A SIMPLIFIED MODEL FOR PREDICTING VERTICAL TEMPERATURE DISTRIBUTION IN A LARGE SPACE

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ABSTRACT

The authors prepared a simplified model for predicting vertical temperature distribution in a large space. In this model, the space is vertically divided into a number of zones. The ascending and descending currents along the vertical wall generated by convective heat flow to the wall are evaluated. When the space is air conditioned, the path of the supply air jet and the air volume entrained from each zone are evaluated. Next, by solving the equations of airflow balance and that of heat balance in each zone, the vertical temperature distribution can be calculated. This calculation model has a useful feature that can be incorporated into dynamic thermal analysis.

This paper describes the model and reports that results calculated using this model agree well with measured results.

INTRODUCTION

In planning the thermal environment and air-conditioning system for a large space, it is necessary to calculate a cooling load for the occupied zone and a temperature for the upper, uncooled area and to predict the vertical temperature distribution during heating.

Especially in large spaces such as atrium buildings, which typically emphasize openness and natural light, glass roofs and walls are widely used for enclosures, so vertical temperature differences tend to become large under the influence of outdoor conditions. Consequently, it becomes more important to predict the vertical temperature distribution for thermal environment and air-conditioning system design than is the case for other large spaces.

In such spaces, the thermal environment varies with time, so dynamic thermal environment analysis, including prediction of vertical temperature distribution, is required.

The authors prepared a simplified model for predicting vertical temperature distribution that can be incorporated into unsteady-state thermal analysis. In this paper, an outline of this model and the results of a study on its accuracy are presented, along with the results of experiments. The above study is made in cases where interior surface temperatures are given. The method of unsteady-state thermal analysis has been reported in Togari et al. (1991).

PREDICTION OF VERTICAL TEMPERATURE DISTRIBUTION IN LARGE SPACES

Many measurements of the patterns of air movement and temperature distribution in large spaces have been reported (Croome and Roberts 1981), and these results are useful for planning the air-conditioning system.

When the prediction of detailed air movement and temperature distribution has been required, scale-model tests or numerical airflow analysis have been used (Linke 1962; Vaturin 1972; Togari and Hayakawa 1987; Nielsen 1975; Murakami et al. 1991).

Although the above methods are useful when the thermal boundary conditions are given or are constant with time, they are not suitable for use in an unsteady-state analysis.

After considering these circumstances, the authors prepared a simplified model for predicting vertical temperature distribution that can be incorporated into unsteady-state heat analysis. In this model, the main components of air movement in large spaces are assumed to be airflows along the vertical wall surfaces and supply airstreams. The airflows are evaluated based on the analysis of the boundary layer on a flat plate (Eckert et al. 1951). The airstreams are calculated based on the analytical method of a free jet (Koestel 1955).

SIMPLIFIED MODEL FOR PREDICTING VERTICAL TEMPERATURE DISTRIBUTION IN A LARGE SPACE

It is well known that in a large space the horizontal temperature distribution is liable to become uniform except for the region of supply airstreams. This was ascertained by an experiment. We divide the object space vertically into n

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zones. Let the uppermost be zone (1) and the lowermost be zone (n).

We assume the following conditions to be the major factors causing vertical temperature differences:

1. In winter, the exterior glass wall is cooled and the descending air current generated (cold draft) makes a vertical temperature difference.
2. The upper part of the large space is heated by the sunshine entering through the skylight, resulting in a considerable vertical temperature difference.
3. The vertical temperature distribution is formed by the cooling of the occupied zone or by an inadequate hot air supply.

In order to make it possible to quantitatively evaluate the influence of the factors mentioned above, our proposed model consists of three parts (Figure 1).

The first is the “wall surface current model” for evaluating the descending (or ascending) current flowing along the vertical wall surface. The second is the “primary airstream evaluation model,” which handles the airstreams discharged from outlets as non-isothermal free jets to evaluate their influence on vertical temperature distribution. Last is the “heat transfer factor \( C_h \)” for evaluating the heat transfer caused by the temperature difference between vertically adjacent zones.

