = 0$. Above this point, we will find $n_1 > n_c$ and $n_2 < 0$, which is of course impossible. So we fall out of the domain of validity of the equations and for any $P > P$ limit, the solution is $n_1 = n_c$ and $n_2 = 0$ (Fig. 5). This value of P_{lim} can be obtained by replacing n_1 by n_c, i.e., $N1 = 1$. In this case, A and B reduce to

$$A = 1,\ \ln(A) = 0,$$

$$B = \frac{1 - e^{-G}}{Ge^{-G}} = \frac{e^{G} - 1}{G},$$

so the equations become

$$\left(G\frac{1 - T}{T}\right)^2 = P_{\text{lim}}\left(\frac{e^{G} - 1}{G} - 1\right),$$

$$P_{\text{lim}} = \frac{G^3}{e^{G} - 1 - G} \cdot \left(\frac{1 - T}{T}\right)^2. \tag{21}$$

The values of P_{lim} are plotted on Fig. 6. It can be seen that P_{lim} is generally small and the solution $n_1 = n_c$ and $n_2 = 0$ is often optimum.

Application to Actual Cases

Let us suppose a volume of 45,000 m^3 and a production of NO_2 due to 30 buses represented in Fig. 7A for a period $\tau_2 = 24$ h. This emission profile does not corre-

Table 2. Comparison of different solutions for three examples.

Case	Solution 1: Classical Sizing n_c	Solution 2: Single Level n	Solution 3: Two Levels n_1	n_2	Consumption Reduction from Solutions 2–3
Zone I	0.200	0.025	0.028	0.024	6%
Zone II	0.300	0.255	0.272	0.138	20%
Zone III	1.000	0.800	0.837	0.231	75%

spond to the profile assumed for the derivation of the equations (see Fig. 1). We therefore must replace it with an equivalent profile. It can be shown from the equations that the profile of Fig. 1 gives the highest peak response for given peak and mean emission values. Therefore we shall consider an equivalent unique rectangular excitation with the same maximum emission and the same total amount of pollutant (i.e., mean emission).

With this profile, we will find a ventilation rate slightly higher than that actually required. We expect a simulation of the pollutant concentration evolution using the actual emission profile to show a peak value slightly lower than X_{all}:

Total amount of pollutant:

$$0.0615 + 0.123 + 0.0126 \times 2 = 0.2097 \text{ m}^3 \text{ of } NO_2;$$

$$\text{Data: } \left.\begin{array}{l} \tau_1 = \dfrac{0.2097}{0.123} = 1.7 \text{ h} \\ \tau_2 = 24 \text{ h} \end{array}\right\} T = 0.07 \qquad G = \frac{n_p \tau_1}{X_{all}} = 1.55;$$

$$\dot{G} = 0.123 \text{ m}^3/\text{h}; \qquad n_p = \frac{\dot{G}}{V} = 2.73 \times 10^{-6}$$

$X_{all} = 3 \text{ ppm} = 3 \times 10^{-6}$;

$P = 0$ (no heating of the air).

Results:
$n_c = 0.910 \text{ h}^{-1}$ $\dot{V}_c = 40950 \text{ m}^3/\text{h}$;
$n = 0.557 \text{ h}^{-1}$ $\dot{V} = 25065 \text{ m}^3/\text{h}$;
$n_1 = 0.586 \text{ h}^{-1}$ $\dot{V}_1 = 26370 \text{ m}^3/\text{h}$;
$n_2 = 0.129 \text{ h}^{-1}$ $\dot{V}_2 = 5805 \text{ m}^3/\text{h}$.

