#6516

NUMERICAL PREDICTION OF COUNTERGRADIENT THERMAL TRANSPORT IN A TURBULENT PLUME

D.J. Bergstrom, Ph.D., P.Eng. **G.D. Stubley, Ph.D., P.Eng.** **A.B. Strong, Ph.D., P.Eng.**

ABSTRACT

This paper examines two turbulence models that could be used for numerical simulations of buoyant airflows in rooms. Specifically, it reports a prediction for the mean temperature field and turbulent heat flux in a plane vertical plume, using both a k-ε eddy viscosity model and an algebraic stress model closure. The numerical solutions were based on the full two-dimensional form of the transport equations, which were solved elliptically. Results showed that the k-ε model was incapable of accurately reproducing the streamwise turbulent heat flux $<u_1\theta>$ in the plume, which is known to be countergradient in the outer region of the flow. In contrast, the algebraic stress model yielded much more realistic predictions for $<u_1\theta>$, although the peak value was approximately 50% higher than the experimental data. One important consequence of the failure of the eddy viscosity model with respect to modeling $<u_1\theta>$ was the inability to reproduce the enhanced level of turbulent transport in a plume due to buoyancy. This study is further evidence of the need to look to second-moment closures for realistic predictions of turbulent airflows when the effect of buoyancy on the turbulent transport is important.

INTRODUCTION

Computational modeling of turbulent flows is becoming increasingly useful for engineering applications. Often, an engineer is specifically interested in the transport of a scalar contaminant. One example that pertains to the design of building systems is the scalar transport associated with airflows in rooms (Murakami et al. 1987, 1991). For air-conditioned rooms, the scalar property of interest is the temperature. Differences in temperature can lead to buoyancy effects, which, in turn, influence the turbulent transport.

In order to predict the scalar transport using time-averaged conservation equations, a turbulence model is required for the turbulent fluxes. Probably the most consistent and realistic turbulence models to date are those based on the transport equations for the second moments, specifically the Reynolds stress $<u_iu_j>$ and scalar flux $<u_i\theta>$. Because these models attempt to represent the "physics" contained in the transport balance, they are potentially capable of including the influence of extra effects, such as buoyancy, on the turbulence field. Simplest among these turbulence models are the so-called *algebraic stress models* (ASM) (Gibson and Launder 1976).

Although they don't have as sound a physical basis, eddy viscosity model (EVM) relations continue to be widely used in engineering applications. Probably the most popular of these is the *k-ε* model. The popularity of the *k-ε* model can be attributed to a number of factors. It is computationally efficient, since it requires the solution of only two additional transport equations. Furthermore, since the EVM hypothesis is structurally similar to the molecular diffusion encountered in laminar flow, the *k-ε* model is relatively easy to implement in standard (i.e., laminar) numerical codes. Finally, the application of the *k-ε* model to wall-bounded flows using either wall functions or low-Reynolds-number formulations is relatively well established in comparison to second-moment closures. All of these factors suggest that the *k-ε* model is an appropriate closure for numerical studies of airflows in rooms. However, this may not be the case; as an EVM closure, the *k-ε* model has significant limitations with respect to the prediction of buoyant airflows.

By way of example, this paper considers a plane buoyant vertical jet, which in the limit of strong buoyancy becomes a plume. A *k-ε* model closure yields an acceptable prediction for the mean velocity and temperature fields in a plane jet. However, when buoyancy is dominant, so that the flow behaves like a plume, the *k-ε* model closure fails to yield a realistic prediction. The explanation lies in the fact that an EVM relation simply cannot accurately model the streamwise turbulent heat flux, $<u_1\theta>$, in a plume. For this flow, the turbulent heat flux is actually "countergradient" to the mean temperature field in the region of the flow away from the centerline. Failure to accurately model the heat flux $<u_1\theta>$, in turn, renders the turbulence model incapable of reproducing the effect of buoyancy, which is to enhance the turbulent transport. In contrast, an ASM closure can be shown to yield a reasonable prediction for

Donald J. Bergstrom is an assistant professor in the Department of Mechanical Engineering, University of Saskatchewan, Saskatoon, Canada. **Gordon D. Stubley** is an associate professor and **Alan B. Strong** is a professor in the Department of Mechanical Engineering, University of Waterloo, Ontario, Canada.

the vertical plume. In particular, an ASM relation is capable of modeling the countergradient turbulent heat flux and the enhanced levels of turbulent transport due to buoyancy.

Recognition of the failure of an EVM relation in regard to a plume is not new. Malin and Spalding (1984) noted the countergradient transport in the streamwise direction for the temperature field in the plume and proposed a novel algebraic heat flux model to eliminate the deficiency of an EVM relation. Their model replaced the streamwise temperature gradient with the temperature variance $<\theta^2>$ divided by a turbulence length scale. Sini and Dekeyser (1987), in using a k-ε model to predict the vertical buoyant jet, resorted to an empirical function of the densimetric Froude number in their EVM relation in order to obtain the correct behavior for a plume. Even with this modification, they noted that an EVM relation failed to reproduce the correct level of $<u_1\theta>$ in the plume. The deficiencies encountered by EVM predictions of a plane plume as described above were also encountered by Durao et al. (1989) for the axisymmetric case. As well as the standard k-ε model, they also considered improved eddy diffusivity relations. Their general assessment was that all the models tested significantly underpredicted the axial turbulent transport.

