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Temperature and humidity control of indoor environmental spaces

M. Zaheer-uddin
Centre for Building Studies, Concordia University, 1455 de Maisonneuve Blvd. West, Montreal H3G 1M8 (Canada)

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Abstract

The temperature and humidity control of indoor environmental spaces is studied. A constant volume heating and humidification system consisting of a boiler, a heating coil, ductwork and a fan, and a two-zone building is considered. The temperature and humidity control models are developed and multivariable controllers are designed. The closed loop system responses subject to step disturbances in outdoor temperatures and relative humidity are given. Also, results showing a typical-day operating performance of the closed loop system are given.

1. Introduction

Heating, ventilating and air-conditioning (HVAC) systems can be broadly classified into (i) constant volume (CV), and (ii) variable air volume (VAV) systems. Despite the fact that the fan energy consumption in CV systems is higher than in VAV systems, the CV systems are used in a variety of buildings either as independent systems or in conjunction with VAV systems [1]. The reason is that CV systems pose no balancing problems and they provide good temperature control. Furthermore, the energy consumption of CV systems can be minimized by modulating the discharge air temperature based on outdoor temperature control concept and/or discriminator relay control action [2].

Generally in practical HVAC systems the discharge air temperature is set at a level that is sufficient to provide heating/cooling to the zones with highest load. A consequence of this is that the discharge air temperature based on highest load is either too warm or too cold for the other zones. Although several alternatives are available to mitigate this situation [2], we explore here the use of multivariable control schemes to continuously modulate the discharge air temperature in order to maintain zone temperatures at the desired setpoints chosen by the occupants of the zones.

A review of the literature shows that analytical studies on control of heating, ventilating and air-conditioning (HVAC) systems are lacking. This is due to the fact that even the simplest HVAC system models are nonlinear and belong to a class of multivariable systems. Computer simulation techniques have been used in programs, namely HVACSIM$^+$ [3] and BLAST [4]. They model the HVAC systems together with a set of control strategies. The control logic is either supplied by the user or chosen from a set of predefined options. The idea behind it is to study the effect of a particular thermostat setpoint profile on energy consumption of a building. However, these open loop strategies are not useful for implementation on real systems which are closed loop.

The control technology in HVAC systems these days is rapidly moving towards the use of direct digital controls (proportional-integral (PI) controllers). However, the design methods used to select or tune the PI constants are still based on the root-locus diagrams [5]. This is valid only when the system or the process under study has a single input and correspondingly a single output. In reality, most HVAC systems have several inputs and, most certainly, several outputs. For instance, zone damper position and the mass flow rate of chilled water in a cooling system can be varied simultaneously to control zone temperature and humidity ratio. Such systems with multiple inputs are known as multivariable control systems. In fact, the classical control design methods such as root-locus and empirical methods [6] cannot be used to design controllers for multiple input systems. Because of this lack of good design methods for multiple input HVAC control systems, the controller tuning is frequently carried out using empirical methods combined with either experience and/or trial and error. As a result

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of this, several control problems have been reported [2].

The present study explores the application of multivariable controller design methods and puts the design process on a much firmer theoretical basis to address the building environment control problem. The advantage is that the PI constants obtained from this study will be based on overall system dynamics and therefore their use is expected to give better temperature regulation.

In this study we explore the design of multiple input controllers for temperature and humidity control in a two-zone building. First we develop an analytical model for a two-zone CV system (Section 2). The control formulation and solution will be given in Section 3. Both open loop and closed loop system responses will be given in Section 4 and typical daily simulation results in Section 5.

2. The system model

Figure 1 shows the schematic diagram of a two-zone constant volume HVAC system. Both zones 1 and 2 are heated and humidified by conditioned air supplied through the diffusers. The return air is recirculated and mixed with fresh outdoor air. The mixed air is then heated in the heating coil. A boiler is used to heat the water which is circulated in the heating coil via a pump and control valve arrangement shown in the Figure. A fan is used to circulate the air in the ductwork. The supply air is humidified before it enters the zones.

There are three controllers C_1, C_2 and C_3 as shown in Fig. 1. The control problem is to maintain the zone temperatures (T_{z1} and T_{z2}) and humidity ratios (W_{z1} and W_{z2}) at their respective setpoints (T_{z1set}, T_{z2set} and W_{z1set}, W_{z2set}). For example, in response to an increase in heating load on the zones the following control actions can be taken:

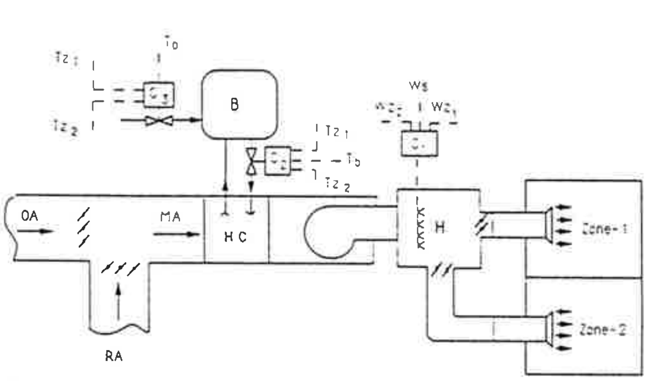


Fig. 1. Schematic diagram of a constant volume space heating system. B = boiler, C_i = controllers, MA = mixed air, OA = outdoor air, RA = return air, HC = heating coil, H = humidifier.

