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A Procedure for Sensitivity Analysis of Airflow in Multi-Zone Buildings

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This paper describes a general procedure for analyzing airflow sensitivities with respect to input parameter variations. The derivation is based on a matrix representation form using the system theoretic approach. The sensitivity analysis provides both valuable insights about the effects of input parameter variations on the accuracy of multi-zone airflow models and necessary information for additional analyses. A case study is used to show the procedure for the airflow sensitivity calculation, and to demonstrate how to use sensitivity information in the error analysis and design assistance.

INTRODUCTION

THERE has been considerable interest over the past few years in the development of models for building airflow analysis, due mainly to energy and indoor air quality considerations. Most of these airflow models are based on the steady-state assumption, and calculate the mean airflow rates due to average values of driving forces [1, 2]. Nevertheless, validations and applications of airflow models have been slow and difficult. One of the major reasons can be attributed to the lack of study on the influence of input parameter variations on the models and the simulation results.

Variations of input parameters arise in several categories. First, the modelling assumptions and approximations employed to transfer a physical building to a multi-zone airflow network introduce errors between the real systems and the conceptual models. Second, in analyzing existing buildings the input data from measurements or estimations have errors due to measurement techniques [3]. Third, when these airflow models are used in the design process, the input parameters are based on estimations using standard procedures and/or manufacturer's specifications. They can be affected by inaccuracy of estimation procedures, manufacturing tolerances, and installation disparities. Fourth, seasoning of building components due to erosions and abrasions can also cause changes of the input parameters. In addition, effects of intentional parameter variations on airflow are analyzed for designing building airflow systems. Changes of input parameters due to any of these or other reasons may cause the predictions of airflow models to be erroneous or even invalid. Therefore, it is important to examine the influence of the input parameter variations on airflow models and their simulation results.

One possible technique to study the influence of par-

ameter variations is the application of *parametric analysis*. In this approach tentative changes in some input parameters are assigned and airflow models are applied to observe the responses. This necessitates considerable data handling and computational efforts. In addition, when the effects of many parameters are concerned, the interpretation of the numerous simulation results can be difficult.

A more feasible approach is the use of *sensitivity analysis* methods. The *sensitivity* (or *sensitivity function* as used in the sensitivity theory [4]) of airflow with respect to individual input parameters is calculated. Then, additional analysis such as error analysis and airflow system design assistance can be carried out based on the sensitivity information. The airflow sensitivities are obtained at a particular point as defined by the values of input parameters and the airflow simulation results based on these input values. This method is also referred to as the *perturbation method*.

The emphasis of this paper is to present a procedure for calculating airflow sensitivities with respect to input parameters. The derivation is based on the matrix representation form of the modelling process for airflow models using the system theoretic approach, developed by the authors [1]. The application of the sensitivity information for error analysis and design assistance is briefly discussed in a case study. In the following sections, a brief review of the system-theoretic modelling representation, the derivation of the airflow sensitivity calculation procedure, sensitivity calculation for a case study, and additional analyses are presented.

SYSTEM-THEORETIC APPROACH FOR AIRFLOW MODELLING

In the system-theoretic approach, a building is modelled by a network of N zones and L openings [1]. The zonal pressures are represented by a column vector $\Phi = [\Phi_1, \Phi_2, \ldots, \Phi_N]^T$, and airflow rates through openings by $Q = [Q_1, Q_2, \ldots, Q_n]^T$.

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For opening l $(l = 1, 2, \ldots, L)$, the volumetric flow equation is expressed by:

$$Q_l = F_l(\Delta P_l) \tag{1}$$

where ΔP_l is the pressure difference across the opening l. The corresponding mass airflow rate is given as $W_l = \gamma_l Q_l = \gamma_l F_l(\Delta P_l)$, where γ_l is the density of the air flowing in the opening l. The ΔP_l is the pressure difference across the opening and can be expressed by a summation of the difference between zonal pressures ($\Pi^T \Phi$) and the total effect of driving forces on the opening (P_f). In the matrix form, this is:

$$\Delta P = \Pi^T \Phi + P_f. \tag{2}$$

The matrix Π in the above equation is the *incidence matrix* for the building interconnections. The elements of Π are given by:

$$\Pi_{il} = \begin{cases} 0 & \text{opening } l \text{ is not connected to zone } i, \\ 1 & \text{opening } l \text{ flows out of zone } i, \\ -1 & \text{opening } l \text{ flows into zone } i. \end{cases} \tag{3}$$

The set of mass balance equations is written as:

$$\Pi \eta F(\Pi^T \Phi + P_f) = 0 \tag{4}$$

where η is a diagonal matrix and its diagonal elements are equal to the density of the air flowing through openings, $\eta_{ll} = \gamma_l$. The zonal pressures are solved by iterative algorithms. In the kth update, the new value of Φ is calculated by:

$$\Phi^{k+1} = \Phi^k + C^k \tag{5}$$

where the correction vector C^k is obtained through:

$$\Pi(\eta D)\Pi^T C^k = -\Pi Q^k \tag{6}$$

and D is a diagonal matrix with

$$D_{ll} = \frac{dF_l(\Delta P_l)}{d\Delta P_l}.$$

The above airflow calculation procedure for the given input parameters is also referred to as a *simulation* in this paper.

DERIVATION OF SENSITIVITY CALCULATION

Values of input parameters used in the above calculation are called the *nominal values*. When there are deviations between the actual and nominal values, the zonal pressures and airflow through openings will differ from the steady-state solutions obtained from multi-zone airflow models. The *sensitivity analysis* examines the relationship between small variations in the input parameters and the potential changes of airflow through buildings.