Five different solutions directly based on these theoretical results are compared in Table 3. The first three solutions use the theoretical values themselves while the two other solutions present slight variations

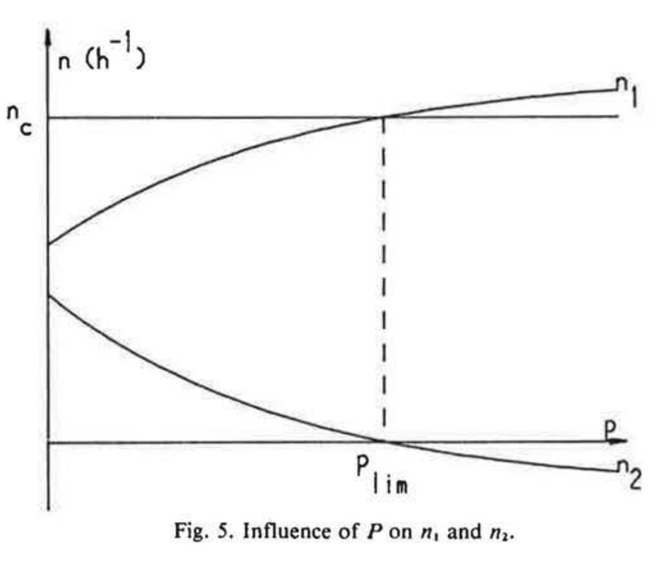

Fig. 5. Influence of P on n_1 and n_2.

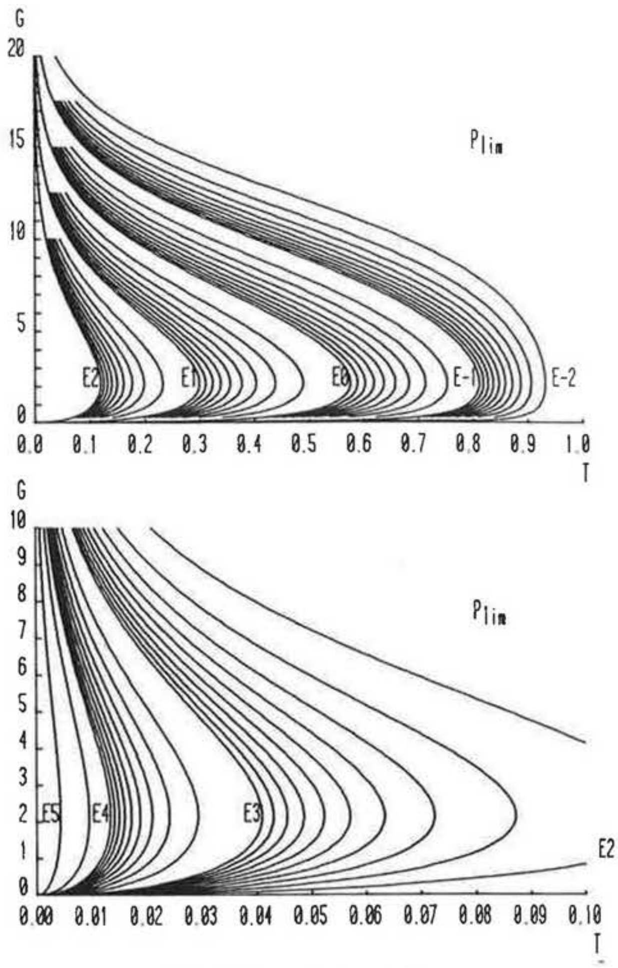

Fig. 6. Determination of P_{lim}.

on the two-level strategy. First, the two levels are rounded for security. This case should be preferred to the theoretical one because n_2 is very sensitive to variations on n_1. Second, the limit values of the ventilation rates are used, i.e., $n_1 = n_c$ and $n_2 = 0$. The time during which the high level ventilation is on extends from 6 to 8 a.m.; thus $\tau_1 = 2$ h and not 1.7 h.

Relative consumptions are estimated considering two different duct sizes: a large one for flows above 40,000 m³/h and a reduced size for flows below 30,000 m³/h. If the duct size is fixed, the consumption varies as n^3 while it is assumed to be proportional to n when the duct size and n are reduced together. It can be seen from Table 3 that the benefit can be very large. Obviously, the last two solutions are the better ones if we reject the theoretical optimiser for security. Since all the choices correspond to different investments, the final choice would require more economical data. Table 3 also refers to Fig. 7 for a simulation of actual evolutions. It can be seen that in all cases, X remains below X_{all}.