(i) the temperature of the supply air can be increased by increasing the mass flow rate of hot water flowing in the heating coil via the controller C_2;

(ii) the temperature of the hot water can be increased by increasing the fuel firing rate in the burner via controller C_3.

Both controllers C_2 and C_3 receive feedback signals from the zone thermostats and the boiler temperature sensor. The humidity levels in the zones are controlled by the controller C_1 which modulates the rate of water vapour injected into the supply air stream. The humidity controller receives feedback signals from the zone humidistat and the humidity sensor located in the supply air distribution box.

The system equations were developed by using the enthalpy and mass balance principles. The enthalpy balance equations on zones 1 and 2 are

$$M_{z1}\frac{dh_{z1}}{dt} = m_{a1}(h_s - h_{z1}) - q_{s1} - q_{l1} \quad (1)$$

$$M_{z2}\frac{dh_{z2}}{dt} = m_{a2}(h_s - h_{z2}) - q_{s2} - q_{l2} \quad (2)$$

where h_{z1} and h_{z2} are the enthalpy of air in zone 1 and 2 respectively; m_{a1} and m_{a2} are the supply airflow rates to the zones; and q_s and q_l are the sensible and latent loads acting on the zones. Likewise the humidity balance equations on the zones are

$$M_{z1}\frac{dW_{z1}}{dt} = m_{a1}(W_s - W_{z1}) - m_{w1} \quad (3)$$

$$M_{z2}\frac{dW_{z2}}{dt} = m_{a2}(W_s - W_{z2}) - m_{w2} \quad (4)$$

In eqns. (1–4) h_s and W_s are the supply air enthalpy and the humidity ratios respectively. The air is sensibly heated in the coil so that we have

$$C_c\frac{dT_c}{dt} = U_2 U_{2max} C_{pw}\epsilon(T_b - T_c) - h_c A_c\left(T_c - \frac{T_{mx}+T_s}{2}\right) \quad (5)$$

where T_s is obtained from

$$m_a C_{pa}(T_s - T_{mx}) = A_c h_c\left(T_c - \frac{T_{mx}+T_x}{2}\right) \quad (6)$$

and T_{mx} is given by

$$T_{mx} = \frac{(1-f)[m_{a1}T_{z1} + m_{a2}T_{z2}] + f(m_{a1}+m_{a2})T_{oa}}{m_a} \quad (7)$$

where f is the percent outdoor air used in the zones. The water vapour is added to the supply air in the distribution box so that the humidity balance is described by the equation,

$$M_{ws}\frac{dW_s}{dt}=m_a(W_{mx}-W_s)+U_1U_{1max} \tag{8}$$

where W_{mx} is given by

$$W_{mx}=\frac{(1-f)[m_{a1}W_{z1}+m_{a2}W_{z2}]+f(m_{a1}+m_{a2})W_{oa}}{m_a} \tag{9}$$

The hot water temperature T_b is obtained by equating the rate of energy stored in the boiler with the input and output heat fluxes,

$$C_b\frac{dT_b}{dt}=U_3U_{3max}\left(1-\frac{\alpha T_b}{T_{bmax}}\right)-U_2U_{2max}\epsilon(T_b-T_c)-a_b(T_b-T_\infty) \tag{10}$$

Equations (1–10) constitute a seventh-order non-linear model. The three control inputs to the system are the rate of water vapour added to the supply air U_1, the mass flow rate of hot water circulating in the heating coil U_2 and the fuel firing rate in the burner U_3. All U values are normalized with respect to their maximum capacities so that they vary between 0 and 1. The outputs to be regulated are the zone temperatures and humidity ratios. The enthalpy of the air in the zones is a function of zone air temperature and humidity ratio,

$$h_z=C_{pa}T_z+W_z(2501+C_{pw}T_z) \tag{11}$$

By neglecting the contributions of the liquid enthalpy of the vapour (which is relatively small) on the zone temperature, we write eqn. (11) as

$$\frac{dh_z}{dt}=(C_{pa}+C_{pw}W_z)\frac{dT_z}{dt} \tag{12}$$

In typical comfort air-conditioning problems, the relative humidity of the zone is maintained between 40–50%. This means that W_z in most cases varies between 0.007 and 0.01. By considering an average value of $W_z=0.0075$ we can substitute eqn. (12) in eqns. (1, 2). By doing so eqns. (1, 2) are decoupled from eqns. (3, 4). In other words, the seventh-order temperature and humidity control problem given by eqns. (1–10) is decoupled into two separate problems: a fourth-order zone temperature control problem and a third-order humidity control problem. Since the temperature control problem is nonlinear, it was linearized about an operating point to design the controllers. The linear model is given by the following equations.