The *input parameters* considered by this sensitivity analysis method can be separated into two types: the driving forces and the opening characteristics. The *driving force parameters* are related to pressure differences across openings, including wind-induced pressures, mechanical ventilation, zonal temperatures and opening locations. The *opening characteristics parameters* are related to flow equations, an example can be the discharge coefficient for the flow equation of an orifice type opening. Variations of

both types of input parameters should not affect the structure of the building airflow systems.

Let the vector θ be the M mutually independent parameters to be studied, θ^p for the driving force related and θ^f for the opening characteristics related. The *sensitivity* of an airflow rate Q_l to the variation of a parameter θ_j is usually defined as the first-order partial derivative: $\partial Q_l / \partial \theta_j$ [4].

Since θ^p and θ^f are related to the airflow in different ways, the calculations are carried out separately before a general form is analyzed. The parameters θ^p are related to the pressure differences across openings. Figure 1 shows the schematic diagram of a general opening l, which connects zones i and j. The expression for the driving force acting on the opening l is:

$$(P_f) = P_l^w + \frac{P_a}{R_a} g \left(-\frac{h_{li}}{T_i} + \frac{h_{lj}}{T_j} \right) - \frac{P_a}{R_a} g \frac{h_l}{T_l} \tag{7}$$

where the wind-induced pressure can be obtained from the given pressure coefficient $(C_p)_l$ and the wind velocity at roof height V_h as: $P_l^w = (C_p)_l \frac{1}{2} \rho_0 V_h^2$. The derivatives of Q with respect to θ^p are calculated as follows:

$$\frac{\partial Q}{\partial \theta^p} = \frac{\partial}{\partial \theta^p} \{F(\Pi^T \Phi + P_f)\} = D\Pi^T \frac{\partial \Phi}{\partial \theta^p} + D \frac{\partial P_f}{\partial \theta^p}. \tag{8}$$

The left side is called the Jacobian matrix Q at θ^p. It is a N by M matrix and its elements are $[\partial Q/\partial \theta^p]_{ij} = \partial Q_i/\partial (\theta^p)_j$. The case is similar for matrices $\partial \Phi/\partial \theta^p$ and $\partial P_f/\partial \theta^p$.

Equation (8) indicates that the relative changes of airflow rates with respect to input parameters θ^p depends on both matrices $\partial \Phi/\partial \theta^p$ and $\partial P_f/\partial \theta^p$. Since the zonal pressures Φ are implicit functions of θ^p through the mass balance equation (4), the Jacobian matrix of Φ can be obtained by applying partial derivatives to Equation (4) with respect to θ^p, it yields:

$$\Pi \frac{\partial \gamma}{\partial \theta^p} \cdot Q\chi + \Pi(\eta D)\Pi^T \frac{\partial \Phi}{\partial \theta^p} + \Pi(\eta D)\frac{\partial P_f}{\partial \theta^p} = 0 \tag{9}$$

where χ is a row vector of 1's. Therefore,

$$g_l = \begin{cases} r_i & \text{if } Q_l > 0 \\ r_j & \text{if } Q_l < 0 \end{cases}$$

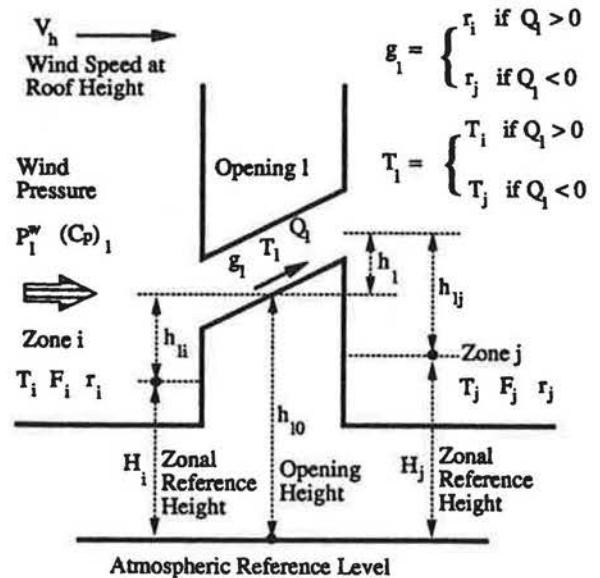$$T_l = \begin{cases} T_i & \text{if } Q_l > 0 \\ T_j & \text{if } Q_l < 0 \end{cases}$$

Fig. 1. Schematic graph of an opening.

$$\frac{\partial \Phi}{\partial \theta^p} = -[\Pi(\eta D)\Pi^T]^{-1}\Pi\left(\frac{\partial \gamma}{\partial \theta^p}\cdot Q\chi + (\eta D)\frac{\partial P_f}{\partial \theta^p}\right). \quad (10)$$

The matrix that requires the inverse operation in this equation is the same as that used in the steady-state airflow calculation (equation 6), and should exist if the steady-state solution of airflow can be obtained.

The Jacobian matrix, $\partial P_f/\partial \theta^p$, is obtained by directly applying partial differentiations to equation (7). The composition of $(P_f)_i$ depends on connections among zones of the building (represented by the Π matrix) and the directions of airflow (represented by the signs of Q). Table 1 shows the elements of this Jacobian matrix for different parameters under various conditions.

