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The Sensitivity Analysis of Building Thermal Network Elements

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The sensitivity theory was applied with the aim of analyzing the influence of thermal resistance and thermal capacity of construction and the influence of a heat source control factor on unsteady thermal transfer in a building. This paper describes a sensitivity function, sensitivity characteristics and the relationship between sensitivities and tolerances. An example of sensitivity analysis of thermal transfer through the shell of a building is also presented. The method may be used to design building elements for an acceptable indoor environment and low energy consumption.

1. INTRODUCTION

ENERGY saving and efforts for reaching thermal comfort lead to the study of the unsteady process in the thermal interaction of the building with the exterior and also with used heating and ventilation systems. The reader can find the matrix system analysis in [1–3]. The detailed description of the analysis method of layered constructions has been presented in [4], with the information about the heat capacity of the interior in [5]. There are many solutions in the field of modelling heat transfer. The method of analogy, which is the most frequently applied method, is described in [6–8].

In Fig. 1 we present the example of the part of the building construction and corresponding thermal network. The construction is formed by: a building shell; its thermal conductivity is called K, thermal capacity C_k, window thermal conductivity K_o, heat transfer coefficient between internal surface and the interior air is α_i, interior construction element thermal capacity C_i, surrounding walls thermal capacity C_s, heat transfer coefficient between them and interior is α_s, heating system control factor G, thermal capacity of heating system C_t, heat transfer coefficient on the heating radiator α_t. These elements are modelled for simplicity as elements with the lumped parameters. Numbers in the circle specify thermal junctions (temperatures: $T_1 = T_e$ in the exterior, T_2 on the internal surface of the shell, $T_3 = T_i$ in the interior, T_4 on the heating radiator, T_5 on the internal walls).

It is possible to assess the changes of temperature within the region of the steady state by the system of differential equations which can be expressed after Laplace transformation in the matrix form [9]:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix} = \begin{bmatrix} K+K_0 & -K & -K_0 & & \\ -K & K+\alpha_i+sC_k & -\alpha_i & & \\ -K_0 & -\alpha_i & K_0+\alpha_i+\alpha_t+\alpha_s+sC_i & -\alpha_t & -\alpha_s \\ & & G-\alpha_t & \alpha_t+sC_t & \\ & & -\alpha_s & & \alpha_s+sC_s \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} \tag{1}$$

Laplace transformation of the temperature change within the interior (T_3), which is evoked by the change of outside temperature, can be determined in the form:

$$T_3 = \frac{\Delta_{13}}{\Delta_{11}} \cdot T_1 = F(s, x_1, x_2, \ldots, x_n) \cdot T_1 \tag{2}$$

where s is Laplace operator and parameters $x_1, x_2, \ldots,$ represent thermal properties of the construction and the heating system.

2. SENSITIVITY DEFINITION

We are often interested in the changes of thermal transfer in practice when thermal properties of particular elements of the system are variable [10]. The practical question is to what extent the thermal transfer is 'sensitive' to the same change of single parameters. If we perform a linear approximation in the region close to the nominal value of parameter x (Fig. 2), it is possible to express the sensitivity of the system function to parameter x as derivation [11],

$$S_x^F(s, x) = \frac{dF(s, x)}{dx}. \tag{3}$$

So-called relative sensitivity is more often used [11]

$$Sr_x^F(s, x) = \frac{dF(s, x)}{dx} \cdot \frac{x_0}{F_0} \tag{4}$$

where x_0 and F_0 are the nominal values. We can express the function $F(s, x)$ after the calculation of algebraic

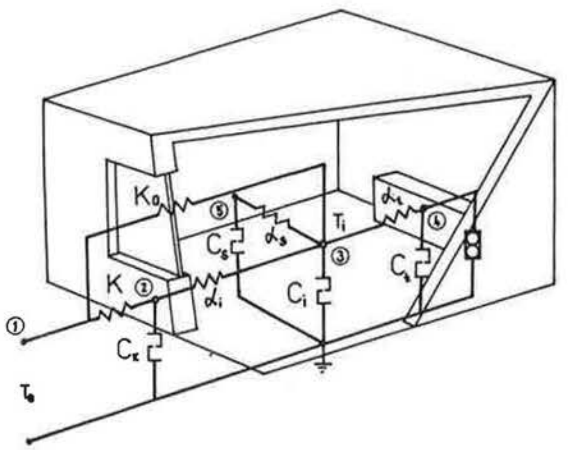

Fig. 1. Part of building construction and thermal network with elements characterizing heat transfer between exterior and interior.

complements in equation (2) as

$$F(s,x)=\frac{N(s,x)}{D(s,x)} \tag{5}$$

where $N(s, x)$ and $D(s,x)$ are polynoms with variable s and variable parameter x.