Temperature control problem

$$\dot{T}+\mathbf{A}T+\mathbf{B}U+\mathbf{E}d_1 \tag{13}$$

$$y_T=\mathbf{C}_1T \tag{14}$$

where

$$T=[\Delta T_{z1}\Delta T_{z2}\Delta T_c\Delta T_b]^T \tag{15}$$

$$U=[\Delta U_2\Delta U_3]^T \tag{16}$$

$$d_1=[\Delta T_a\Delta m_{w1}\Delta m_{w2}\Delta T_\infty]^T \tag{17}$$

and the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}_1$, $\mathbf{E}$ have the following structure

$$\mathbf{A}=\begin{bmatrix}a_{11}&a_{12}&a_{13}&0\\a_{21}&a_{22}&a_{23}&0\\a_{31}&a_{32}&a_{33}&a_{34}\\0&0&a_{43}&a_{44}\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}0&0\\0&0\\b_{31}&0\\b_{41}&b_{42}\end{bmatrix} \tag{18}$$

$$\mathbf{C}_1=[1\ 1\ 0\ 0],\ \mathbf{E}=\begin{bmatrix}e_{11}&e_{12}&0&0\\e_{21}&0&e_{23}&0\\e_{31}&0&0&0\\0&0&0&e_{44}\end{bmatrix} \tag{19}$$

Humidity control problem

$$\dot{W}=\mathbf{F}W+\mathbf{G}V+\mathbf{H}d_2 \tag{20}$$

$$y_w=\mathbf{C}_2W \tag{21}$$

$$W=[\Delta W_{z1}\Delta W_{z2}\Delta W_s]^T \tag{22}$$

$$V=\Delta U_1 \tag{23}$$

$$d_2=[\Delta m_w\Delta W_{oa}]^T \tag{24}$$

and the matrices $\mathbf{F}$, $\mathbf{G}$, $\mathbf{H}$ and $\mathbf{C}_2$, have the following structure

$$\mathbf{F}=\begin{bmatrix}f_{11}&0&f_{13}\\0&f_{22}&f_{23}\\f_{31}&f_{32}&f_{33}\end{bmatrix},\ \mathbf{G}=\begin{bmatrix}0\\0\\g_3\end{bmatrix} \tag{25}$$

$$\mathbf{C}_2=[1\ 1\ 1],\ \mathbf{H}=\begin{bmatrix}h_{11}&0\\h_{21}&0\\0&h_{32}\end{bmatrix} \tag{26}$$

The matrices in eqns. (18–26) were computed for a system with parameters defined in Table 1.

3. The design of temperature and humidity controllers

Several methods are available for the design of multivariable controllers [7]. Here, we use the linear quadratic (LQ) state feedback technique which was also used in our previous study [8]. As shown in

TABLE 1. System parameters

Variable		Magnitude and units
A_c	coil area	15 m^{-2}
a_b	boiler surface heat loss coefficient	12 kJ h^{-1} °C
C_b	boiler thermal capacity	594 kJ h^{-1} °C
C_c	coil thermal capacity	180 kJ °C^{-1}
C_{pa}	specific heat of air	1.0 kJ °C^{-1}
C_{pw}	specific heat of water	4.2 kJ C^{-1}
f	fraction of outdoor air	0.15
h	heat transfer coefficient	220.6 kJ h^{-1} m^{-2} °C
m_a	fan capacity	2356 kg h^{-1}
M_{ws}	mass of air in mixing box	18.7 kg
M_z	mass of air in the zone	374 kg
U_{1max}	humidifier capacity	24 kg h^{-1}
U_{2max}	maximum water flow rate	1350 kg h^{-1}
U_{3max}	burner capacity	68562 kJ h^{-1}
q_s	sensible load	Variable
q_l	latent load	Variable

that study, if accurate output control about a setpoint is desired, it will be necessary to add integral control action to the state feedback controller. In the absence of integral action, good output regulation cannot be achieved. In other words, as the external disturbance increases, the error between the zone temperature and the setpoint temperature would increase. Since thermal comfort of the occupants of a zone is influenced more by the magnitude of the zone temperature fluctuations than relative humidity, we conclude that it is appropriate to design the temperature controller with integral action and use state feedback controller to regulate the relative humidity.

3.1. *Zone temperature controller with integral action*

To the model equations given by eqns. (13, 14) a servocompensator of the type

$$\xi = \Omega\xi + \theta e \tag{27}$$

is added, where e is the error between T_z and T_{zset} for each zone and Ω and θ are the matrices that are fully determined for the class of disturbances acting on the system. Here, we assume that disturbances such as T_{oa}, m_w and W_{oa} can be represented by step functions and therefore we are interested in rejecting the effects of such disturbances in order to hold zone temperatures at their respective setpoints. The minimization of a performance index, namely,

$$J = \int_0^\infty \left\{ [T'\xi'] \begin{bmatrix} \mathbf{Q}_1 & 0 \\ 0 & \mathbf{Q}_2 \end{bmatrix} \begin{bmatrix} T \\ \xi \end{bmatrix} + U'RU \right\} \mathrm{d}t \tag{28}$$

for the system of eqns. (13, 14, 27) yields a new control law

$$U^* = -[\mathbf{K}_1\mathbf{K}_2]\begin{bmatrix} T \\ \xi \end{bmatrix} \tag{29}$$

where $\mathbf{K}_1$ and $\mathbf{K}_2$ are the gain matrices.

3.2. *Humidity controller*

The humidity controller design is similar to the temperature controller except that it has no integral action. The performance index to be minimized is

$$J_2 = \int_0^\infty \{\Delta W'Q\Delta W + \Delta V'R\Delta V\}\, \mathrm{d}t \tag{30}$$

which yields a state feedback controller of the type

$$V = -K_3 W \tag{31}$$

where K_3 is the state feedback gain vector. It may be noted that for both temperature and humidity control problems we have chosen an infinite time solution to the LQ problem. The reason for this is that we assume eqns. (1–10) can be used as a good basis to describe the constant volume system. Some parameters in eqns. (1–10), such as heat transfer coefficients, in practice are likely to change. Such changes can be taken care of by periodically updating the system parameters. With a periodic update of the system parameters, the controller gains can be recomputed. Such adaptive optimal control schemes can be easily applied to the problem discussed in this study.