For the opening characteristics related input parameters, the sensitivity of Q with the respect to θ^f can be represented by the Jacobian matrix:

$$\frac{\partial Q}{\partial \theta^f} = \frac{\partial}{\partial \theta^f}F(\Pi^T\Phi + P_f) = D\Pi^T\frac{\partial \Phi}{\partial \theta^f} + \frac{\partial F}{\partial \theta^f}. \quad (11)$$

The elements of $\partial F/\partial \theta^f$ are the derivatives of the flow equations to the flow equation related parameters, such as:

$$\left(\frac{\partial F}{\partial \theta^f}\right)_{ij} = \frac{\partial(K_i\Delta P_i^{n_i})}{\partial K_i}$$

if $F_i(\ ) = K_i\Delta P_i^{n_i}$ and $\theta_j^f = K_i$. The Jacobian matrix Φ can be calculated in a similar manner as in equations (9 and 10):

$$\frac{\partial \Phi}{\partial \theta^f} = -[\Pi(\eta D)\Pi^T]^{-1}\Pi\eta\frac{\partial F}{\partial \theta^f}. \quad (12)$$

For a mixed set of input parameters of driving forces and opening characteristics,

$$\theta = \begin{bmatrix} \theta^p \\ \theta^f \end{bmatrix},$$

the zonal pressures and airflow are functions of these parameters, defined by the mass balance equation (4) and the opening flow equations. The general form of Φ Jacobian matrix can be obtained by combining the analysis performed on θ^p and θ^f, as:

$$\frac{\partial \Phi}{\partial \theta} = -[\Pi(\eta D)\Pi^T]^{-1}\Pi\Lambda \quad (13)$$

where Λ is composed of:

Table 1. Calculation formulas for basic Jacobian matrices

Elements	Calculation Formulas
$\dfrac{\partial(P_f)_1}{\partial T_1}$	$\Pi_{11}\dfrac{\rho_1 g}{T_1}\left\{h_{11} + u\left[\dfrac{\Pi_{11}+\xi(Q_1)}{2}\right]\times h_1\right\}$
$\dfrac{\partial(P_f)_1}{\partial h_{11}}$	$(-\rho_1 + \rho_i)\, g\cdot u\left[\dfrac{\Pi_{11}-\xi(Q_1)}{2}\right]$
$\dfrac{\partial(P_f)_1}{\partial h_{1o}}$	$(-\rho_1 + \rho_j)\, g$
$\dfrac{\partial(P_f)_1}{\partial(P^w)_1}$	Π_{o1}
$\dfrac{\partial(P_f)_1}{\partial(C_p)_1}$	$\Pi_{o1}\dfrac{(P^w)_1}{(C_p)_1},\quad$ or $\quad\Pi_{o1}\dfrac{\rho_0}{2}V_h^2$
$\dfrac{\partial(P_f)_1}{\partial V_h}$	$2\Pi_{o1}\dfrac{(P^w)_1}{V_h},\quad$ or $\quad\Pi_{o1}(C_p)_1\rho_0 V_h$
$\dfrac{\partial F_1}{\partial(\theta^f)_j}$	derivatives of flow equation to $(\theta^f)_j$
$\dfrac{\partial\gamma_1}{\partial T_1}$	$-\dfrac{\rho_1}{T_1}\quad$ if $\gamma_1 - \rho_1$

Notes:
1. The function $u()$ equals to the nearest integer value toward negative infinite,

2. $\Pi_{o1} = -\sum_{i=1}^{N}\Pi_{11}$ and ρ_0 is the density of the outdoor air,

3. $\xi(v) = 1$ if $v > 0$; $\xi(v) = -1$ if $v < 0$; $\xi(v) = 0$ if $v = 0$.

$$\Lambda = \left[\frac{\partial \gamma}{\partial \theta^p} \cdot Q\chi + (\eta D)\frac{\partial P_f}{\partial \theta^p} \eta \frac{\partial F}{\partial \theta^f}\right]. \qquad (14)$$

The sensitivity matrix can, then, be calculated from:

$$\frac{\partial Q}{\partial \theta} = \frac{\partial}{\partial \theta}\{F(\Pi^T\Phi + P_f)\} = \left[0 \ \frac{\partial F}{\partial \theta^f}\right]$$

$$+ D\Pi^T \frac{\partial \Phi}{\partial \theta} + \left[D\frac{\partial P_f}{\partial \theta^p} \ 0\right]. \quad (15)$$

Matrices $\partial P_f/\partial \theta^p$, $\partial F/\partial \theta^f$, and $\partial \gamma/\partial \theta^p$ are basic Jacobian matrices, and can be calculated directly by employing the procedures in Table 1.

The above equations (13–15) are the final formula for calculating sensitivities of airflow with respect to input parameters. The input data for the calculation can be obtained from the building characteristics (Π and η), from the steady-state solution of a multizone airflow model (D), and from calculation formulas of Table 1 ($\partial P_f/\partial \theta^p$, $\partial F/\partial \theta^f$, and $\partial \gamma/\partial \theta^p$). The calculation can be easily incorporated into the existing steady-state airflow models using network approaches.

Usually, we are interested in the percentage change of airflow due to parameter variations. This is called the *semi-relative sensitivity* and can be obtained through $[(\partial Q)/Q]/\partial \theta$. In addition, the changes of the airflow out of one zone or into the entire building are more informative. The sensitivities of the total airflow out of zones or into the building are calculated by summations in:

$$\frac{\partial Q_i^T}{\partial \theta_j} = \sum_l \frac{\partial Q_l}{\partial \theta_j}, \qquad i = 0, 1, \ldots, N, \qquad (16)$$

where the summation is over all the opening l so that air in opening l is flowing away from zone i.