It is significant that the k-ε model using EVM relations for the turbulent heat flux should break down in as basic a flow as the vertical plume. One important implication for numerical studies of airflows in rooms is that although the k-ε model is computationally attractive in the case of buoyant flows, it has significant limitations. On the other hand, an ASM closure, which retains much of the computational advantage of the k-ε model, is much more successful at including the effect of buoyancy on the turbulent transport.

Since a prediction for a plane vertical plume using an ASM has been reported elsewhere (Bergstrom et al. 1990), this paper focuses specifically on the role of the streamwise turbulent heat flux, especially with respect to its contribution to the buoyancy production of turbulence kinetic energy. Both EVM and ASM closures are considered. The next section describes the mathematical model equations. The numerical solution of these equations is described next and then the results are compared to the experimental data of Ramaprian and Chandrasekhara (1989).

THE MATHEMATICAL MODEL

The mathematical model is based on the mean transport equations, the turbulence model relations for the Reynolds stress and turbulent heat flux, and the transport equations for the turbulent scale parameters k, ε, and $<\theta^2>$. A complete description of the equations and the associated turbulence models can be found in Bergstrom (1987).

For incompressible flow of a Newtonian fluid, the mean transport equations representing conservation of mass, momentum, and thermal energy can be written as follows using cartesian tensor notation:

$$\frac{\partial}{\partial x_i}(\rho U_i) = 0, \tag{1}$$

$$\begin{aligned} \rho\, \partial U_i/\partial t + \rho\, U_j \frac{\partial U_i}{\partial x_j} &= -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}(\mu \frac{\partial U_i}{\partial x_j}) \\ &+ \rho\, g_i - \rho \frac{\partial}{\partial x_j}\langle u_i u_j\rangle , \end{aligned} \tag{2}$$

$$\partial \Theta/\partial t + U_j \frac{\partial \Theta}{\partial x_j} = \frac{\partial}{\partial x_j}(\Gamma \frac{\partial \Theta}{\partial x_j}) - \frac{\partial}{\partial x_j}\langle u_j \theta\rangle , \tag{3}$$

where U_i and u_i represent the mean and fluctuating velocity components, respectively, and Θ and θ denote the mean and fluctuating values of the temperature, respectively. The notation $< >$ is used to indicate the time-averaged value. With respect to fluid properties, μ is the dynamic viscosity and Γ is the molecular diffusivity. The variation of the density, ρ, with temperature is approximated by

$$\rho = \rho_o(1 - \beta\Delta\Theta) \tag{4}$$

where $\Delta\Theta = \Theta - \Theta_o$ is the local deviation of the temperature field from the reference value Θ_o, and β is the coefficient of thermal expansion. Finally, g_i is the gravitational force vector.

The turbulent transport in the mean equations is associated with the gradients of the second moments, i.e., the Reynolds stress $<u_iu_j>$ and the turbulent heat flux $<u_i\theta>$. In order to close these equations, turbulence model relations are required for $<u_iu_j>$ and $<u_i\theta>$. Two different models were considered: EVM relations using a k-ε model closure and ASM relations requiring solution of transport equations for k, ε, and $<\theta^2>$. The two different model relations for the Reynolds stress and turbulent heat flux are presented in Tables 1 and 2, respectively. With regard to the k-ε model, different values have been proposed for the turbulence model coefficients c_μ and σ_t in buoyant flows. However, as will be shown in the results, modifying the value of c_μ and σ_t does not change the essential nature of an EVM closure. The values adopted in the present study, i.e., $c_\mu = 0.09$ and $\sigma_t = 0.7$, represent typical values for turbulent free shear flows.

The ASM closure essentially follows that of Gibson and Launder (1971). The model relations used for the pressure scrambling terms are those originally put forward by Launder (1975). Improved models for these terms in the second-moment transport equations are being actively pursued by Launder's group (Launder 1988) and elsewhere. The algebraic relations represent approximations to the transport equations for the second moments. In the present analysis, the net transport of $<u_iu_j>$ and $<u_i\theta>$ has been

TABLE 1
Turbulence Model Relations for the Reynolds Stress

Flux	$k-\epsilon$	ASM
$\langle u_i u_j \rangle$	$-\nu_t(\partial U_i/\partial x_j + \partial U_j/\partial x_i)$ $+2/3\,\delta_{ij}k$ $\nu_t = c_\mu k^2/\epsilon$	$\phi(k/\epsilon)[(1-c_2)(P_{ij} - 2/3\,\delta_{ij}P_K)$ $+(1-c_3)(G_{ij} - 2/3\,\delta_{ij}G_K)] + 2/3\,\delta_{ij}k$ $P_{ij} = -\langle u_i u_k \rangle \partial U_j/\partial x_k - \langle u_j u_k \rangle \partial U_i/\partial x_k$ $G_{ij} = -\beta\langle u_i\theta\rangle g_j - \beta\langle u_j\theta\rangle g_i$ $P_K = -\langle u_i u_j\rangle \partial U_i/\partial x_j \quad G_K = -\beta\langle u_i\theta\rangle g_i$ $\phi = [(P_K + G_K)/\epsilon - 1 + c_1 R]^{-1}$
	$c_\mu = 0.09$	$c_1 = 2.2 \quad c_2 = c_3 = 0.55$