When the vertical temperature gradient is large, the influence of the radiant heat transfer between walls becomes large. We treat heat radiation and convection separately. We will report on this point in another paper in conjunction with a method of dynamic thermal analysis including prediction of vertical temperature distribution.

### Wall Surface Current Model

Taking a glass surface of the window as an example, when heating a room, heat flows out from the room to the outside through the window and at the same time air cooled in the vicinity of the glass surface forms the descending flow along the glass (cold draft).

Such a current along the vertical wall is modeled as follows. In this paper, interior surface temperatures are assumed to be known.

### Cooled Surfaces

1. **Generation of Descending Current** (see Figure 2a) Let us take the case where the surface temperature \( T_w(1,K) \) of wall \( K \) is lower than the zone air temperature \( T(I) \). It is assumed that heat flow, \( q_w(I,K) \), occurs from zone (I) to the wall and at the same time a descending current occurs along the wall (wall area \( A_w(I,K) \), air volume \( V_{out}(I,K) \)).

![Figure 1] Figure 1 Outline of the simplified prediction model (case where a space is divided into five zones).

![Figure 2] Figure 2 Wall surface current model.
Let the average temperature of the descending current be \( T_d(I, K) \). As this temperature drop is caused by the heat flow to the wall, heat balance equations are represented as follows:

\[
q_w(I, K) = \alpha_c(I, K) \cdot A_w(I, K) \cdot \{ T_w(I, K) - T(I) \} + \gamma \cdot V_{\text{out}}(I, K) \cdot \{ T_d(I, K) - T(I) \}
\]

The average temperature, \( T_d(I, K) \), of the descending current is approximately represented with the following expression (see Appendix A):

\[
T_d(I, K) = 0.75 \cdot T(1) + 0.25 \cdot T_w(I, K)
\]

Substituting this relation into Equation 1, the following equation is obtained:

\[
V_{\text{out}}(I, K) = 4 \cdot \alpha_c(I, K) \cdot A_w(I, K) / C \cdot \gamma
\]

That is, using the convective heat transfer coefficient, \( \alpha_c \), of the wall, the air volume of the generated descending current is obtained.

**Composition of Descending Currents (see Figure 2b)**

As in zone (1), the descending current flowing from zone (I-1) along the wall (air volume \( V_{\text{MD}}[I-1, K] \)), temperature \( T_d[I-1, K] \) is added to the descending current generated in this zone. Temperature \( T_d(I, K) \) and air volume \( V_d(I, K) \) of the "composite descending current" are obtained from the following equation, where \( C \cdot \gamma \) of each descending current component is omitted on the assumption that they are nearly equal to each other.

\[
V_m(I, K) = V_{\text{MD}}[I-1, K] + V_{\text{out}}(I, K)
\]

\[
T_m(I, K) = \{ V_{\text{MD}}[I-1, K] \cdot T_m[I-1, K] + V_{\text{out}}(I, K) \cdot T_d(I, K) \} / V_m(I, K)
\]

**Judgment of Airflow Pattern (see Figure 2c and Table 1)**

From the relationship between the air temperature in zone (I) and that in zone (I+1) associated with the composite descending current temperature \( T_d(I, K) \), we calculated the air volume \( V_{\text{MD}}(I, K) \) entering zone (I) and the air volume \( V_{\text{MD}}(I, K) \) going down along the wall.

**Heated Surfaces**

**Calculation for the Heated Surface Starts from the Lowermost Zone (n)**

The calculation procedure is the same as in the case of cooled surfaces.

**Primary Airstream Evaluation Model**

Taking as an example the case where a supply outlet is mounted in the wall and hot air is discharged horizontally, we explain the basic philosophy of handling the supply airstream.

As shown in Figure 1, we divide the room vertically into five zones with the assumption that the supply air outlet is located in zone (4). This is the same condition as that of the experiment to be described later.

Hot air supplied from the outlet at zone (4) induces the surrounding air to continuously increase its volume, moves upward by a buoyant force, and flows into zone (3). The difference between the total volume of the primary airstream flowing into zone (3) and the supply air volume is the entrained air volume \( V_{\text{E}}[4, 1] \) from zone (4).