SENSITIVITY CALCULATION FOR A CASE STUDY

The airflow sensitivity analysis with respect to input parameters has been presented. The input to this analysis are the input data for and the solutions of the steady-state airflow models, and the set of parameters of interests. In the following section, the example from [1] is used to demonstrate the sensitivity calculation procedure.

The case study building, its opening characteristics, and the incidence matrix Π are shown in Figure 2(a)–(c). By using the calculations in equations (1–6), the steady-state solutions are obtained (Figure 2(d)).

Let $\theta = [T_0 \ T_4 \ h_{30} \ C_{pl} \ P_4^w \ V_h \ K_4 \ n_3 \ n_1]^T$ be an arbitrary set of input parameters of interests. The first six parameters are related to driving forces, while the last three are related to opening characteristics. The calculations are as follows. Table 2 shows the expressions of individual elements in the basic Jacobian matrices $\partial P_f/\partial \theta^p$, $\partial F/\partial \theta^f$ and $\partial \gamma/\partial \theta^p$, obtained by using Table 1. The Λ matrix can be calculated according to equation (14).

Matrix $[\Pi(\eta D)\Pi^T]^{-1}$ can be obtained from the last iteration of the steady-state calculation in equation (6). Then the Jacobian matrix of zonal pressures, $\partial \Phi/\partial \theta$, can be obtained from equation (13). Finally, using equation (15), the sensitivities of airflow Q with respect to the set of parameters θ can be calculated.

The calculation procedure has been implemented in a computer program. Table 3 shows the results of the sensitivity calculation. The sensitivities of airflow rates through openings with respect to the nine parameters are shown in the first eight rows. Displayed in the next eight rows are the semi-relative sensitivities. The semi-relative sensitivities of total airflow out of individual zones and into the entire building due to input parameter variations are shown in the last five rows.

The values in Table 3 indicate the tendency of airflow changes as a result of small input parameter variations around their nominal values. For example, the cell on row 1 and column 1 indicates that if the outside temperature were to increase one unit, i.e. 1°C, the airflow rate in the opening one would tend to increase 0.438 m³/hr, or 0.53% of the nominal value (82.6 m³/hr) as indicated by the cell on row 9 and column 1. The same 1°C increase of the outside temperature would cause the airflow rate out of room 1 to increase 0.164% and the natural ventilation rate into the entire building to decrease 0.174%, as indicated by the cells on row 5 from the bottom and column 1 and the last row and column 1, respectively.

The sensitivities calculated by the above procedure indicate the potential influence of parameters on the airflow at **only** one specific point defined by the nominal values of the input parameters. However, when there are real changes in the input parameters (e.g. the wind speed increases), the above sensitivity analysis procedure has to be performed again for the new situation.

ERROR ANALYSIS AND DESIGN ASSISTANCE

Airflow sensitivities provide information about potential changes in airflow due to input parameter variations. This information can be used for further analysis such as error analysis and design assistance. The *error analysis* provides information about errors in airflow calculations due to errors in input parameters. Given general information regarding error ranges about input data, this analysis can calculate the accuracy of the predictions by airflow models. In the assistance for building airflow system designs, the sensitivity information is used for designing building airflow systems to achieve desired specifications. In this section, the case study is used to show briefly the procedures of these two additional analyses.

The potential changes of airflow due to input parameter variations in one or more input parameters can be estimated based on the airflow sensitivity values. If a single parameter θ_j increases by $\Delta\theta_j$ to a new value of θ_j^* the airflow changes can be calculated by:

$$\Delta Q = \frac{\partial Q}{\partial \theta_j} \times \Delta\theta_j. \qquad (17)$$

When two or more parameters have small variations, the predicted airflow changes can be obtained by summations of individual effects, as:

$$\Delta Q = \sum_j \frac{\partial Q}{\partial \theta_j} \times \Delta\theta_j. \qquad (18)$$

When the standard deviations of input parameters, σ_{θ_j},
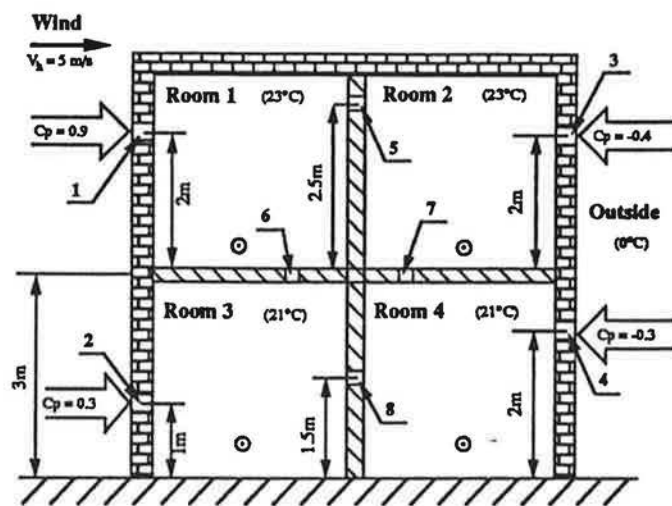
(a) A Four Room Building

$$\Pi = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 \end{bmatrix}$$

(c) Π Matrix

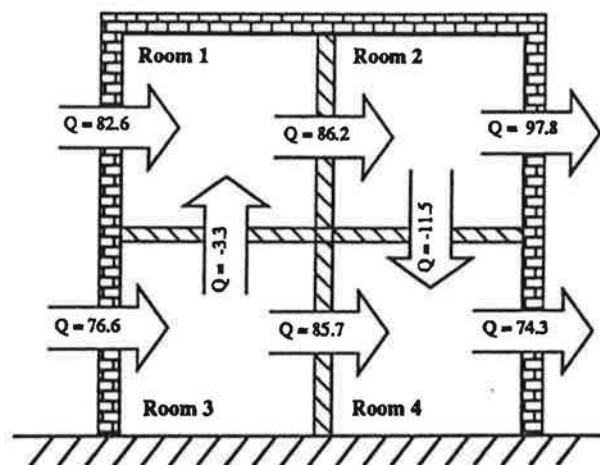| #	n	K	#	n	K
1	.65	.005	5	.5	.015
2	.65	.008	6	.5	.020
3	.65	.007	7	.5	.020
4	.65	.009	8	.5	.015