TABLE 2
Turbulence Model Relations for the Turbulent Heat Flux

Flux	$k-\epsilon$	ASM
$\langle u_i\theta \rangle$	$-\gamma_t \partial\Theta/\partial x_i$ $\gamma_t = \nu_t/\sigma_t$	$\phi_\theta(k/\epsilon)[P_{i\theta} + (1-c_{3\theta})G_{i\theta} - c_{2\theta}\langle u_k\theta\rangle \partial U_i/\partial x_k]$ $P_{i\theta} = -\langle u_i u_k\rangle \partial\Theta/\partial x_k - \langle u_k\theta\rangle \partial U_i/\partial x_k$ $G_{i\theta} = -\beta\langle\theta^2\rangle g_i$ $P_\theta = -2\langle u_i\theta\rangle\, \partial\Theta/\partial x_i$ $\phi_\theta = [(1/2R)(P_\theta/\epsilon_\theta - 1) + (1/2)(P_K/\epsilon + G_K/\epsilon - 1) + c_{1\theta}]^{-1}$
	$\sigma_t = 0.7$	$c_{1\theta} = 3.0 \quad c_{2\theta} = c_{3\theta} = 0.5$

TABLE 3
Transport Equations for the Turbulence Scale Parameters

Convection	Diffusion	Production	Dissipation
$Dk/Dt =$	$\partial/\partial x_j[(\nu_t/\sigma_K)\partial k/\partial x_j]$	$-\langle u_i u_j\rangle \partial U_i/\partial x_j - \beta\langle u_i\theta\rangle g_i$	$-\epsilon$
$D\epsilon/Dt =$	$\partial/\partial x_j[(\nu_t/\sigma_\epsilon)\partial\epsilon/\partial x_j]$	$+c_{1\epsilon}(\epsilon/k)(P_K + G_K)$	$-c_{2\epsilon}\epsilon(\epsilon/k)$
$D\langle\theta^2\rangle/Dt =$	$\partial/\partial x_j[(\nu_t/\sigma_\theta)\partial\langle\theta^2\rangle/\partial x_j]$	$-2\langle u_j\theta\rangle\partial\Theta/\partial x_j$	$-\epsilon_\theta$
$R = (\langle\theta^2\rangle/\epsilon_\theta)/(k/\epsilon) = 0.5$ $\sigma_K = 1.0 \;\; \sigma_\epsilon = 1.3 \;\; \sigma_\theta = 1.4 \;\; c_{\epsilon 1} = 1.44 \;\; c_{\epsilon 2} = 1.92$			

modeled in terms of the corresponding transport of k and $<\theta^2>$. This represents a potentially more general relation than one that neglects convection and diffusion altogether.

The most important distinction between the two models pertains to the dependence of the second-order fluxes on the mean field gradients $\partial\Theta/\partial x_j$ and $\partial U_i/\partial x_j$. An EVM relation implies a dependence on a single mean field gradient; an ASM relation introduces a much more complex dependence on the mean flow field. In the present study, attention will be focused on the mean temperature field and the performance of the turbulence model relation for the heat flux $<u_i\theta>$. If we consider the streamwise turbulent heat flux $<u_1\theta>$, an EVM relation implies that $<u_1\theta>$ is solely dependent on the mean streamwise temperature gradient $\partial\Theta/\partial x_1$. In contrast, the ASM relation includes a dependence on both temperature and velocity gradients, specifically, $\partial\Theta/\partial x_1$, $\partial\Theta/\partial x_2$, $\partial U_1/\partial x_1$, and $\partial U_1/\partial x_2$. The algebraic relation also includes a dependence on the other flux component $<u_2\theta>$, as well as the Reynolds stress components $<u_1u_1>$ and $<u_1u_2>$. As such, the algebraic relations for the turbulent heat flux and Reynolds stress are strongly coupled.

Evaluation of the turbulence model relations for the second moments also required solution of transport equations for the turbulence-scale parameters. The EVM relation required solution of transport equations for the turbulence kinetic energy, k, and its dissipation rate, ε. For an ASM closure, a transport equation for the temperature fluctuation $<\theta^2>$ was also solved. The thermal dissipation rate, $\varepsilon_\theta = 2\Gamma <\partial\theta/\partial x_j\, \partial\theta/\partial x_j>$, was obtained from assumption of a constant turbulence time scale ratio, R. The transport equations for the turbulence-scale parameters are presented in Table 3, together with the relation for R. For the free shear flow being considered, the high-Reynolds-number form of the turbulent transport equations was adopted.

THE NUMERICAL SOLUTION

The numerical solution method followed the finite-volume formulation of Raithby et al. (1986). A staggered grid arrangement was adopted. The upwind weighted approximations of Raithby et al. were used to discretize the differential transport equations. A pseudo-transient formulation based on a finite time step was used to introduce relaxation into the numerical solution. The discrete equation set was solved iteratively, using the SIMPLEC algorithm to solve for the velocity-pressure field. The solution was considered to be converged when the normalized mass residual was less than 10^{-5} per control volume. Additional details of the solution method are given in the thesis of Bergstrom (1987).