Discussion

The model described above is applicable when the emission sources are evenly distributed so that the spatial

Table 3. Relative consumptions of different solutions compared to the classical one. Two different sizes of the duct section are assumed, depending on the level of the ventilation flow.

Solution Type	Ventilation Flow(s)	Additional Assumption for Consumption Estimate	Consumption	
			Full Duct Size	Reduced Duct Size
Classical solution (Fig. 7B)	$\dot{V}_c$ = 40,950 m³/h	–	1	–
Single-level solution (Fig. 7C)	$\dot{V}$ = 25,065 m³/h	–	0.23	0.61
Two-level solution theoretical values	$\dot{V}_1$ = 26,370 m³/h $\dot{V}_2$ = 5,805 m³/h	–	–	0.06
Two-level solution corrected for security (Fig. 7D)	$\dot{V}_1$ = 30,000 m³/h $\dot{V}_2$ = 10,000 m³/h	τ_1 = 2 h (from 6 to 8 a.m.)	–	0.12
Two-level solution with limit values (Fig. 7E)	$\dot{V}_1$ = 40,950 m³/h $\dot{V}_2$ = 0 m³/h	τ_1 = 2 h (from 6 to 8 a.m.)	0.08	–

variations of X remain small. If this is not the case, the assumption of perfect air mixing is no longer accurate. Attempting to include these spatial variations would increase the complexity of the model considerably.

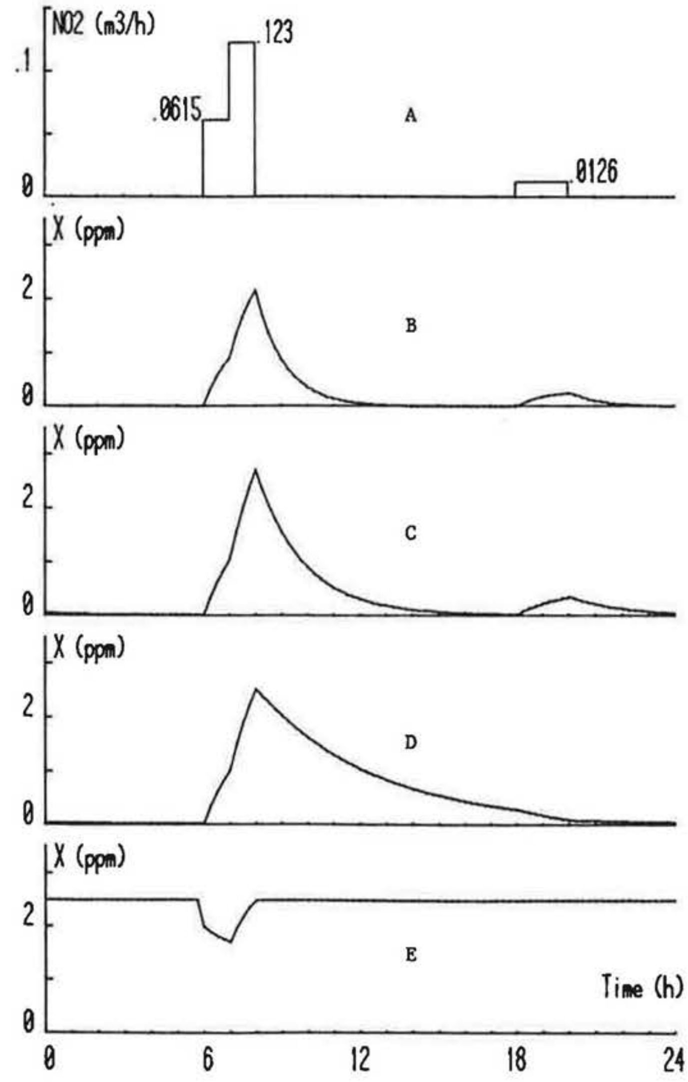

Fig. 7. Application to an actual case.