The primary air moves further up while entraining air from zone (3) \( V_{\text{E}}[3, 1] \) and zone (2) \( V_{\text{E}}[2, 1] \) and finally enters zone (1). In this case, it is assumed that zone (1) is to be provided with the sum of supply air volume \( V_{\text{S}} \) (temperature \( T_{\text{S}} \)) and the entrained air volumes \( V_{\text{E}}[3, 1] \) and \( V_{\text{E}}[4, 1] \).

The final number of primary air and transfer air volumes between zones varies with the supply air conditions (air volume, temperature, type of outlet, etc.). Thus the influence of supply conditions on vertical temperature distribution can be evaluated using this model.

We assume that supplied air behaves nearly as a free air jet in a large space. To calculate the path of the primary airstream and entrained air volumes, we used the analytical method for a non-isothermal jet. Non-isothermal jet analyses have been reported by Koestel (1955) and Kubota (1973), the former assuming that temperature in the space is uniform and the latter assuming a linear vertical temperature distribution. In our model, each zone temperature \( T(I) \) is used as a boundary condition for jet analysis. The primary airstream evaluation model is described further in Appendix B.

**Heat Transfer by Temperature Difference between Vertically Adjacent Zones**

Heat transfer by the temperature difference between vertically adjacent zones is handled as follows, assuming two cases, one that the upper zone temperature is high and the other that the upper zone temperature is low.

**When the Upper Zone Temperature Is High**

For the parameter that specifies heat transfer between adjacent zones, the heat transfer factor \( C_h(I) \) is introduced. The following equation is used to calculate heat flow, \( q_h(I) \), from zone (I-1) to zone (I):

\[
q_h(I) = C_h(I) \cdot A_h(I) \cdot (T(I-1) - T(I))
\]

where \( A_h(I) \) = the boundary area between zone (I-1) and zone (I).

The heat transfer factor, \( C_h(I) \), is defined as "the value obtained when a stable temperature stratification is formed." \( C_h(0) = 2.3 \text{ W/m}^2\cdot\text{°C} \) is used here mainly.

**When the Upper Zone Temperature Is Low**

If the upper zone temperature is lower than the lower zone temperature, mixing due to the density difference becomes active. Therefore we assume \( T(I-1) = T(I) \).
TABLE 1
Judgment on Airflow Pattern in Case of Descending Current Along the Vertical Wall

<table>
<thead>
<tr>
<th>$T_m(I, K)$</th>
<th>$V_{IN}(I, K)$</th>
<th>$V_{WE}(I, K)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq T(I)$</td>
<td>$V_m(I, K)$</td>
<td>$0$</td>
</tr>
<tr>
<td>$T(I) &gt; T_m(I, K)$</td>
<td>$V_m(I, K) \cdot \frac{T_m(I, K) - T(I+1)}{T(I)-T(I+1)}$</td>
<td>$V_m(I, K) - V_{IN}(I, K)$</td>
</tr>
<tr>
<td>$T_m(I, K) \leq T(I+1)$</td>
<td>$0$</td>
<td>$V_m(I, K)$</td>
</tr>
</tbody>
</table>

Air Volume Balance in Each Zone

The air volume balance in zone $(I)$ (see Figure 3) is represented by the following equation:

$$0 = \sum_{K=1}^{n} (\gamma \cdot V_{IN}(I, K) - \gamma \cdot V_{OUT}(I, K)) + \sum_{L=1}^{m} \sum_{K=1}^{n} (\gamma \cdot V_{E}(L, I) - \gamma \cdot V_{E}(I, L)) + \gamma \cdot V_{C}(I+1) - \gamma \cdot V_{C}(I)$$

$V_{C}$ represents the air volume considered to be transferred between vertically adjacent zones through their boundary and is derived from Equation 7. When the flow direction is upward, the air volume is positive.