In contrast to most previous numerical predictions for a buoyant jet, the present study retained the full two-dimensional form of the transport equations and solved them elliptically. Thin shear layer approximations were not introduced. The elliptic solution of turbulent shear flows, especially buoyant shear flows, often leads to numerical problems associated with the violation of realizability constraints during intermediate steps of the solution process. Computational checks were introduced into the algorithm to ensure that "temporary" violations of realizability did not prematurely terminate the solution process. At the same time, care was taken to ensure that the checks did not influence the final solution fields, once steady state had been achieved.

The solution domain consisted of the half-plane of a vertical plane plume, as shown in Figure 1. The boundary conditions are also summarized in Figure 1. The plume was considered to be discharged from a slot of width D located in a smooth insulated wall. The centerline represented an axis of symmetry. At the outer edge, the mean velocity gradient was set equal to zero, while the temperature was set equal to the ambient value. A zero gradient outflow boundary condition was implemented at the downstream edge of the solution domain.

In order to ensure grid independence of the similarity profiles, solutions were obtained on grids of different sizes and mesh densities. Specifically, a solution was first obtained on a grid using 45 × 35 control volumes and extending 80 and 31 slot widths in the streamwise and cross-stream directions, respectively. A second solution was

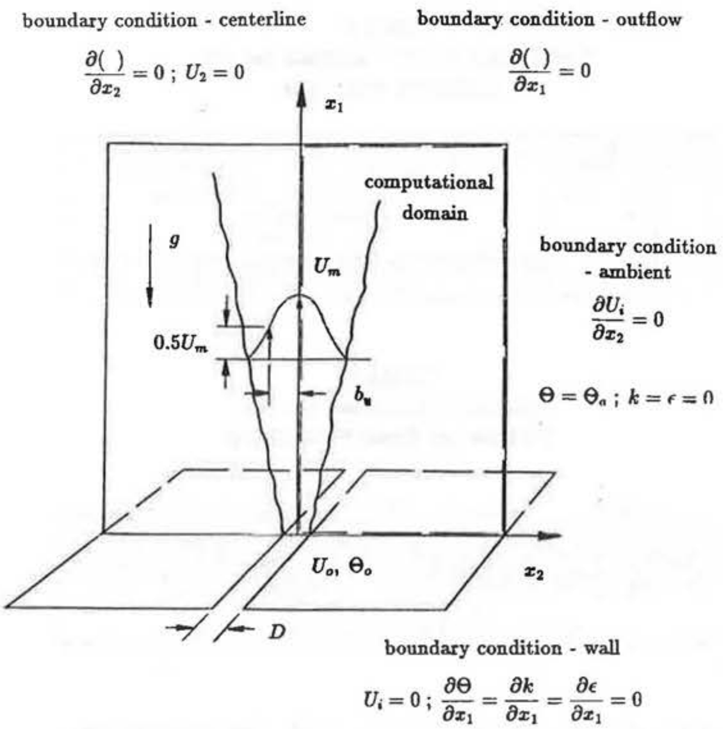

Figure 1 *A plane vertical turbulent plume (Bergstrom et al. 1990).*

then obtained on a refined grid, using 85 × 55 control volumes. Comparison of the similarity profiles obtained on the two grids indicated a maximum difference of 2%. The solution presented below pertains to the refined grid and is considered to be, for practical purposes, grid independent.

RESULTS

The computational model described above was used to predict a plane vertical plume. The relative influence of buoyancy compared to inertial forces is given by the densimetric Froude number, $Fr = U^2/g\beta\Theta D$. In this case, a discharge Froude number of $Fr_o = 8$ ensured that buoyancy forces were dominant. The flow behaved as a plume over most of the solution domain. Typical values of the discharge conditions were as follows: $U_o = 0.05$ m/s, $\Theta_o = 20$°C, and $D = 0.005$ m.

Chen and Rodi (1980) used similarity analysis to determine the mean field behavior of a plume in the fully developed region of the flow. The resultant similarity relations describe the evolution of the mean velocity, U_m, and temperature, Θ_m, along the centerline, i.e.,

$$U_m / U_o = B_u \, Fr^{-1/3}, \tag{5}$$

$$\Theta_m / \Theta_o = B_\theta \, Fr^{1/3} (x_1 / D)^{-1}, \tag{6}$$

and the corresponding lateral growth or spread rates, i.e.,

$$S_u = db_u / dx_1, \tag{7}$$

$$S_\theta = db_\theta / dx_1, \tag{8}$$

where b_u and b_θ are, respectively, the half-widths of the velocity and temperature field. In the equations above, x_1 is the streamwise flow direction.

The predicted values of the similarity parameters for the plume are presented in Table 4 for both EVM and ASM closures, together with the values recommended by Chen and Rodi (1980) and the more recent measurements of Ramaprian and Chandrasekhara (1989) (hereafter denoted RC). In regard to the mean temperature field, the EVM closure predicted a thermal spread rate that is approximately 40% lower than the experimental value. In comparison, the ASM closure did much better, although the level of the thermal spread rate is still about 13% less than the experimental value. Predictions based on the solution of the full differential equations for $<u_i u_j>$ and $<u_i\theta>$, e.g., Malin and Younis (1990), have also obtained a relatively low level for the thermal spread rate in the plume when the standard model constants are adopted.

Bergstrom et al. (1990) present a comprehensive discussion of the ASM prediction for the velocity and temperature fields in the plume. In the present paper, attention will be focused on the prediction for the turbulent heat flux and its role in reproducing the effect of buoyancy on the overall level of turbulent transport.