The relative sensitivity (4) is in fact a sensitivity function because variables s, x occur here. If we substitute nominal values $s = j\omega_0$, $x = x_0$, we will get concrete number values expressing sensitivities.

The sensitivities can be calculated advantageously from the expression:

$$Sr_x^F(s,x_i)=x_i\left(\frac{N'}{N}-\frac{D'}{D}\right) \tag{6}$$

thus avoiding the more difficult derivation of the rational functions.

As it is shown above, we are interested in the analysis of thermal damping: in the amplitude of thermal changes in the construction and in the time delay (phase displacement) of thermal swings in constructions.

By means of Fourier transformation or eventually by fast Fourier transformation we can decompose the time dependence of the thermal change to the system of harmonic constituents, which are characterized by the corresponding amplitude and phase. We do not have to calculate the sensitivities of amplitudes and phases to changing parameters as particularly assessed derivations

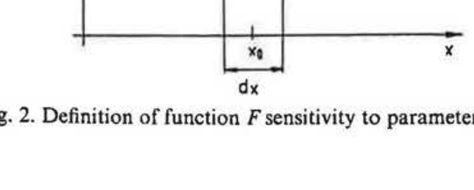

Fig. 2. Definition of function F sensitivity to parameter x.

of module and phase functions derived from the system function $F(j\omega, x)$. We can however respect the fact that the real part of sensitivity function (4) after substituting $s = j\omega$ expresses directly the amplitude sensitivity of temperatures. The imaginary part expresses sensitivity of phase delay to the change of parameter x. We can express it by the formula:

$$Sr_{x_i}^F(j\omega)=Re\,Sr_{x_i}^F+j\,Im\,Sr_{x_i}^F=\frac{\mathrm{d}|F(j\omega,x_i)|}{\mathrm{d}x_i}\cdot\frac{x_i}{|F(j\omega,x_i)|}+j\,\frac{\mathrm{d}\{\arg F(j\omega,x_i)\}}{\mathrm{d}x_i}. \tag{7}$$

The graphs of functions

$$Re\,Sr_x^F(j\omega) \tag{8}$$

$$Im\,Sr_x^F(j\omega) \tag{9}$$

will be called sensitivity characteristics.

SENSITIVITIES AND TOLERANCES

The tolerance of system function $F(x_i)$, which determines thermal transfer, is a differential defined by expression:

$$\Delta F=\sum_{i=1}^{n}S_{x_i}^F\cdot\Delta x_i \tag{10}$$

while the following formula applies for the relative tolerance:

$$\frac{\Delta F}{F}=\sum_{i=1}^{n}Sr_{x_i}^F\cdot\frac{\Delta x_i}{x_i}. \tag{11}$$

We know that the resultant signs of particular tolerances can be different and that influences can partially compensate each other. So-called 'the worst case' can occur when all particular tolerances with the same signs are added, e.g. with positive signs. It can be expressed by the equation:

$$\frac{\Delta F}{F}=\sum_{i=1}^{n}\sqrt{\left(Sr_{x_i}^F\cdot\frac{\Delta x_i}{x_i}\right)^2}. \tag{12}$$

The analysis of the worst case of the tolerances influence has considerable significance in the different applications in building design, e.g. it makes it possible to remove the undesirable consequences of the production scatters of building construction parameters.

4. APPLICATIONS

As an example the sensitivities of all elements were calculated in the model which is shown in Fig. 1. Cal-

Table 1. Kinds of walls

Number	Category	Wall thickness	Material
1	lightweight	0.11	wood thermal insulation Sololit
2	middle heavyweight	0.6	brick
3	heavyweight	1.2	stone