Heat Balance in Each Zone

The heat balance equation in zone $(I)$, except the uppermost zone $(1)$ and the lowermost zone $(n)$, is as follows:

$$0 = \sum_{K=1}^{m} C \cdot r \cdot V_{E}(I, K) \cdot (T_{M}(I, K) - T(I)) + \sum_{K=1}^{n} C \cdot r \cdot V_{IN}(I, K) \cdot (T_{S}(I) - T(I)) + \sum_{L=1}^{m} C \cdot r \cdot V_{E}(I, L) \cdot (T(I+1) - T(I)) + C \cdot r \cdot V_{E}(I, I) \cdot (T(I+1) - T(I))$$

The first term of Equation 8 represents heat flow by the descending and ascending currents obtained as a result of heat flow to and from the vertical wall. The last two terms are used in the case of $V_{C}(I) > 0$ and $V_{C}(I+1) > 0$.

For zone $(1)$, the term of the convective heat transfer from the ceiling and, for zone $(n)$, the term of the convective heat transfer from the floor are added to the equation.

Calculation Procedure

Figure 4 shows the calculation procedure when the interior side surface temperature $T_{W}(I, K)$ is known.

At first, the assumed values are set as air temperatures $T(I)$ in five zones. The airflows along the vertical walls can be calculated from the “wall surface current model.” The final entering zone of the supply airstream and entrained air volumes are calculated from the “primary airstream evaluation model.” By solving the air volume balance equation (7) and heat balance equation (8) for each zone, temperatures $T(I)$ are calculated for all zones.

When the differences between the assumptive values of $T(I)$ and their calculated values become sufficiently small, temperatures $T(I)$ are regarded as correct values.
Boundary conditions are given by the experiment, including supply air conditions, interior surface, etc.

Assumption of zone temperature $T(I)^*$

Wall surface currents calculation
(Figure-2, Table-1)

Primary airstream calculation
(Appendix-2)

Air flow balance equation (7)

Heat balance equation (8)

Calculation of zone temperature $T(I)$

$$T(I) = T(I)^*$$

Figure 4 Calculation procedure.

**SCALE-MODEL EXPERIMENTS**

As shown in Figure 5, scale-model tests were performed using a test room $3 \text{ m} \times 3 \text{ m} \times 2.5 \text{ m}$ (high), which was located in an airflow test laboratory. The test room was made of insulated boards except for one glass wall, and the ceiling and floor insulation were removable.

Seven cases were tested (see Table 2): two where only the outside conditions changed, three where hot air was supplied, and two where cool air was supplied.

![Test room, used in this study, in airflow testing laboratory.](image)

The supply air conditions were set up assuming that the test room used was a one-fifth scale model of a large space. A supply outlet was mounted in the wall opposite to the glass wall, $625 \text{ mm}$ above the floor. Two outlets were used, one $500 \text{ mm}$ wide and another $250 \text{ mm}$ wide, both $74 \text{ mm}$ long. The supply air volume was in a range between $100$ and $150 \text{ m}^3/\text{h}$. The Reynolds number of the supply air was on the order of $10^4$. Reduction ratios in the scale-model tests were assumed as follows:

1) Size $N_f = 1/5$
2) Archimedes number of supply air $N_{Ar} = 1.0$
3) Temperature difference $N_r = 1.0$
4) Velocity $Nu = 0.45$

A return inlet was installed at the edge in the same wall as the outlet, $250 \text{ mm}$ above the floor or $250 \text{ mm}$ below the ceiling.

Interior air and surface temperatures were measured at 160 points located on two sections perpendicular to each other, as shown in Figure 6. Air temperatures were measured using Cu-Co thermocouples.

Heat flow sensors coated with aluminum foil were installed as shown in Figure 6, and the values measured were referenced to infer the convective heat transfer coefficient, $\alpha$, used for the calculations (see Table 3). The heat flow sensors used were $0.7 \text{ mm}$ thick with a heat resistance of $0.0017 \text{ m}^2{\circ}\text{C}/\text{W}$, which corresponded to about $3\%$ of the heat resistance of the glass wall.