The predicted similarity profiles for the transverse turbulent heat flux $<u_2\theta>$ using EVM and ASM closures were compared to the experimental data of RC in Figure 2. Although the shape of the $<u_2\theta>$ profile for both models agrees well with the experimental data, the peak value of the EVM prediction is approximately 20% too low, while the ASM prediction is approximately 30% too high. The experimental uncertainty quoted by RC for the turbulent heat flux was 10%. However, their experimental profile for $<u_2\theta>$ was as much as 20% lower than the profile implied by an energy balance based on the mean temperature and velocity fields.

The predicted profiles for the streamwise turbulent heat flux $<u_1\theta>$ using both EVM and ASM closures are compared to the experimental data of RC in Figure 3. The EVM closure failed to predict both the level and direction of $<u_1\theta>$. In contrast, an ASM closure did much better, although the peak value is now approximately 50% too high.

TABLE 4
Prediction for the Similarity Parameters in a Vertical Plume

Study	B_u	B_θ	S_u	S_θ
$k-\epsilon$	2.3	3.6	0.080	0.082
ASM	2.0	2.8	0.097	0.113
data RC [8]	2.13	2.56	0.11	0.133
data CR [13]	1.9	2.4	0.12	0.13

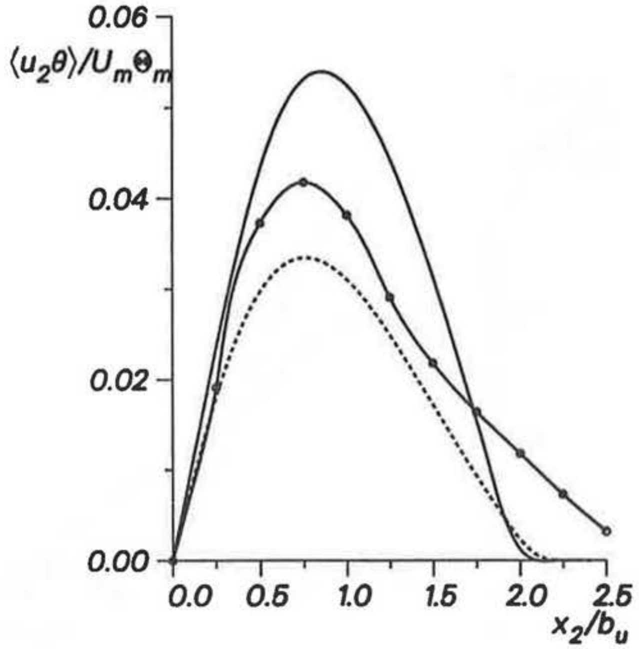

Figure 2 *Similarity profile for transverse turbulent heat flux in plume (Bergstrom et al. 1990).* ---- k-ε *model;* ——— *ASM;* —⊖— *Ramaprian and Chandrasekhara (1989) data.*

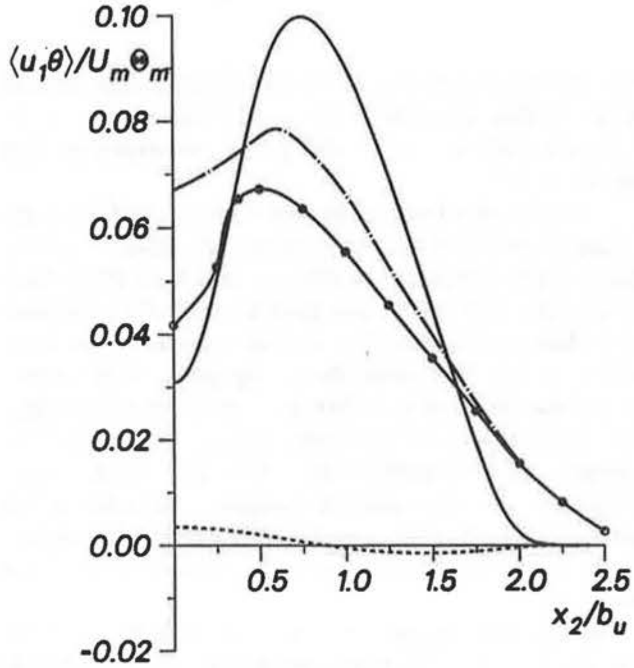

Figure 3 *Similarity profile for streamwise turbulent heat flux in plume (Bergstrom et al. 1990).* ----, k-ε *model;* ——— *ASM;* ——·—— k-W *model (Malin and Spalding 1984);* —⊖— *Ramaprian and Chandrasekhara (1989) data.*

The EVM failed to predict the streamwise turbulent transport because it depends entirely on the mean streamwise temperature gradient $\partial\Theta/\partial x_1$. Experimental studies have shown that, whereas $<u_1\theta>$ remains positive throughout the plume, $\partial\Theta/\partial x_1$ changes sign, indicating that the turbulent transport in the outer region of the jet is countergradient. Figure 4 plots the streamwise temperature gradient for the plume based on the similarity relations for the mean temperature field. From Equations 5 and 6, and assuming a Gaussian profile for Θ, i.e.,

$$\Theta/\Theta_m = \exp(-a\eta^2), \tag{9}$$

where $a = -\ln(0.5)$ and $\eta = x_2/b_\theta$, then the expression for $\partial\Theta/\partial x_1$ becomes

$$\frac{\partial\Theta}{\partial x_1} = S_\theta(\Theta_m/b_\theta)\exp(-a\eta^2)(2a\eta^2 - 1). \tag{10}$$