(b) Opening Characteristics

(d) Calculated Airflow Rates (m³/hr)

Fig. 2. Case study.

are given, the standard deviations of airflow as a result of the indetermination in input parameters can be obtained as:

$$\sigma_Q = \sqrt{\sum_j \left[\left| \frac{\partial Q}{\partial \theta_j} \right| \times \sigma_{\theta_j} \right]^2}. \tag{19}$$

In order to verify the results of calculations in equations (17 and 18), the predictions of airflow change potentials based on airflow sensitivities are compared with the airflow changes obtained through multiple simulations

using airflow models. Let ΔQ^* be the actual airflow change between the calculated airflow by substituting $\theta_j + \Delta\theta_j$ into the airflow models and the nominal values of airflow. The relation between the prediction and the actual simulation result, for single input parameter, is illustrated in Fig. 3.

Both equations (17 and 18) use the linear extrapolation to approximate the potential changes of airflow due to parameter variations. Theoretically, equations (17 and 18) represent only the first-order terms in the power

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Table 2. Basic Jacobian matrices of the case study

$$\frac{\partial P_f}{\partial \theta^p} = \begin{bmatrix}
\rho_0 g \frac{h_{10}}{T_0} & \rho_0 g \frac{h_{20}}{T_0} & -\rho_0 g \frac{h_{30}}{T_0} & -\rho_0 \frac{g_{40}}{T_0} & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_4 g \frac{h_{44}}{T_4} & 0 & 0 & -\rho_4 g \frac{h_{84}}{T_4} & 0 & 0 \\
0 & 0 & (-\rho_2 + \rho_0) g & 0 & 0 & 0 & 0 & 0 \\
\frac{P_1^v}{C_{p1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
\frac{P_1^v}{V_h} & \frac{P_2^v}{V_h} & -\frac{P_3^v}{V_h} & -\frac{P_4^v}{V_h} & 0 & 0 & 0 & 0
\end{bmatrix}^T$$

$$\frac{\partial F}{\partial \theta^f} = \begin{bmatrix}
0 & 0 & 0 & \frac{Q_4}{K_4} & 0 & 0 & 0 & 0 \\
0 & 0 & n_3 \frac{Q_3}{\Delta P_3} & 0 & 0 & 0 & 0 & 0 \\
n_1 \frac{Q_1}{\Delta P_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T$$

$$\frac{\partial \gamma}{\partial \begin{bmatrix} T_0 \\ T_4 \end{bmatrix}} = \begin{bmatrix}
-\frac{\rho_0}{T_0} & -\frac{\rho_0}{T_0} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{\rho_4}{T_4} & -\frac{\rho_f}{T_4}
\end{bmatrix}^T$$

where

$$\theta = \begin{bmatrix} T_0 & T_4 & h_{30} & C_{p1} & P_4^v & V_h & K_4 & n_3 & n_1 \end{bmatrix}^T$$

$$\theta^p = \begin{bmatrix} T_0 & T_4 & h_{30} & C_{p1} & P_4^v & V_h \end{bmatrix}^T \quad \text{and} \quad \theta^f = \begin{bmatrix} K_4 & n_3 & n_1 \end{bmatrix}^T$$

expansions of the airflow with respect to input parameters. The expansion for single parameter variations is expressed as:

$$\Delta Q = \frac{\partial Q}{\partial \theta_j} \Delta \theta_j + \frac{1}{2} \frac{\partial^2 Q}{\partial \theta_j^2} \Delta \theta_j^2 + o(3) \qquad (20)$$

and for variations of two or more parameters in:

Fig. 3. Indication of discrepancy between sensitivity calculation and simulation.

$$\Delta Q = \sum_j \frac{\partial Q}{\partial \theta_j} \Delta \theta_j$$

$$+ \frac{1}{2} \left\{ \sum_j \frac{\partial^2 Q}{\partial \theta_j^2} \Delta \theta_j^2 + \sum_i \sum_{\substack{j \\ j \neq i}} \frac{\partial^2 Q}{\partial \theta_i \partial \theta_j} \Delta \theta_i \Delta \theta_j \right\} + o(3) \qquad (21)$$

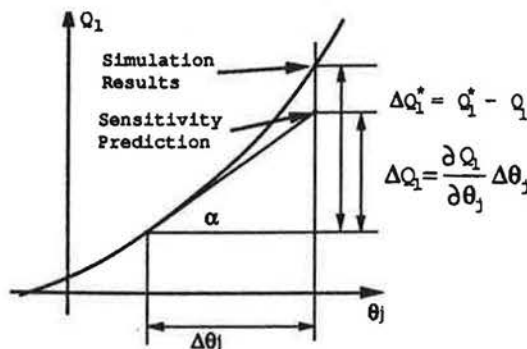where o(3) are higher order errors, which are normally smaller than the first and second order terms.