**COMPARISON BETWEEN CALCULATED AND MEASURED VALUES**

In calculating the vertical temperature distribution, the measured interior side surface temperatures and supply air conditions were used as the boundary conditions. A space
### TABLE 2
Experimental Cases and Their Conditions

<table>
<thead>
<tr>
<th>Case Name</th>
<th>Supply Air Condition</th>
<th>Outlet size (mm, mm)</th>
<th>Air volume (m³/h)</th>
<th>Velocity (m/s)</th>
<th>Temperature (°C)</th>
<th>Momentum (kg·m/s²)</th>
<th>Inlet Location</th>
<th>Outside Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-11</td>
<td>Natural convection (Ceiling heated and floor cooled)</td>
<td>42 (14)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N-10</td>
<td>Natural convection (Heating outside)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H-100</td>
<td>500 x 74</td>
<td>150 1.13</td>
<td>40.7</td>
<td>0.053</td>
<td>Zone(5)</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H-101</td>
<td>500 x 74</td>
<td>150 1.13</td>
<td>40.7</td>
<td>0.053</td>
<td>Zone(1)</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H-110</td>
<td>250 x 74</td>
<td>135 2.03</td>
<td>40.4</td>
<td>0.086</td>
<td>Zone(5)</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-200</td>
<td>250 x 74</td>
<td>135 2.03</td>
<td>12.1</td>
<td>0.095</td>
<td>Zone(5)</td>
<td>3.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-210</td>
<td>250 x 74</td>
<td>100 1.50</td>
<td>13.8</td>
<td>0.052</td>
<td>Zone(5)</td>
<td>3.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 3
Convective Heat Transfer Coefficients, \( \alpha \) (W/m²·°C), Used in This Study

<table>
<thead>
<tr>
<th>Natural convection</th>
<th>Hot air supplied</th>
<th>Cooling air supplied</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ceiling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat flow</td>
<td></td>
<td></td>
</tr>
<tr>
<td>upward : ( \alpha_c = 4.6 )</td>
<td>upward : ( \alpha_c = 9.3 )</td>
<td>Heat flow</td>
</tr>
<tr>
<td>downward : ( \alpha_c = 2.3 )</td>
<td>Influence of primary air</td>
<td>downward : ( \alpha_c = 2.3 )</td>
</tr>
<tr>
<td><strong>Floor</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat flow</td>
<td></td>
<td></td>
</tr>
<tr>
<td>upward : ( \alpha_c = 4.6 )</td>
<td>upward : ( \alpha_c = 4.6 )</td>
<td>Heat flow</td>
</tr>
<tr>
<td>downward : ( \alpha_c = 2.3 )</td>
<td>Influence of primary air</td>
<td>downward : ( \alpha_c = 2.3 )</td>
</tr>
<tr>
<td><strong>Glass wall</strong></td>
<td>( \alpha_c = 3.5 )</td>
<td>Zone(1) : ( \alpha_c = 9.3 )</td>
</tr>
<tr>
<td><strong>Insulated wall</strong></td>
<td>( \alpha_c = 3.5 )</td>
<td>Zone(1) : Zone(2) : Zone(5) : Zone(5) : ( \alpha_c = 3.5 )</td>
</tr>
</tbody>
</table>

**Figure 6** Measuring items and positions.
was divided vertically into five zones. The uppermost part was zone (1). Zone (5) was assumed to be the occupied zone. The glass wall was designated wall 1 (K=1) and the other three vertical walls were designated wall 2 (K=2). The heat transfer factor used was \( C_f = 2.3 \).

**When the Outside of the Test Room Is Heated (Experimental Case: N-10)**

We kept the outside temperature of the test room at about 12°C for more than 24 hours to make all the temperatures uniform, including the inside temperature of the walls. Afterward, we heated it to 42°C and measured the temperature distributions. As Figure 7 shows, the surface temperatures of the glass wall were higher than other surface temperatures. The room air temperature was raised mainly by heat flow through the glass wall.