Using the recommended value of 0.13 for S_θ (Chen and Rodi 1980), the curve shown in Figure 4 was obtained. From the analysis, the positive value of the streamwise gradient of the mean temperature in the outer region of the flow is due to the dominance of spreading over the axial decay. Also shown in Figure 4 is the streamwise mean temperature gradient based on an ASM closure, which compares favorably with the similarity relation. An ASM closure is capable of modeling the countergradient heat flux because the ASM relation includes a dependence on the production terms $-<u_1u_2>\ \partial\Theta/\partial x_2$ and $-u_2\theta>\ \partial U_1/\partial x_2$, both of which are significant in a shear flow characterized by strong mean field gradients in the transverse flow direction.

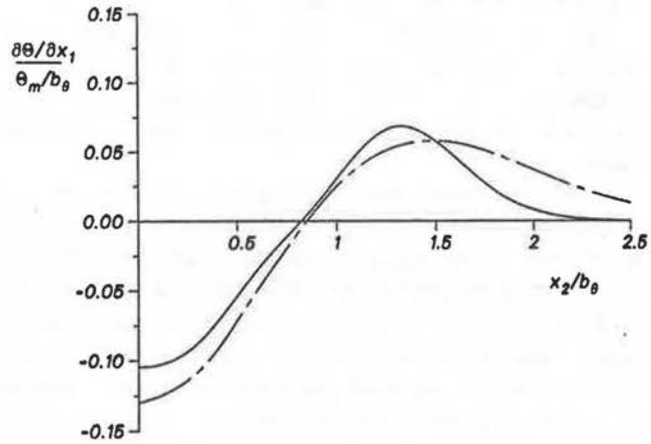

Figure 4 *Mean streamwise temperature gradient in plume (Bergstrom et al. 1990).* ——·—— *similarity relation, Equation 10;*——— ASM.

In their numerical study of buoyant jets, Malin and Spalding (1984) also recognized that an eddy viscosity model would be inappropriate for $<u_1\theta>$ because the profile for the heat flux would then necessarily change sign across the plume. Instead, they proposed the following relation:

$$\langle u_1\theta\rangle = C_H(k\langle\theta^2\rangle)^{1/2} \tag{11}$$

where $C_H = 0.55$ is an empirical constant chosen to optimize the prediction for a pure plume in a uniform environment. Their prediction for $<u_1\theta>$ in a plane plume is also given in Figure 3; it compares favorably with the experimental profile, except for the relatively high value at the centerline. (Of course, the good comparison is to be expected, since the value of C_H was optimized for this specific flow.)

Consider now the consequence for the mean field of the failure of an EVM relation to accurately model $<u_1\theta>$. Results of previous studies (Hossain and Rodi 1982) have shown that a k-ε model does yield a reasonable prediction for a plane vertical heated jet, at least in terms of the mean field behavior. The prediction for the streamwise component of the turbulent heat flux in a heated jet is incorrect, but this result is not of any consequence for the mean flow. However, for a vertical plume, a flow in which the effect of buoyancy on the turbulent transport is significant, the failure to correctly model the streamwise turbulent heat flux has an important consequence for the overall flow prediction.

In the transport equation for k, the buoyancy production, $G_K = -\beta <u_1\theta>$, represents the additional generation of turbulence kinetic energy due to the density fluctuations associated with the streamwise heat flux $<u_1\theta>$. The k-ε model prediction for the buoyancy production across the width of the plume was compared to the experimental measurements of RC (Ramaprian and Chandrasekhara 1989) in Figure 5. Also shown is an ASM prediction. It is evident that a k-ε model does not realistically model the buoyancy production. On the other hand, an ASM relation that successfully predicts the countergradient thermal transport also reproduces the approximate level of the buoyancy production term. The peak level predicted by the ASM closure is approximately 50% higher than the experimental value.

The numerical and experimental profiles for the turbulence kinetic energy in the plume are shown in Figure 6. Because direct measurements were not available, the experimental value of k was estimated from the relation $k = 3/4\,(<u^1u^1> + <u^2u^2>)$, using the normal Reynolds stress components measured by RC. The k-ε model prediction was too low; an ASM prediction was typically within 10% of the experimental value. One of the principal reasons for the deficiency in the k-ε model prediction is the failure to include the influence of buoyancy on the turbulent transport.

As noted above, Sini and Dekeyser (1987) used a modified k-ε model to predict a vertical buoyant plane jet. A realistic prediction for the spread rates and decay constants for the plume was only obtained by introducing an empirical modification to the value of the eddy viscosity ν_t in the EVM relations adopted. Their prediction for k is also shown in Figure 6. It is substantially low and very close to the profile predicted by a k-ε model. This can be attributed to the fact that an unrealistic value of $<u_1\theta>$ precluded a reasonable prediction for G_K and hence k. The important point to be noted is that the failure of a turbulence model can be "repaired" in terms of the mean field behavior without ensuring a realistic prediction for the turbulence field. Such a "fix" cannot be regarded as a valid and generally useful predictive model.