The accuracy of equations (17 and 18) depends mainly on the degree of linearity of Q as a function of the parameter θ_j and the magnitude of parameter variations. For the multiple parameter variations, the accuracy of equation (18) is further influenced by the second-order cross partial differentials among the varying parameters.

For the case study, the calculations in equation (17) for selected single parameters are listed in Table 4, together with the relative discrepancy between the prediction by equation (17) and the actual airflow simulations. The parameter variations are 3°C for temperatures, 0.5 m for opening locations, and 10% for the rest. Each cell in Table 4 contains two numbers, the first one is the predicted potential airflow change due to one parameter

Table 3. Sensitivity analysis results of the case study

Sensitivity of Individual Air Flow Rates

	$\dfrac{\partial Q_i}{\partial T_0}$	$\dfrac{\partial Q_i}{\partial T_4}$	$\dfrac{\partial Q_i}{\partial h_{30}}$	$\dfrac{\partial Q_i}{\partial Cp_1}$	$\dfrac{\partial Q_i}{\partial P_4^w}$	$\dfrac{\partial Q_i}{\partial V_h}$	$\dfrac{\partial Q_i}{\partial K_4}$	$\dfrac{\partial Q_i}{\partial n_3}$	$\dfrac{\partial Q_i}{\partial n_1}$
Q_1(82.6)	0.438	-0.016	0.808	67.93	-1.391	25.40	851.5	21.17	157.77
Q_2(76.6)	-0.714	-0.028	1.725	-32.20	-2.975	11.73	1821.9	45.20	-74.78
Q_3(97.8)	-1.055	0.258	5.834	14.36	3.133	18.08	-1918.4	152.91	33.36
Q_4(74.3)	0.123	-0.052	-3.067	24.20	-7.813	22.03	4784.6	-80.39	56.22
Q_5(86.2)	-0.478	-0.500	1.519	19.63	-2.148	20.02	1315.2	39.81	45.61
Q_6(-3.3)	0.625	0.483	-0.643	54.01	0.640	7.52	-391.8	-16.85	125.45
Q_7(-11.5)	-0.573	0.792	4.286	-5.23	5.245	-1.92	-3211.8	112.32	-12.16
Q_8(85.7)	-0.451	0.449	1.219	18.97	-2.568	20.11	1572.8	31.93	44.06

Semi-Relative Sensitivity of Individual Air Flow Rates

	$\dfrac{(\partial Q_i)/Q_i}{\partial T_0}$	$\dfrac{(\partial Q_i)/Q_i}{\partial T_4}$	$\dfrac{(\partial Q_i)/Q_i}{\partial h_{30}}$	$\dfrac{(\partial Q_i)/Q_i}{\partial Cp_1}$	$\dfrac{(\partial Q_i)/Q_i}{\partial P_4^w}$	$\dfrac{(\partial Q_i)/Q_i}{\partial V_h}$	$\dfrac{(\partial Q_i)/Q_i}{\partial K_4}$	$\dfrac{(\partial Q_i)/Q_i}{\partial n_3}$	$\dfrac{(\partial Q_i)/Q_i}{\partial n_1}$
Q_1(82.6)	0.530	-0.020	0.979	82.28	-1.684	30.77	1031.5	25.67	191.11
Q_2(76.6)	-0.933	-0.037	2.252	-42.05	-3.885	15.32	2379.2	59.03	-97.66
Q_3(97.8)	-1.079	0.264	5.967	14.69	3.204	18.49	-1962.2	156.39	34.12
Q_4(74.3)	0.165	-0.070	-4.131	32.60	-10.521	29.67	6443.0	-108.25	75.71
Q_5(86.2)	-0.554	-0.580	1.762	22.78	-2.491	23.22	1525.5	46.18	52.90
Q_6(-3.3)	18.926	14.620	-19.488	1636.55	19.390	228.06	-11874.0	-510.74	380.00
Q_7(-11.5)	-4.994	6.900	37.325	-45.61	45.676	-16.77	-27970.9	978.24	-105.00
Q_8(85.7)	-0.526	0.524	1.421	22.12	-2.995	23.45	1834.0	37.24	51.38

Semi-Relative Sensitivity of Individual Zonal Ventilation Rates

	$\dfrac{(\partial Q_i^T)/Q_i^T}{\partial T_0}$	$\dfrac{(\partial Q_i^T)/Q_i^T}{\partial T_4}$	$\dfrac{(\partial Q_i^T)/Q_i^T}{\partial h_{30}}$	$\dfrac{(\partial Q_i^T)/Q_i^T}{\partial Cp_1}$	$\dfrac{(\partial Q_i^T)/Q_i^T}{\partial P_4^w}$	$\dfrac{(\partial Q_i^T)/Q_i^T}{\partial V_h}$	$\dfrac{(\partial Q_i^T)/Q_i^T}{\partial K_4}$	$\dfrac{(\partial Q_i^T)/Q_i^T}{\partial n_3}$	$\dfrac{(\partial Q_i^T)/Q_i^T}{\partial n_1}$
Q_1^t(89.5)	0.164	-0.020	0.979	82.28	-1.684	30.77	1031.5	25.650	191.11
Q_2^t(97.8)	-1.079	0.264	5.967	14.69	3.204	18.49	-1962.1	156.396	34.12
Q_3^t(85.7)	-0.526	0.524	1.421	22.12	-2.995	23.45	1834.3	37.247	51.39
Q_4^t(85.7)	-0.526	0.864	1.421	22.12	-2.995	23.45	1834.3	37.247	51.38
Q_0^t(159.1)	-0.174	-0.028	1.592	22.45	-2.744	23.34	1680.0	41.713	52.14

Note: Units for air flow rates are m³/hr.