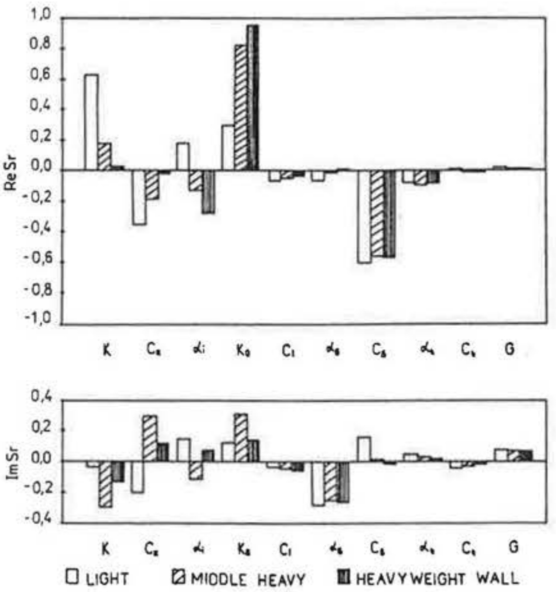

Fig. 3. Sensitivities of three different kinds of walls.

culations were performed for three different categories of shell. Materials and input values for the computer are in Tables 1 and 2.

Resultant values are shown in Fig. 3, where amplitude and phase sensitivities are drawn. In this figure the elements of significant influence on thermal transfer are evident.

The thermal conductivity of windows and thermal capacity of walls which surround the interior are significant for both kinds of sensitivities *Re Sr* and *Im Sr*, and to a lesser degree the capacity of the shell and its thermal conductivity. The other elements have no substantial significance. It is possible to observe the graphic overflowing of sensitivities between construction elements changing their values.

As expected the sensitivity is a variable depending on nominal values. We can see the dependence in the graphs, see Fig. 4(a–j).

The theory of sensitivities makes it possible to obtain the change of the temperature profile in the desirable node by the change of the specific element of the construction. It is not necessary to repeat the analysis; the results can be assessed from the values of amplitude sensitivity *Re Sr* and phase displacement sensitivity *Im Sr*.

The calculation of the temperature in the interior caused by the change of outside temperature as a result of increasing the U-value of the building shell by 10% was performed for verification of this method. The analysis was first carried out for values in Table 2 for the

Table 2. Input values

	C_k	C_i (Wh · K^{-1})	C_t	C_s	K	α_i	K_0 (W · K^{-1})	α_t	α_s	G
1	202									
2	2210	50	100	600	11	84.5	4.2	20	338	30
3	6350									

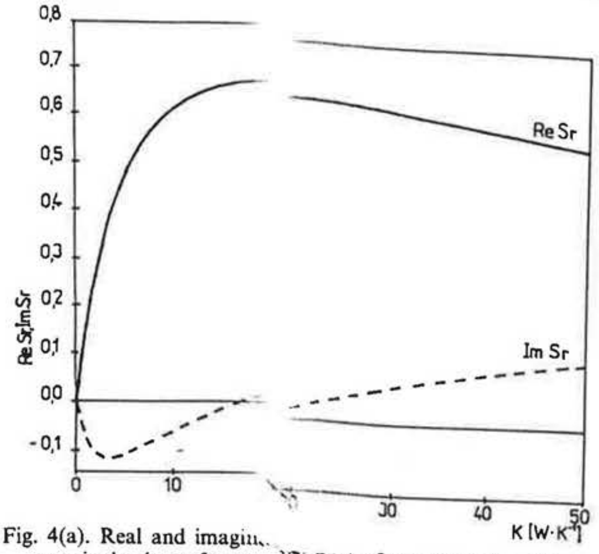

Fig. 4(a). Real and imaginary part of sensitivity for different nominal values of particular elements. Sensitivity to K.

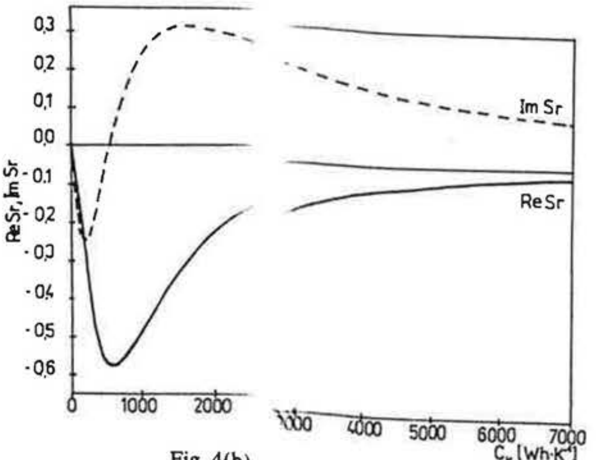

Fig. 4(b). Sensitivity to C_K.