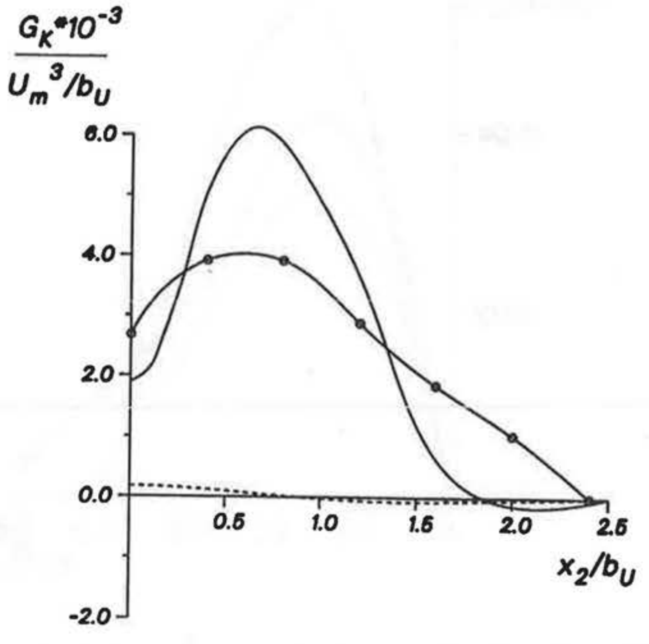

Figure 5 *Buoyancy production of turbulence kinetic energy in plume (Bergstrom et al. 1990).* ---, k-ε *model;* ——— *ASM;* —⊖— *Ramaprian and Chandrasekhara (1989) data.*

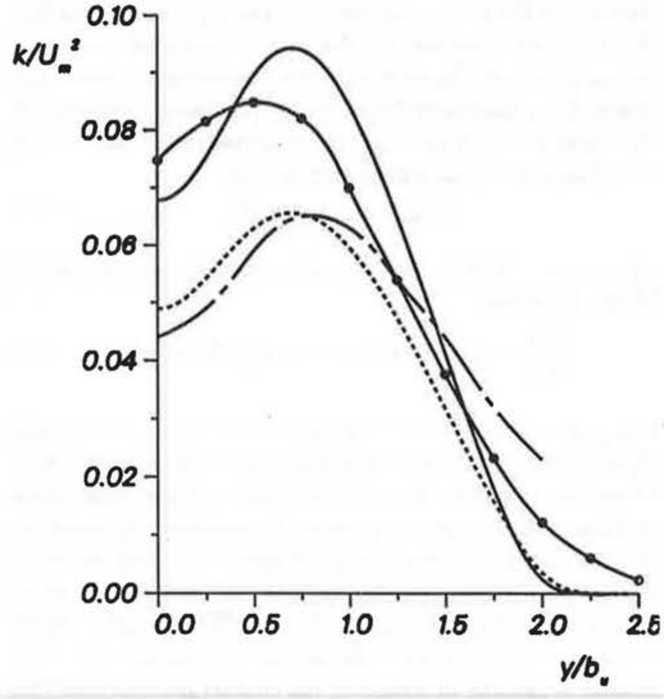

Figure 6 *Similarity profile for turbulence kinetic energy in plume (Bergstrom et al. 1990).* ---, k-ε *model;* ——— *ASM;* ——·—— *modified* k-ε *model (Sini and Dekeyser 1987);* —⊖— *Ramaprian and Chandrasekhara (1989) data.*

CONCLUSIONS

As an example of a buoyant airflow, this study considered a numerical simulation of a plane vertical plume, which is characterized by strong destabilizing buoyancy. The full two-dimensional form of the transport equations was retained, and the resultant equation set solved elliptically. The predictions for the mean temperature field and turbulent heat fluxes were compared to the experimental data of Ramaprian and Chandrasekhara (1989).

Both eddy viscosity and algebraic stress models were used to close the mean transport equations. The EVM closure based on the k-ε model failed to provide an accurate prediction for the plume. Specifically, an EVM relation was incapable of predicting the streamwise turbulent heat flux, which is countergradient in the outer region of the flow. For a vertical plume, Ramaprian and Chandrasekhara (1989) have shown this term to represent a significant fraction of the total streamwise transport of thermal energy.

As a direct consequence of this deficiency in the EVM relation for $\langle u_1\theta \rangle$, the k-ε closure also failed to reproduce the correct level of turbulence kinetic energy, k. In particular, the predicted value of the buoyancy production term in the transport equation for k was negligible, which is physically incorrect. This low level of turbulence kinetic energy partly accounts for the low level of the predictions for the momentum and thermal spread rates using an EVM closure. A clear implication is that numerical models intended for buoyant airflows in rooms may encounter specific flow configurations where the standard k-ε model is inadequate to predict the overall scalar transport.

The ASM model was shown to yield much better results. The nonequilibrium algebraic stress model introduced a complex dependence of $\langle u_1\theta \rangle$ on both the mean velocity and temperature gradients (in both coordinate directions). This enabled an ASM to obtain a much more realistic prediction for $\langle u_1\theta \rangle$ and the associated level of turbulent transport in the plume.

ACKNOWLEDGMENTS

Financial support was provided by the Natural Sciences and Engineering Research Council of Canada.