Figure 8 shows a calculated result of airflows and temperatures. As the glass surface is highly heated, the temperature \( T_{w}(l,1) \) of the ascending current along the glass surface becomes higher than the zone air temperature \( T_{z} \). Therefore, the entire draft volume ascends along the uppermost part, zone (1). However, at insulated walls (K=2) where the temperature differences between the zone air and the wall surface are small, vertical airflow volumes along the wall are comparatively small.

Thus, the useful feature of the “wall surface current model” is that the differences in thermal characteristics of the vertical walls are represented as the differences of airflow patterns along the wall.

Calculated results of vertical temperature gradients are shown compared with the measured results in Figure 9. Although the calculated values are slightly higher than the measured values, the characteristic of vertical temperature distribution is well represented.

![Figure 7](image)

**Figure 7** Vertical temperature distribution measured at 17:00 in experiment case (N-10).

**In the Case of Hot Air Supply**

Outside temperatures of the test room were kept between 11°C and 12°C. Afterward, hot air was supplied to the test room, and the resultant temperature rise and temperature distributions were measured (experimental case: H-100). As shown in Figure 10, hot air supplied horizontally moved upward by buoyant force and vertical temperature differentials became large.

Figure 11 shows calculated airflows and temperatures. The hot air supplied from an outlet in zone (4) moves upward and finally flows into zone (1). The primary airstream entrains at about 400 m³/h from each of zones (2) to (4) and finally flows into zone (1) with an air volume of about 1400 m³/h and the temperature of zone (1) becomes higher than others. As the airflow rate from zone (1) to zone (2) is large, the temperature of zone (2) nearly equals the temperature of zone (1). As for zone (5), air flows into it mostly from zone (4) in a small quantity. Consequently, the temperature of zone (5) is kept comparatively low.

Figure 12 shows comparisons between the measured and calculated values. The calculated patterns of vertical temperature distributions are in good agreement with the measured ones, but at warm-up time (11:00 and 12:00) calculated temperatures are slightly higher than measured temperatures.

In Figure 12, measured values for zone (1), which is the final entering zone of primary air in this case, are averages of measurements at all nine points of the same height (see Figure 5). Other measured values are averages of those measured at points that were not exposed to the primary airstream.

Figure 13 shows a comparison in the steady-state condition of case H-110 with a half-width outlet. The calculated values agree well with the measured values. As the momentum of the supply air becomes large in this case, it is calculated that the supply air jet reaches the glass wall in zone (2).
Outside temperature
11.6°C 31.5°C 36.9°C 38.8°C 39.3°C 40.1°C 41.0°C

Figure 9 Comparisons between calculated values and measured values in case where outside is heated (N-10). In the execution of calculation, interior surface temperature distribution at each time is given by the experiment.

Figure 10 Vertical temperature distribution measured at 12:00 in experiment case (H-100).

Figure 11 Calculated result of airflows and temperatures at 12:00 (H-100).

Figure 15 shows a steady-state temperature distribution measured in experiment case C-210. As shown in Figure 16, the calculated vertical temperature gradient agrees well with the measured one.

As mentioned above, the influences of supply air conditions and of return inlet location can be evaluated.

In the Case of Occupied Zone Cooling

We performed two tests under different supply air conditions, assuming cooling of the occupied zone. The space cooling load was given by the temperature difference between the inside and outside of the test room.

The throw of the supply air jet was considerably long in this case. Therefore, even after the supply airstream reached the lowermost part, zone (5), the current flowing along the floor and the glass surface was still strong, and so
zone (3) and zone (4) were considered to be cooled. But in the "primary airstream evaluation model," the influence of the supply airstream after reaching zone (5) is not evaluated.

Therefore, in order to ascertain the reliability and limits of this model, it is necessary to examine the throw and the central velocity of the supply air jet.

In the Case Where the Ceiling Is Heated and the Floor Is Cooled

In this case, we insulated the walls, including the glass wall (whose outside was covered with insulation 75 mm thick). We removed the ceiling and floor insulation and prepared under-floor cooling (about 14°C) and outside heating (about 40°C) (experiment case: N-11).