4. Simulation results

Before examining the responses of the closed loop systems, it is instructive to check the open loop responses of the temperature and humidity control systems. With control inputs set arbitrarily at $U_1 = 0.27$, $U_2 = 0.32$, and $U_3 = 0.42$ and constant disturbances, namely $T_{oa} = -2$ °C, $W_{oa} = 0.002$, $T_\infty = 20$ °C, $m_{w1} = 1.75$ kgh^{-1} and $m_{w2} = 1.5$ kgh^{-1} acting on the zones the system responses were simulated (Figs. 2(a)–d)).

It is apparent from Fig. 2(a) that the zone temperature responses exhibit exponentially rising characteristics which is typical of a zone undergoing heating. Since the control inputs and the disturbances are constant, the zone temperatures eventually reach steady state. The steady-state time is 5–6 hours. Also shown in Figs. 2(b), (c) and (d) are T_b, T_c and the humidity ratio responses. The results in Figs. 2(a)–(d) not only show the open loop responses of the system when acted upon by

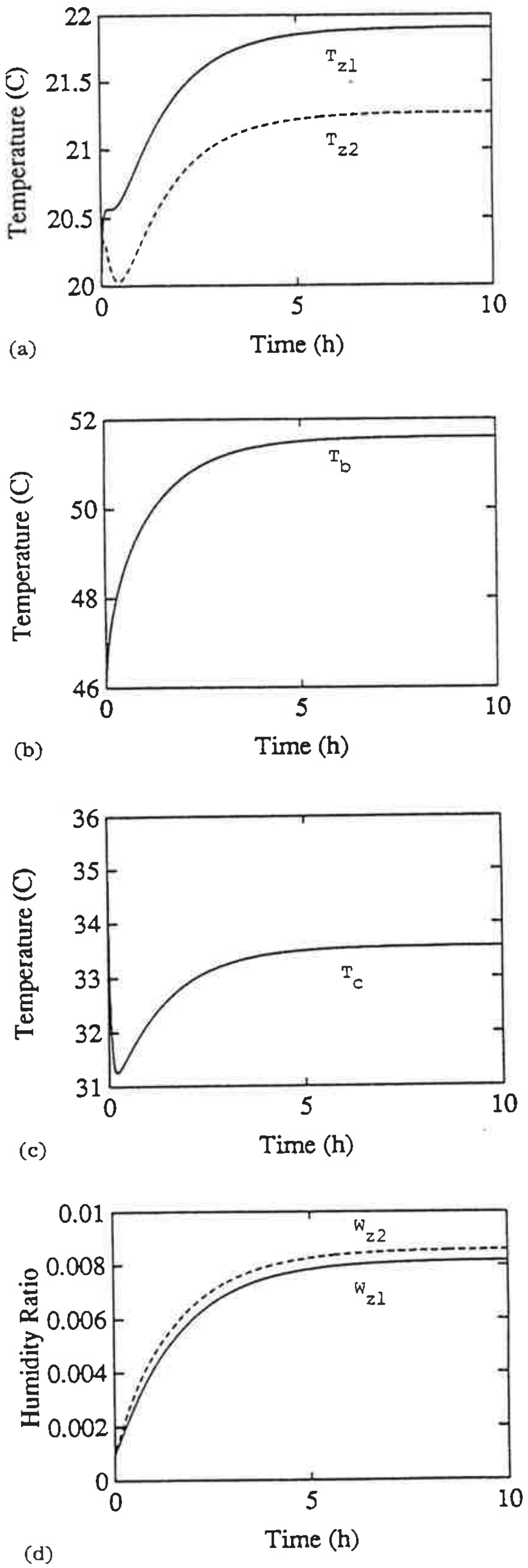


Fig. 2. Open loop system responses subject to constant inputs. (a) T_{z1}, T_{z2} responses; (b) T_b response; (c) T_c response; (d) W_{z1}, W_{z2} responses.

constant disturbances, but their shapes also indicate that the model equations and the magnitude of the system parameters chosen (Table 1) are appropriate.

4.1. Simulation of the closed loop systems

The closed loop implementation schemes for the temperature and humidity control systems are depicted in Figs. 3(a) and (b). As shown in Fig. 3(a) the temperature control system has two feedback loops. An inner feedback loop and an outer servocompensator loop. Both feedback loops work together to regulate the zone temperatures close to the setpoint temperatures. In contrast, the closed loop system for the humidity control (Fig. 3(b)) does not have the servocompensator loop. A consequence of this is that the relative humidity of the zones will not reach the humidistat setpoint. The offset or the error is likely to be small and is not a major concern from the viewpoint of thermal comfort.