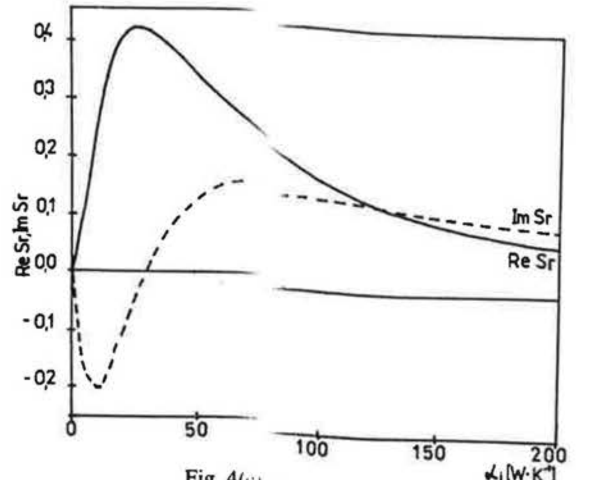

Fig. 4(c). Sensitivity to α_i.

lightweight wall (Fig. 3), then *Re Sr* and *Im Sr* were found. We have developed a program which assesses *Re Sr* and *Im Sr* for the nominal value. The program also draws the sensitivity graphs in dependence on the nominal values of the chosen element. Calculated values are:

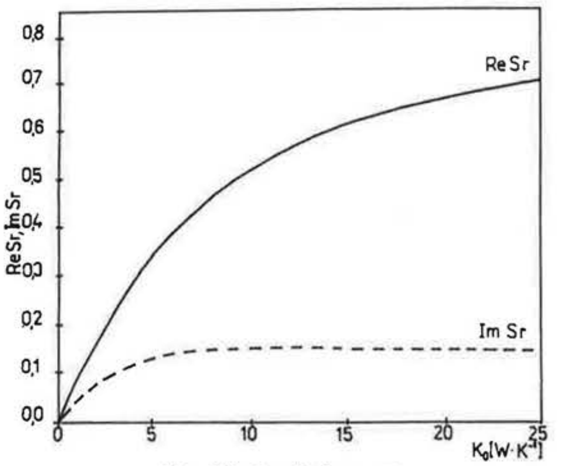

Fig. 4(d). Sensitivity to K_0.

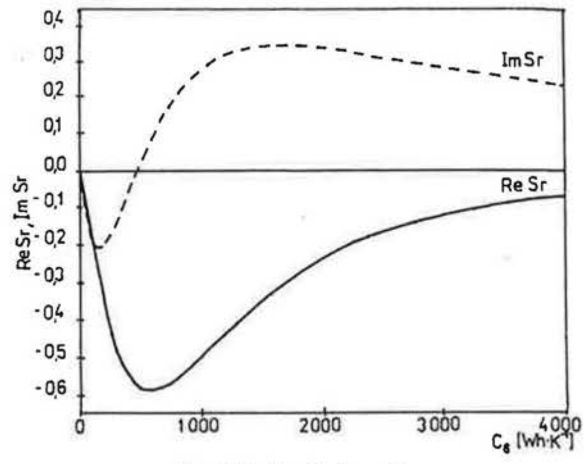

Fig. 4(g). Sensitivity to C_S.

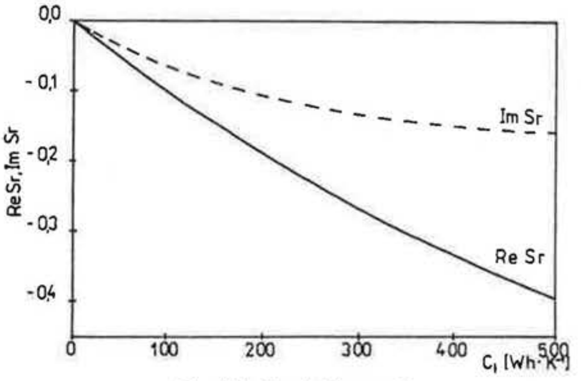

Fig. 4(e). Sensitivity to C_l.