Table 4. Error analysis of case study with single parameter variations

θ	$\Delta\theta$	Q_1^t 89.5	Q_2^t 97.8	Q_3^t 85.7	Q_4^t 85.7	Q_0^t 159.1
T_0	3°C	0.44/ 7.36	-3.17/ 0.25	-1.35/-0.26	-1.35/-0.26	-0.83/-2.12
T_4	3°C	-0.05/-3.78	0.77/ 2.13	1.35/ 2.01	2.22/ 0.59	-0.13/-1.43
h_{30}	0.5	0.44/ 0.84	2.92/ 0.69	0.61/ 2.10	0.61/ 2.11	1.27/ 0.95
Cp_1	10%	6.63/ 1.79	1.29/ 1.95	1.71/ 4.41	1.71/ 4.41	3.22/ 2.20
P_4^w	10%	0.73/ 0.97	-1.52/-0.07	1.25/ 2.05	1.25/ 2.05	2.12/ 1.10
V_h	10%	13.77/-0.61	9.04/-1.86	10.06/-0.80	10.06/-0.80	18.57/-1.00
K_4	10%	0.83/ 4.97	-1.73/ 3.73	1.42/ 6.14	1.42/ 6.14	2.41/ 5.10
n_3	10%	1.49/ 0.63	9.94/ 0.05	2.08/ 4.95	2.08/ 4.95	4.31/ 0.95
n_1	10%	11.12/-2.05	2.17/-1.76	2.86/ 2.37	2.86/ 2.37	5.39/-1.27

Note: Units for air flow rates are m³/hr.

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variation. The second number is the relative discrepancy, $(\Delta Q - \Delta Q^*)/\Delta Q^*$, with positive numbers indicating over predictions. For example, the cell of T_0 and Q'_1 is 0.44/7.36, which indicates that if the outdoor temperature were to rise 3°C, the airflow out of the room 1 would increase by 0.44 (m³/hr). This prediction over estimates by 7.36% compared to the difference of airflow between two actual simulations.

Table 4 shows very good agreement between predictions and simulations. Most of the discrepancies are under 3%, only very few are larger than 5%. The same analysis for other individual parameters shows similar results.

One main advantage of using the sensitivity analysis method is that it requires much less computational efforts than the simulations using airflow models. In the implementation of the case study, each simulation needs approximately 10 to 15 iterations to converge to a relative precision of 10^{-5}, while the calculation using sensitivities

is performed in one run without iterations. For the calculations of Table 4, the method using airflow sensitivities needs less than 5% of the computing time required by the simulation method.

Table 5 shows similar analyses (as in Table 4) for variations of two or more parameters. Most of the relative discrepancies are around 5% and a few (6 out of 100) are over 10%. Only for a few cases (7 out of 100), are there very large discrepancies, but these only happen when the relative changes of airflow $\Delta Q'/Q'$ are very small (less than 1% of the nominal airflow rates).

Equation (19) describes quantitatively the propagation of errors from input parameters to the airflow simulation results. It can be used to obtain the standard deviations (STDs) of airflow for given error ranges of input parameters. In the course of verifying this relation, a *Monte-Carlo method* [5] is employed. In this method, the statistics of airflow are calculated from multiple simulations using combinations of randomly chosen input

Table 5. Error analysis for multiple parameter variations

Varying Parameters	Sensitivity Prediction/Relative Discrepancy				
	Q_1^t	Q_2^t	Q_3^t	Q_4^t	Q_0^t
T_0, T_4	0.39/ 9.71	-2.39/ 0.41	-0.01/-30.78	0.87/ 0.28	-0.96/ -1.89
T_0, h_{30}	0.88/ 11.95	-0.25/-60.27	-0.74/ -9.71	-0.74/ -9.71	0.44/ 67.11
T_0, C_{p1}	7.07/ 4.70	-1.87/ -1.53	0.36/ 64.18	0.36/ 64.18	2.39/ 5.33
T_0, V_h	14.21/ 1.95	5.88/ -3.12	8.70/ 0.51	8.70/ 0.51	17.74/ -1.12
T_0, K_4	1.27/ 6.41	-4.89/ 0.60	0.06/-335.2	0.06/-335.2	1.58/ 6.67
T_0, n_1	11.56/ -3.52	-1.00/ 14.51	1.51/ 3.92	1.51/ 3.92	4.57/ -5.11
C_{p1}, V_h	20.40/ -3.54	10.33/ -2.82	11.76/ -1.04	11.76/ -1.04	21.79/ -2.02
C_{p1}, K_4	7.46/ 2.17	-0.43/ -4.28	3.12/ 3.63	3.12/ 3.65	5.62/ 2.03
C_{p1}, K_3	7.73/ 2.28	8.62/ 1.71	3.24/ 3.74	3.24/ 3.74	6.40/ 2.09
C_{p1}, n_1	17.75/ -8.14	3.46/ -7.62	4.57/ -1.02	4.57/ -1.02	8.61/ -6.86
V_h, K_4	14.61/ -0.95	7.31/ 0.86	11.47/ -1.51	11.47/ -1.51	20.98/ -1.91
V_h, n_1	24.89/-11.88	11.21/ -6.66	12.92/ -4.33	12.92/ -4.33	23.96/ -6.32
K_4, n_4	1.52/ 7.35	-3.16/ 5.17	2.59/ 9.61	2.59/ 9.61	4.40/ 7.73
K_4, n_1	11.95/ -3.42	0.44/ -9.40	4.28/ 0.30	4.28/ 0.30	7.80/ -2.50
T_5, C_{p1}, V_h, K_4	21.67/ -1.16	5.44/ 0.55	11.83/ -0.63	11.83/ -0.62	23.36/ -3.13
T_0, T_1, T_2, T_3, T_4	-0.54/ 0.74	-0.86/ 1.06	-0.64/ 0.94	-0.64/ 0.94	-1.06/ 0.85
$C_{p1}, C_{p2}, C_{p3}, C_{p4}$	6.89/ 2.14	4.52/ 1.48	5.03/ 2.04	5.03/ 2.04	9.29/ 1.91
K_1, K_2, K_3, K_4	7.74/ 1.76	8.34/ 1.96	6.65/ 3.40	6.65/ 3.40	12.40/ 3.29
n_1, n_2, n_3, n_4	11.84/ -4.23	11.96/ -3.27	7.90/ 0.54	7.90/ 0.54	14.90/ -0.12
$T_0, T_4, h_{30}, C_{p1}, V_h, K_4, n_3, n_1$	34.67/-14.92	21.24/-11.73	18.72/ -5.46	19.60/ -6.14	34.21/-10.63