On the other hand, one may wonder if global dilution is the proper means for decontaminating a room when local pollution occurs. The designer should try a local removal solution. If this is not possible, it is probably more economical to install a mixing fan than to increase the ventilation rate until this zone reaches an acceptable concentration. Second, we assumed that the control cycle is exactly in phase with the emissions. Once again, this was done to simplify the equations since the desired control cycle should probably be close to the in-phase solution assumed.

Conclusion

As a starting point for this study, we considered that in the case of intermittent pollution, a static calculation of the ventilation requirements is not accurate. Therefore we produced a dynamic evaluation, and found three different modes of operation corresponding to three zones in the nomograph. In the third zone, we concluded that a two-level ventilation system should be applied. A new parameter appears in the determination of these two levels, representing energy consumption or running costs considerations. Finally, we found a limit value to this new parameter above which the two flow rates reduce to the classical static sizing on the one hand, and zero on the other hand.

The simple static sizing formula is used extensively in the literature. We have emphasized that this approach might lead to inaccurate results for numerous situations. However, its domain of application reveals wider than expected before conclusions of the present study were known. The calculations seem sophisticated, but we successfully computed the equations on a programmable handheld calculator. On this calculator, the entire process takes approximately 1 min and can easily be applied to real cases.

Of course, some improvements are possible. For instance, the assumption of perfect mixing of the air was made. A rough estimation of the real air mixing should greatly improve the process. Another improvement should be the possibility of recycling air through a filter. Even in this rather simple form, the method proved to be of great help in an accurate sizing of ventilation systems.

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Appendix A

The equation governing the single level case is

$$N = \frac{1 - e^{-GN}}{1 - e^{-GN/T}}. \quad (6)$$

Normally, G and T are known and the equation must be solved for N.

To draw the nomograph, it is easier to compute T as a function of G and N

$$T = \frac{-GN}{\ln\left(1 - \frac{1 - e^{-GN}}{N}\right)}$$

If the term between brackets happens to be equal to zero, it can be seen from Eq. (6) that $T = 0$. In any other case, we must have

$$\frac{1 - e^{-GN}}{N} \leq 1,$$

$$1 - e^{-GN} \leq N.$$

This condition will always be satisfied for $G \leq 1$. If $G > 1$, it will be fulfilled only if N is greater or equal to a minimum value that is the solution of

$$\frac{1 - e^{-GN}}{N} = 1.$$

Figure 8 shows the validity field of N for two different ranges of G.

In order to grasp the physical meaning of this condition, let us consider a fixed quantity of pollutant whose emission time becomes negligible with respect to the loading period τ_2. This yields

$$T \to 0; \quad n_p\,\tau_1 \text{ constant but less than } X_{all}\ (G < 1).$$

Therefore, $N \to 0$, but as $n_p \to \infty$, n remains bounded. For instance, in the limit case $\tau_1 = 0$ (case of a Dirac function), we obtain

$$n = -\frac{1}{\tau_2}\ln\left(1 - \frac{n_p\,\tau_1}{X_{all}}\right),$$

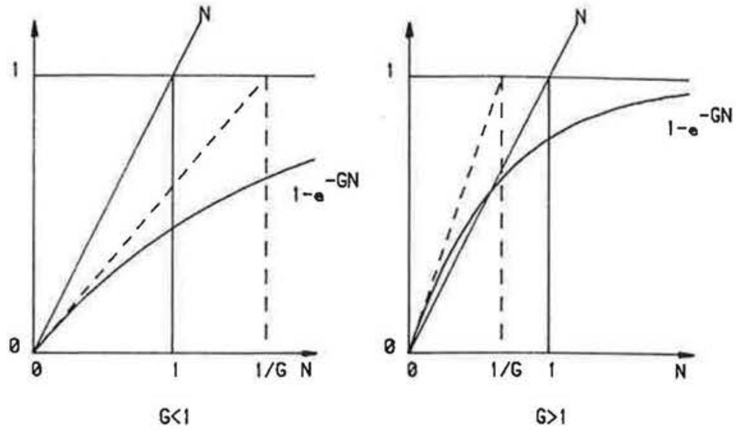

Fig. 8. Validity field of N for $G < 1$ and $G > 1$.