NOMENCLATURE

b	=	half-widths
B_u, B_θ	=	decay constants for plume
D	=	slot width
D/Dt	=	material derivative, $D(\ )/Dt = \partial(\ )/\partial t + U_j\,\partial(\ )/\partial x_j$
Fr_o	=	discharge Froude number, $\mathrm{Fr}_o = U_o^2/g\beta\Theta_o D$
g_i	=	gravitational vector
G_K	=	buoyancy production rate of k
G_{ij}	=	buoyancy production rate of $\langle u_iu_j \rangle$
$G_{i\theta}$	=	buoyancy production rate of $\langle u_i\theta \rangle$
k	=	specific turbulent kinetic energy
P	=	mean pressure
P_K	=	shear production rate of k
P_θ	=	production rate of $\langle \theta^2 \rangle$
P_{ij}	=	shear production rate of $\langle u_iu_j \rangle$
$P_{i\theta}$	=	mean production rate of $\langle u_{i\theta} \rangle$
R	=	turbulence time scale ratio
S	=	spread rate, $S = db/dx_1$
t	=	time
u_i	=	fluctuating velocity
$\langle u_iu_j \rangle$	=	Reynolds stress
$\langle u_i\theta \rangle$	=	turbulent heat (or scalar) flux
U_i	=	mean velocity
x_i	=	distance from origin

Greek Symbols

β	=	coefficient of thermal expansion
Γ	=	molecular diffusivity
Γ_t	=	turbulent diffusivity
δ_{ij}	=	Kronecker delta
ε	=	dissipation rate of k
ε_θ	=	dissipation rate of $\langle \theta^2 \rangle$
θ	=	fluctuating temperature
Θ	=	mean temperature difference between local and ambient value, i.e., for temperature T, $\Theta = T - T_a$
μ	=	molecular dynamic viscosity
ν	=	molecular kinematic viscosity
ν_t	=	turbulent kinematic viscosity
ρ	=	mean fluid density
σ_t	=	turbulent Prandtl number
σ_K	=	turbulent Prandtl number for k
σ_ε	=	turbulent Prandtl number for ε
σ_θ	=	turbulent Prandtl number for $\langle \theta^2 \rangle$

Subscripts

a	=	ambient value
i,j,k	=	Cartesian indices
m	=	centerline value
o	=	discharge value at slot
u	=	variable referenced to velocity field
θ	=	variable referenced to temperature (or scalar) field

Abbreviations

ASM	=	algebraic stress model
EVM	=	eddy viscosity model
RC	=	Ramaprian and Chandrasekhara (1989)

REFERENCES

Bergstrom, D.J. 1987. A computational study of plane buoyant free jets using algebraic stress-flux models. Ph.D. thesis, University of Waterloo, Waterloo, Ontario.

Bergstrom, D.J., A.B. Strong, and G.D. Stubley. 1990. An algebraic stress model prediction of a plane vertical plume. *Numer. Heat Transfer* 18: 263-281.

Chen, J.C., and W. Rodi. 1980. *Turbulent buoyant jets—A review of experimental data, HMT*, vol. 4. Oxford: Pergamon Press.

Durao, D.F.G., J.C.F. Pereira, and J.M.P. Rocha. 1989. Evaluation of k-ε turbulence model for predicting buoyant free round jets. Proceedings of 26th National Heat Transfer Conference, Philadelphia, Pa.

Gibson, M.M., and B.E. Launder. 1976. On the calculation of horizontal non-equilibrium turbulent shear flows under gravitational influence. *J. Heat Transfer, ASME* 98(C1): 81-87.

Hossain, M.S., and W. Rodi. 1982. A turbulence model for buoyant flows and its application to vertical buoyant jets. In *Turbulent Buoyant Jets and Plumes, HMT*, Vol. 6, W. Rodi, ed. Oxford: Pergamon Press.

Launder, B.E. 1975. On the effect of a gravitational field on the turbulent transport of heat and momentum. *J. Fluid Mech.* 67: 569-581.

Launder, B.E. 1988. The prediction of force field effects on turbulent shear flows via second-moment closure. Invited paper, 2d European Turbulence Conference, Berlin.

Malin, M.R., and B.G. Spalding. 1984. The prediction of turbulent jets and plumes by use of the k-W model of turbulence. *Physico Chemical Hydrodynamics* 5(2): 153-198.

Malin, M.R., and B.A. Younis. 1990. Calculation of turbulent buoyant plumes with a Reynolds stress and heat flux transport closure. *Int. J. Heat Mass Transfer* 33: 2247-2264.

Murakami, S., S. Kato, and Y. Suyama. 1987. Three dimensional numerical simulation of turbulent airflow in a ventilated room by means of a two-equation model. *ASHRAE Transactions* 93(2): 621-641.

Murakami, S., S. Kato, and H. Nakagawa. 1991. Numerical prediction of horizontal nonisothermal 3-D jet in room based on the *k-ε* model. *ASHRAE Transactions* 97(1): 38-48.

Raithby, G.D., P.F. Galpin, and J.P. Van Doormaal. 1986. Prediction of heat and fluid flow in complex geometries using general orthogonal coordinates. *Numer. Heat Transfer* 9: 125-142.

Ramaprian, B.R., and M.S. Chandrasekhara. 1989. Measurements in vertical plane turbulent plumes. *J. Fluids Engrg* 111: 69-77.

Sini, J.F., and I. Dekeyser. 1987. Numerical prediction of turbulent plane jets and forced plumes by use of the k-ε model of turbulence. *Int. J. Heat Mass Transfer* 30(9): 1787-1801.