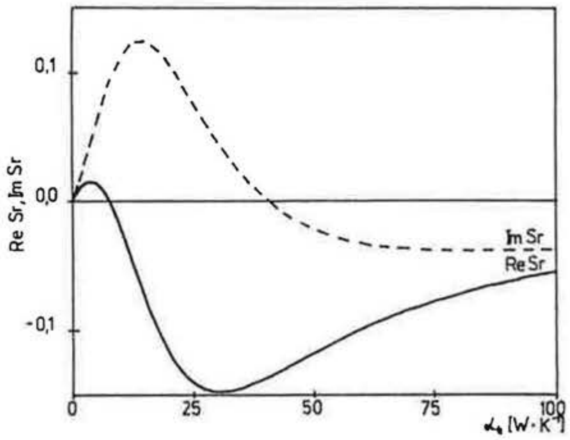

Fig. 4(h). Sensitivity to α_t.

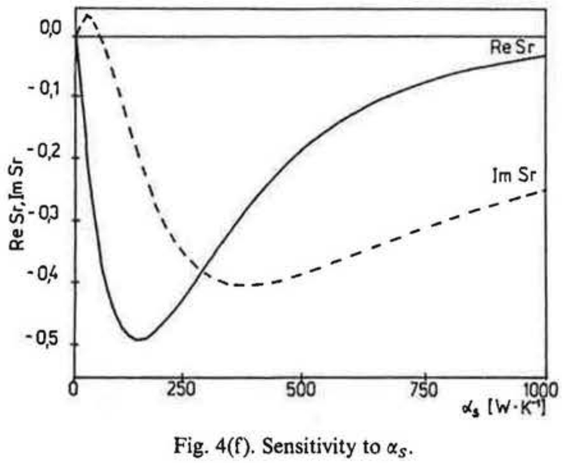

Fig. 4(f). Sensitivity to α_S.

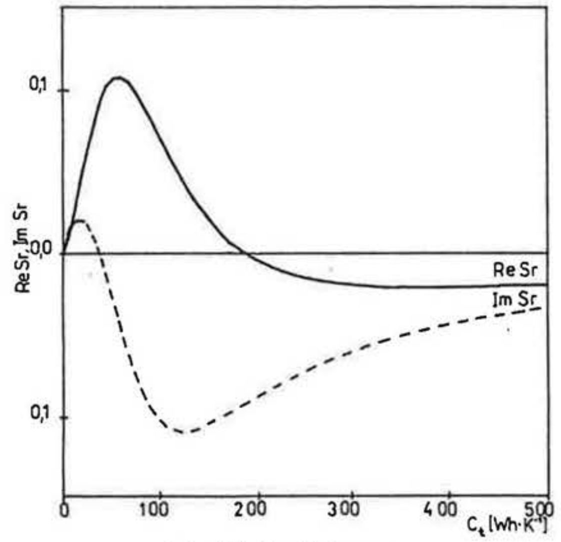

Fig. 4(i). Sensitivity to C_t.

$$Re\,Sr = 0.638 \quad Im\,Sr = -0.040.$$

The new temperature dependence corresponding to the changed U-value was determined by equation (11) from these values. The original temperature curve of indoor air temperature and the newly assessed curve are depicted in Fig. 6. The comparison was made between this function and the values obtained from the analysis at the changed value of wall conductivity.

The newly obtained values of both methods do not differ more than 1%. It means a good argeement of results were reached using the described method of sensitivity analysis and analysis method. Inaccuracy increases with the increasing change of element value because the function F is not linear.

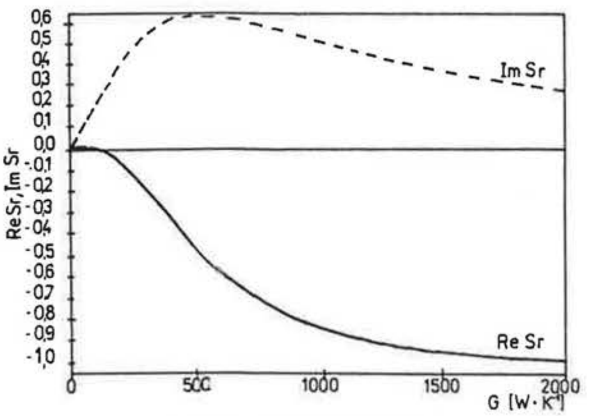

Fig. 4(j). Sensitivity to G.