The closed loop responses of the temperature control system are depicted in Figs. 4(a)–(d). The results show how the control system would respond during a cold day with $T_{oa} = -5$ °C and with the magnitude of the disturbances m_{w1}, m_{w2} and T_∞ similar to those used in open loop tests. It is apparent from Fig. 4(a) that the controller is able to bring the zone temperatures starting from arbitrary initial values such as $T_{z1}(0) = 21.7$ °C and $T_{z2}(0) = 21.1$ °C to their respective setpoints $T_{z1set} = 21.9$ °C and $T_{z2set} = 21.26$ °C in about 30 minutes. In fact the

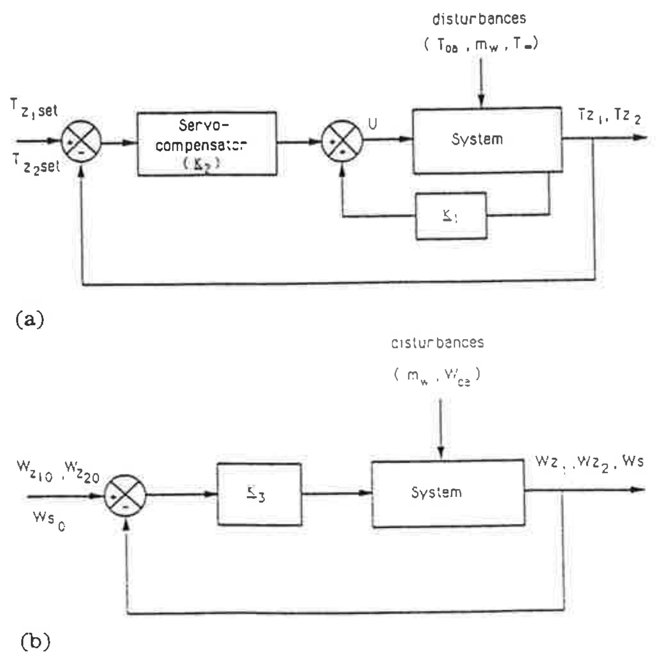


Fig. 3. Block diagram of the closed loop control system. (a) Temperature control; (b) humidity control.

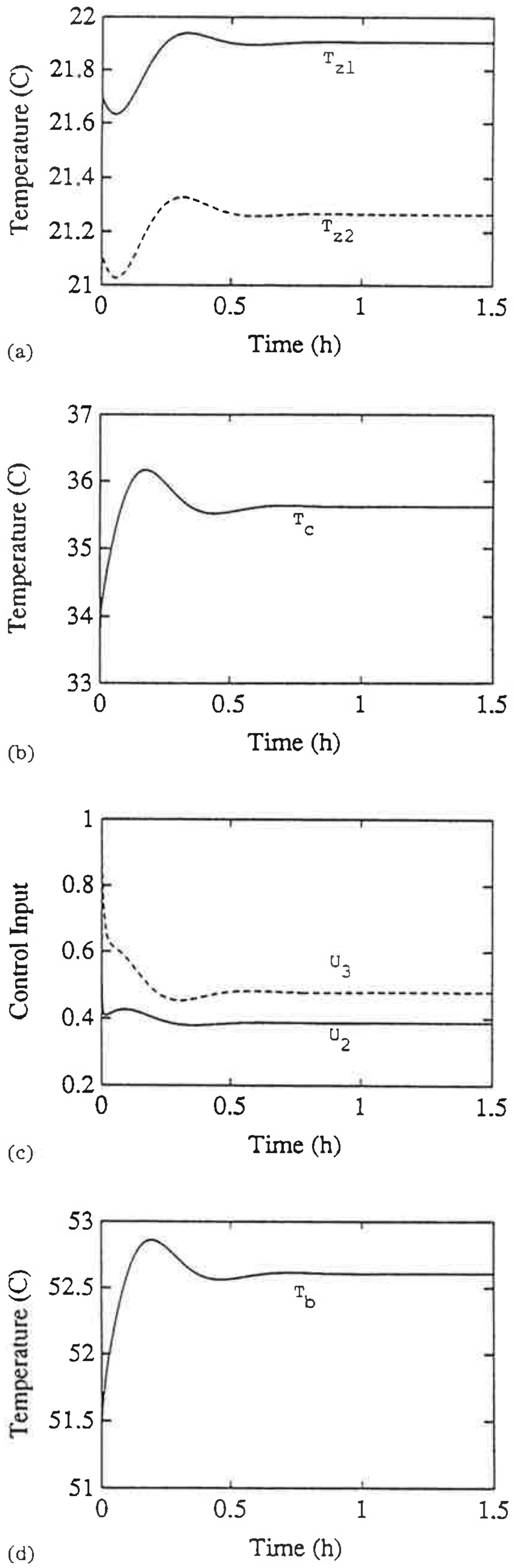


Fig. 4. Closed loop system responses to a step disturbance in T_a. (a) T_{z1}, T_{z2} responses; (b) T_c response; (c) U_2, U_3 responses; (d) T_b response.