which can be large but remains bounded as long as $(n_p\,\tau_1)/X_{all} < 1$. When $n_p\,\tau_1$ reaches X_{all}, n becomes infinite and remains infinite for any $n_p\,\tau_1 > X_{all}$. n is also very large for small values of τ_1/τ_2 when $G > 1$ as $N = (n\,X_{all})/n_p$ no longer tends to zero.

Appendix B

We found the equation

$$n_2 = -\frac{1}{(\tau_2 - \tau_1)}\ln A, \quad (8)$$

where

$$A = \frac{N1 + e^{-GN1} - 1}{N1\, e^{-GN1}}. \quad (9)$$

The existence of the logarithm requires $A \geq 0$ and as its denominator is always positive:

$$N1 + e^{-GN1} - 1 \geq 0;$$

$$F(N1) = \frac{1 - e^{-GN1}}{N1} \leq 1.$$

An analysis similar to the one of Appendix A can be applied; this condition is always true if $G \leq 1$, while it is only true for $G > 1$ if $N1$ is greater or equal to the solution of

$$F(N1) = \frac{1 - e^{-GN1}}{N1} = 1.$$

The meaning of this condition is clear. It is only possible to find an adequate value of n_2 if n_1 bounds $(X_1 - X_2)$ to X_{all}. $F(N1) = 1$ supposes $X_2 = 0$ and $X_1 = X_{all}$ and thus provides minimum of n_1. Remember that this value is lower limit as it will require $n_2 = \infty$. Now if $G > 1$, this limit is strictly positive. It is exactly zero when $G = 1$ and when $G < 1$ it becomes possible to find an adequate finite value for n_2 even with $n_1 = 0$. In this last case, the equation reduces to

$$A = 1 - G. \tag{9'}$$

Appendix C: Nomenclature

Symbol	Definition	Units
A	intermediate variable	–
B	intermediate variable	–
c	specific heat of the air	J/kg °K
G	dimensionless emission rate	–
$\dot{G}$	source strength	m^3/h(or g/h)
K	coefficient of the W equation	Wh
K_0	coefficient of the W equation	–
K_2	coefficient of the W equation	h
L	characteristic parameter of the ventilation network	Pa sec^2/m^6
n	single level ventilation rate	h^{-1}
N	dimensionless ventilation rate (single level)	–
n_1	high level ventilation rate	h^{-1}
n_2	low level ventilation rate	h^{-1}
N_1	dimensionless ventilation rate (high level)	–
n_c	classical ventilation rate	h^{-1}
n_p	pollutant emission rate	h^{-1}(or g/m^3h)
P	consumption considerations parameter	–
Q	energy consumption over one cycle	Wh
T	dimensionless emission duration	–
V	volume of the room	m^3
$\dot{V}$	volume of air change	m^3/h
W	total power demand	W
W_h	heating power demand	W
W_f	fan power demand	W
X	instantaneous pollutant concentration	–(or g/m^3)
X_1	peak concentration	–(or g/m^3)
X_{max}	maximum assymptotic concentration	–(or g/m^3)
X_{all}	maximum allowed concentration	–(or g/m^3)
X_0	outside concentration	–(or g/m^3)
τ	time	h
τ_0	time constant	h
τ_1	emission duration	h
τ_2	cycle period	
ϱ	specific mass of the air	kg/m^3
Δ_t	temperature difference	°K
Δ_p	pressure drop	Pa
η_h	efficiency of the heating system	–
η_f	efficiency of the fan	–

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