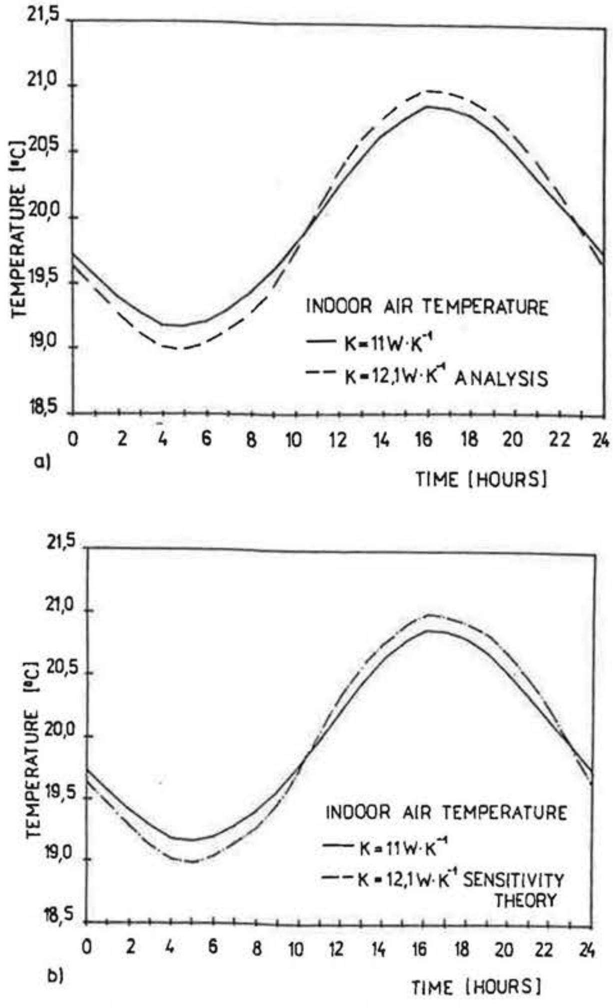

Fig. 6. Indoor air temperature for original conditions ($K = 11$ $W \cdot K^{-1}$) and new evaluated temperature (for $K = 12.1$ $W \cdot K^{-1}$) by means of (a) analysis and (b) sensitivity theory.

5. CONCLUSION

The proposed method of sensitivity is advantageous for finding elements which substantially influence the thermal dynamic behaviour of the construction. If we perform the thermal dynamic analysis we have to use the accurate input values for these elements. The absolute value of tolerances depends on the value of sensitivities and besides it on the change of particular parameter, it means the resultant value of tolerance is influenced by parameters which show large change of their values. The thermal conductivity of walls, the thermal capacity of walls and the interior can be involved in these parameters. For example, increasing or decreasing of the moisture in the building shell causes the change of the thermal conductivity and the thermal capacity of walls in the range approximately $\pm 15\%$. Variable number of persons in the interior and different furnishings raise the change of the thermal capacity of the interior even by 100%. Although in our case the sensitivity on this parameter is low with larger nominal value the sensitivity increases very fast.

The mentioned method may even be used instead of synthesis. This means the calculation of searched values of elements for desirable amplitude and phase displacement. However, it is necessary to assume that the theory of linear sensitivities gives good results only for small changes of element values in the model. For larger changes we have to use gradual calculation of sensitivities in dependence on the changed nominal value, or use the theory of non-linear sensitivities that will have considerable significance for building construction.

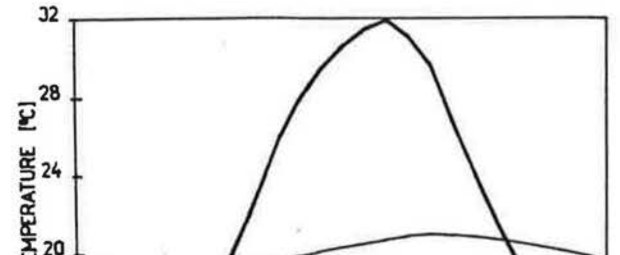

Fig. 5. Outdoor and predicted indoor air temperature.

The mutual ratio of sensitivities between particular elements is changed according to the model varieties. The distribution of sensitivities is different for the transfer function which is developed for the building model, characterizing the change of temperature in the interior according to the change of thermal flow from the heating system (constant outside temperature) and the most outstanding elements are the elements which describe the heating system.

It is necessary to mention the property of a sensitivity invariance theorem representing the fact that the sum of sensitivities is constant. We have exemplified the validity of this theorem. The validity of the invariance theorem causes an interesting phenomenon: if one sensitivity increases at the change of the component value, the others decrease.

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