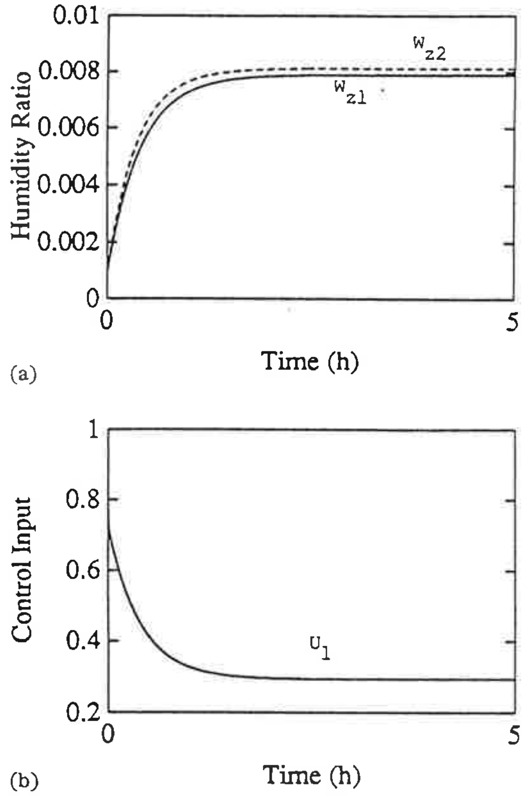


Fig. 5. Closed loop humidity control responses. (a) W_{z1}, W_{z2} responses; (b) U_1 response.

zone temperatures reach to within 0.05 °C of the setpoints in less than 15 minutes. This is despite the fact that the heating load acting on the zones was suddenly increased by way of a 5 °C step change in T_{oa}. Since a 5 °C change in outdoor temperature represents a 20% increase in the heating load, we note that the controller is able to reject the effects of a 20% sudden increase in the heating loads in about 15 minutes. How the coil temperature T_c and the boiler temperature T_b varied during this time are shown in Figs. 4(b) and (d) respectively, and the magnitude of the control inputs U_2 and U_3 in Fig. 4(c). It is apparent from Fig. 4(c) that in response to an increase in heating load, the temperature of the hot water and its mass flow rate in the heating coil have been increased by the controllers C_2 and C_3. This is indicated by the mass flow rate valve position U_2 which momentarily increases to 0.84 and gradually decreases to 0.39 under steady state. Likewise, the fuel firing rate in the burner increases to 91% instantaneously and then gradually decreases to 47.7% and is held constant in order to reject the effects of constant sustained disturbances acting on the zones. The consequence of higher values of U_2 and U_3 means that the boiler temperature (Fig. 4(d)) and the coil temperature (Fig. 4(b)) also increase. Since constant disturbances are acting on the zones, T_b and T_c will eventually reach steady state as shown in the Figures.

Figures 5(a) and (b) show the responses of the humidity control system. The disturbances acting on the zones were assumed as follows: $W_{oa} = 0.002$, $m_{w1} = m_{w2} = 2$ kgh^{-1}. Given this rate of moisture loss from the zones, how the controller C_1 maintains the zone humidity ratios is depicted in Fig. 5(a). It may be noted that the controller is able to bring W_{z1} and W_{z2} from the given initial conditions $W_{z1}(0) = W_{z2}(0) = 0.001$ to very close to the humidity setpoints ($W_{z1set} = 0.0082$, $W_{z2set} = 0.0087$). The actual final values of W_{z1} and W_{z2} are 0.0079 and 0.0082 respectively. Thus it is apparent that there is a small finite error which is in the acceptable range. The response times are reasonable as it can be noted that the zone humidity ratios reach a value of 0.005 in about 20 minutes. How the humidity control input U_1 must be varied during this time is depicted in Fig. 5(b). The rate of moisture added to the supply air rises rapidly during the initial few seconds and then gradually decreases to a final value of 0.293.

5. Typical daily simulation results

The results depicted in Figs. 4 and 5 show that the temperature and humidity controllers are able to reject step disturbances efficiently. However, it is also important to evaluate the performance of the controllers over a longer period of time such as a typical day with realistic changes in outdoor air temperatures and relative humidity. For this purpose, we consider a typical daily outdoor air temperature profile as shown in Fig. 6(a). This represents the temperature variations that occur during a typical day in October in Montreal (45.2° N). By approximating this profile by a series of small step functions we can test the performance of the temperature controller over the 24-hour period. The results of this exercise are shown in Figs. 6(b) and (c).

Figure 6(b) shows the temperature responses over the 24-hour period. It is apparent that the zone temperatures are held constant at their respective setpoints even though the outdoor temperature undergoes a 16 °C variation during the day. The temperatures of the boiler and the coil also vary in response to the load on the zones (Fig. 6(b)). Both T_b and T_c are increasing as the outdoor temperature decreases and when the outdoor temperature becomes warmer during the afternoon the temperatures of the boiler and the coil begin to decrease as shown in the Figure. The zone tem-