Note: Units for air flow rates are m³/hr.

Table 6. Comparisons between sensitivity and Monte-Carlo approaches

Varying Parameters	STD	STD of Q_0^t		
		SENSI[1] (m³/hr)	MONTE[2] (m³/hr)	RELNS[3] (percent)
$T_0\ T_1\ T_2\ T_3\ T_4$	3°C	1.21	1.18	2.58%
$h_{10}\ h_{20}\ h_{30}\ h_{40}$	0.5m	3.40	3.19	6.9%
$C_{p1}\ C_{p2}\ C_{p3}\ C_{p4}$	10%	4.84	4.88	-0.8%
$K_1\ K_2\ K_3\ K_4$	10%	6.82	6.48	5.2%
$n_1\ n_2\ n_3\ n_4$	10%	8.13	7.80	4.2%
V_h	10%	18.81	19.86	-5.2%
All of Above		22.43	22.08	1.5%

Note:
1. Results based on sensitivity approach, equation (19),
2. Results using the Monte-Carlo method,
3. Relative discrepancy between the above two approaches.

parameters. Results based on both the sensitivity approach (equation 19) and the Monte-Carlo method are shown in Table 6. Taking the first row for example, if a STD of 3°C exists in outdoor and zonal temperatures, the calculation using equation (19) predicts a STD of 1.21 m³/hr in the ventilation rate into the building. The Monte-Carlo method based on 100 airflow simulations, with each zonal temperature independently and randomly chosen according to a normal distribution of 3°C STD, yields a STD of 1.18 m³/hr. The similar comparisons for other selected groups of parameters in Table 6 indicate compatible results from both methods, however the sensitivity approach requires much less computational effort.

Information of airflow sensitivities can also be used to assist the design of building airflow systems. Design goals can be to achieve specified ventilation rates, to obtain specifications for retrofitting of existing buildings, to limit airflow into or out of a zone, etc. For the case study building, we assume the nominal values of input parameters represent a protocol design of a building and the typical outdoor weather conditions during the Winter. The design goal is to reduce the building natural ventilation rate from 159 m³/hr down to 100 m³/hr by airtightening the building.

The variations of the four openings on the building envelope are of interest. The sensitivity information of these four openings should be obtained and used for guiding the selection of new opening sizes. Calculations show that the sensitivity of the airflow rate into the building with respect to the K values of these four openings are:

$$\left\{ \frac{\partial Q_0'}{\partial K_j} \times K_j \right\} = [35.41\ 32.72\ 31.82\ 24.06]. \quad (22)$$

Since values of all these four sensitivities are positive, their K values should be reduced to achieve any reduction in the ventilation rate. Assuming $\beta\%$ variations in the K values will bring about the required reduction, the following equation holds:

$$100 - 159.1\ (\text{m}^3/\text{hr}) = \left\{ \sum_j^4 \frac{\partial Q_0'}{\partial K_j} K_j \right\} \times \beta\%. \quad (23)$$

This yields $\beta = (100 - 159.1)/124.02 * 100 = -47.68$. That is, each of the K values should be reduced by 47.68% to bring the building ventilation rate down to 100 m³/hr. The airflow simulation with the reduced new K values yields an actual ventilation rate of 90.98 m³/hr. This indicates the new K values based on sensitivity information overestimated the ventilation reduction by 15%. Therefore, we correct the β value to $47.68\% * (1-15\%) \approx 40\%$. One more simulation shows that the airflow into the building is reduced to 102.5 m³/hr. By using the sensitivity information in combination with the airflow simulations, we have been able to find the new K values quickly to achieve the design goal.

CONCLUSION

A general procedure of sensitivity analysis for airflow in multi-zone buildings has been presented with a case study. The airflow sensitivities indicate potential changes of airflow due to variations in mutually independent input parameters, at the point defined by the nominal values of input parameters and the airflow simulation results. By utilizing the matrix representation form a multi-zone airflow model, a general form of the sensitivity calculation is derived.

The sensitivity analysis provides useful information for

the error analysis and design assistance, furnishes further insights into the accuracy of existing airflow models, and requires minimum computational efforts. The calculation procedure can be easily integrated into the existing steady-state airflow models, such as the COMIS's model [6]. Further research is needed to use the sensitivity information for more comprehensive procedures in the error analysis and building airflow system design assistance.

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