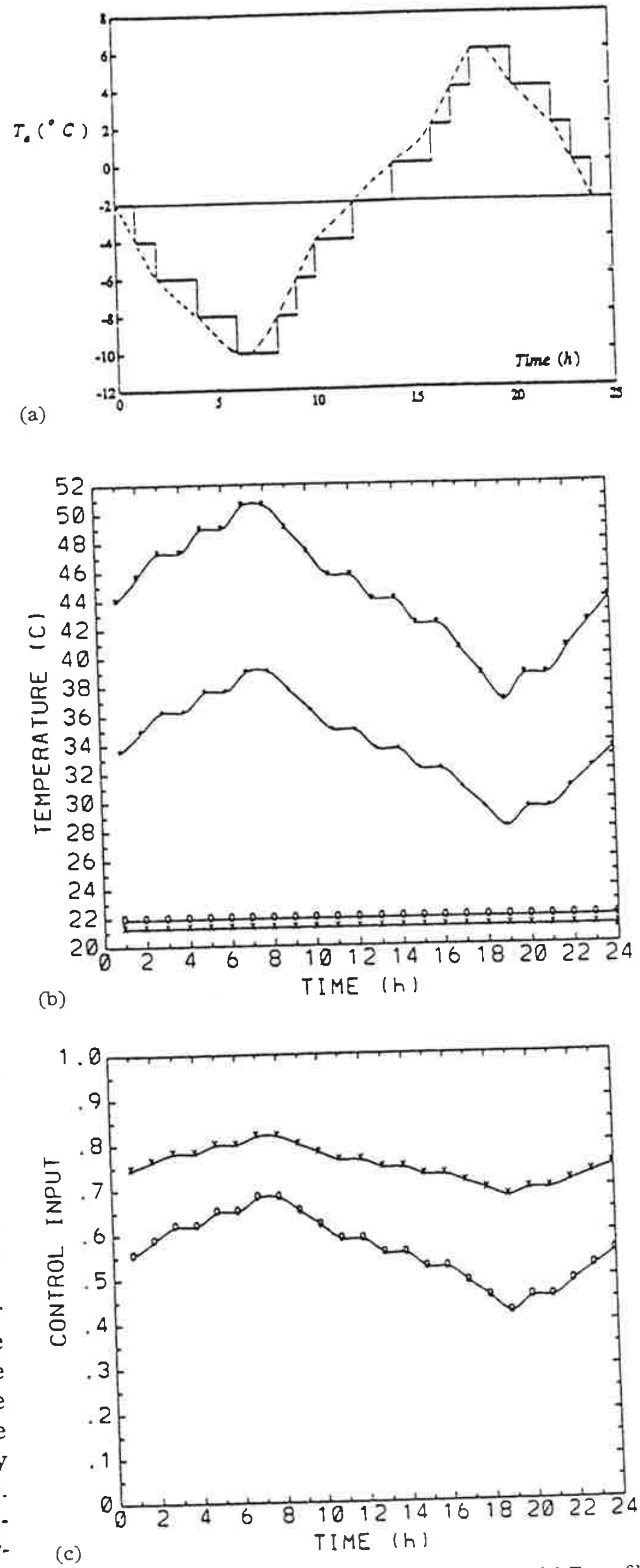
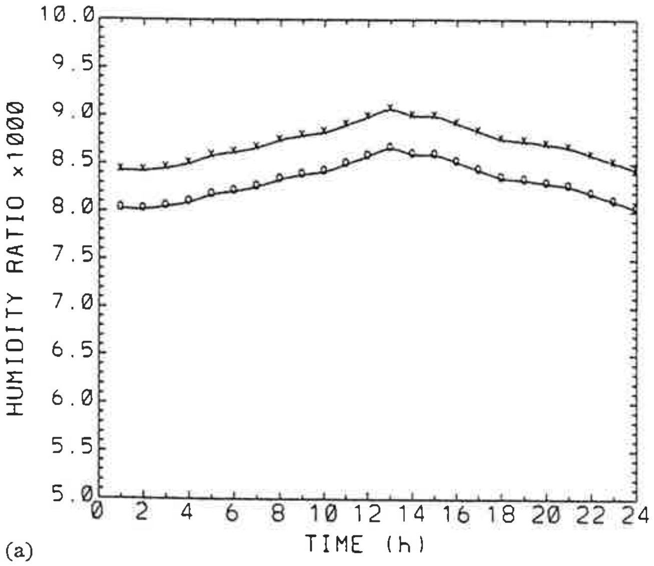
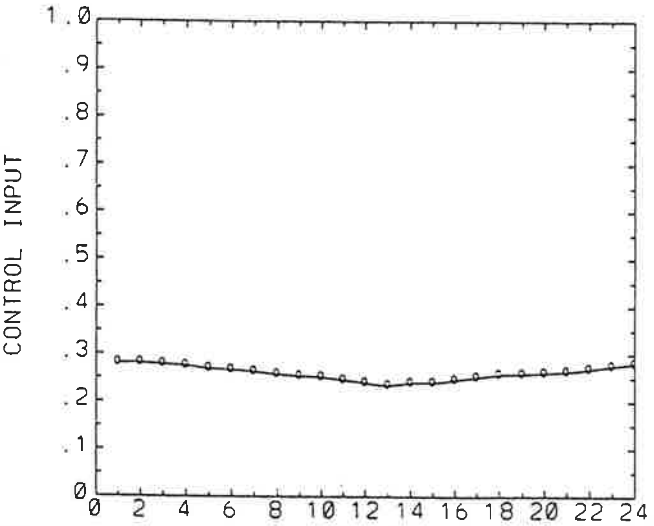


Fig. 6. Typical daily temperature control responses. (a) T_a profile; (b) T_{z1}, T_{z2}, T_b, T_c responses, —o— T_{z1}, —×— T_{z2}, —*— T_b, —+— T_c; (c) U_2, U_3 responses, —o— U_2, —×— U_3.



(a)



(b)

Fig. 7. Typical daily humidity control responses. (a) W_{z1}, W_{z2} responses, —o— W_{z1}, —×— W_{z2}, (b) U_1 response.

perature regulation is achieved by continuously modulating the mass flow rate of hot water U_2 and the fuel firing rate U_3 (Fig. 6(c)). Thus we conclude that the daily outdoor temperature variations of the type shown in Fig. 6(a) pose no difficulties as the controller is able to maintain good zone temperature control throughout the day.

Apart from the changes in outdoor temperatures, the environmental zones also receive solar radiation fluxes and internal heat loads. All these disturbances acting on the zones interact with the building envelope. We have shown in a previous study [9] that the effects of such disturbances can be rejected by designing servocompensators of the type presented in this paper.

As far as the humidity control is concerned, we have also studied the response characteristics of the humidity controller by simulating a typical day operation. For this purpose, a typical daily outdoor humidity profile was used. It is apparent from Fig. 7(a) that the zone humidity ratios reach to within 6% of the respective humidistat setpoints. How the humidity controller modulated the rate of moisture addition to the supply air during this day is shown in Fig. 7(b). The rate of moisture addition shown in Fig. 7(b) indicates that the humidifier output was varied between 25–30% of its rated capacity in order to maintain the zone humidity ratios (Fig. 7(a)) close to the humidistat setpoints ($W_{z1set}=0.0082$ and $W_{z2set}=0.0087$).

6. Limitations

Although the results shown in this study are for a two-zone building, it is evident from the foregoing results that the same analysis could be extended to buildings with more than two zones. However, this would slow down the system responses considerably. In other words, the zone temperatures would take a much longer time to settle towards the setpoints. To overcome this problem it would be necessary to not only control the discharge temperature according to the highest load but also control the zone air dampers to regulate the zone temperatures. Work is in progress to design controllers for such systems also known as VAV systems.

7. Conclusions

Enthalpy and mass balance principles were used to develop a seventh-order nonlinear model for a constant volume heating and humidifying system. By neglecting the liquid enthalpy of the water vapour, the seventh-order control system is divided into a fourth-order temperature control system and a third-order humidity control system. Two multivariable controllers were designed, one for each system for a two-zone building. It has been shown that by continuously modulating the discharge air temperature, it is possible to regulate the zone temperatures and humidity ratios close to the setpoints when the zones are acted upon by disturbances due to variations in outdoor temperatures and humidity ratios that typically occur during a cold day.

Acknowledgement

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Nomenclature

$\mathbf{A}$	system matrix
A_c	outside area of the coil
a_b	heat loss coefficient of the boiler exterior surfaces
a_{ij}	matrix elements
$\mathbf{B}$	matrix
b_{ij}	elements of matrix $\mathbf{B}$
$\mathbf{C}_1$ and $\mathbf{C}_2$	output matrices
C_{pa}	specific heat of air
C_b	thermal capacity of the boiler
C_c	thermal capacity of the coil
C_{pw}	specific heat of water
$\boldsymbol{d}_1$, $\boldsymbol{d}_2$	disturbance vectors
$\mathbf{E}$	disturbance matrix
e	output error
e_{ij}	elements of matrix $\mathbf{E}$
$\mathbf{F}$	matrix
f_{ij}	elements of matrix $\mathbf{F}$
f	outdoor air fraction
$\mathbf{G}$	matrix
g_i	elements of matrix $\mathbf{G}$
$\mathbf{H}$	disturbance matrix
h_{ij}	elements of matrix $\mathbf{H}$
h_c	heat transfer coefficient
h_s	enthalpy of supply air
h_z	enthalpy of zone air
J_1, J_2	performance indices
$\mathbf{K}_1$, $\mathbf{K}_2$, K_3	feedback gains
M_z	mass of air in the zone
M_{ws}	mass of air in the distribution box
m_a	mass flow rate of the supply air
m_{w1}, m_{w2}	mass flow rate of water vapour
$\mathbf{Q}$	weighting matrix on state variables
$\mathbf{Q}_1$, $\mathbf{Q}_2$	weighting matrices
q_s	sensible heat loss rate
q_l	latent heat loss rate
R	weighting on control input
T	temperature vector
T_b	boiler temperature
T_c	coil temperature
T_s	supply air temperature
T_{oa}	outdoor air temperature
T_{∞}	plenum temperature
T_{mx}	mixed air temperature
T_{bmax}	maximum temperature of the boiler
T_z	zone air temperature
T_z	zone air temperature
T_{zset}	zone setpoint temperature
t	time
U	control vector, normalized
U_1	rate of moisture addition (normalized with respect to U_{1max})
U_2	mass flow rate of hot water (normalized with respect to U_{2max})
U_3	fuel firing rate (normalized with respect to U_{3max})
U_{1max}	humidifier capacity
U_{2max}	maximum flow rate of hot water
U_{3max}	rated capacity of the burner
V	control input (equal to ΔU_1)
ΔU	variations in U about the operating point
ΔT	variations in temperatures
$\boldsymbol{W}$	humidity ratio, vector
W_s	humidity ratio, supply air
W_{oa}	outdoor air humidity ratio
W_z	zone air humidity ratio
W_{zset}	humidistat setpoint
y_T	output, temperature control system
y_W	output, humidity control system
ξ	state variable for the output integrator
Δ	variations from the operating point
Ω	compensator matrix
θ	compensator matrix
ϵ	effectiveness

Subscripts

1	referring to zone 1
2	referring to zone 2

Superscripts

$\cdot$	derivative with respect to time
*	optimal value
/	transpose
T	transpose

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