Figure 19 shows the measurement result. A stable temperature stratification was formed in the lower part of the test room. Wall temperatures were kept slightly higher than air temperatures at the same height. As is observed in the calculated result, shown in Figure 20, there is less flow between vertical zones along the wall surface, and heat transfer between zones depends primarily on the heat transfer factor $C_a(t)$. As is seen in Figure 21, in the case of the heat transfer factor $C_a(t) = 2.3 \text{ w/m}^2\text{.}^\circ\text{C}$ (2 kcal/h·m²·°C), the vertical temperature distribution is well reproduced.
In the Case Where the Return Inlet Is Located in Zone (1) (H-101)

As shown in Figure 22 and Figure 23, temperature difference $\Delta \theta_{44}$ between zone (1) and zone (4) changes little in the range of $C_a = 0$ to 8. On the contrary, a temperature difference of $\Delta \theta_{45}$ between zone (4) and zone (5) in the lower part is considerably influenced by the value of $C_a$. When $C_a = 2.3$ W/m²°C (2 kcal/hr-m²°C), the calculated vertical temperature difference $\Delta \theta_{45}$ becomes close to the measured value (difference of 2.9°C). Setting $C_a$ at 0 results in overestimating temperature difference $\Delta \theta_{45}$.
Figure 24 shows the calculated result of airflows and temperatures. Air transfer volumes between zones are considerably larger from zone (1) to zone (4) because of the influence of the supply airstream. In zone (5), air transfer volumes between other zones are small. This is the reason why there is a difference in the influence of the $C_p$ value on $\Delta \theta_{14}$ and on $\Delta \theta_{45}$.

**In the Case Where the Return Inlet Is Located in Zone (5) (H-100)**

When the return inlet is in the lower part, zone (5), air transfer occurs between zones (4) and (5) (see Figure 11), reducing the influence of the value of $C_p$ (Figure 25).

Although setting $C_p$ at 0 results in a slight overestimation of the vertical temperature difference, the difference from the measured values is around 0.5°C.

**The Meaning and Role of $C_p$**

As discussed above, when air transfer is active between vertically divided zones, the value of $C_p$ has less effect on vertical temperature difference. On the contrary, when

![Diagram](image-url)
Figure 21 Comparison between measured values and calculated values. In this case (N-11), heated ceiling, cooled floor, and insulated other walls.

Figure 22 Influences of $C_a$ values on vertical temperature distribution (H-101).

Figure 23 Influences of $C_a$ values on temperature differences between vertically divided zones (H-101).

Figure 24 Calculated result of airflows and temperatures in steady-state (H-101).

CONCLUSIONS

We proposed a simplified model for predicting the vertical temperature distribution in a large space that can be incorporated in unsteady-state thermal analysis. In most
In order to apply the model to an actual large space, it is still necessary to clarify the nature and numeric value of the heat transfer factor, $C_a$.

- The surface heat transfer coefficient, $\alpha_s$, used here was estimated using the measured values of heat currents. It is usually difficult to estimate the value of this coefficient, including the influence of air-conditioning air currents.
- This model is a macroscopic model for predicting vertical temperature distribution, and it is very important to clarify its limits of applicability. As mentioned in the case of occupied zone cooling, the throw of the supply airstream is considered an important factor.

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NOMENCLATURE

- $I/L$ = zone number in a large space
- $K$ = number of vertical wall
- $T(l)$ = air temperature in zone $(l)$
- $A_w(l, K)$ = area of vertical wall $K$ in zone $(l)$

Notation

- $D_o$ = equivalent diameter
- $X_p$ = centerline velocity constant, dependent on outlet type and discharge pattern
- $X$ = $X$-coordinate
- $Y$ = $Y$-coordinate
- $S$ = distance from the outlet
- $\phi$ = angle between $u_m$ and $X$-axis
- $u_m$ = centerline velocity
- $t_m$ = centerline temperature
- $\Delta t$ = temperature difference between centerline and surrounding ($t = t_m$)
- $r$ : distance from centerline
- $u$ = velocity at $r$
- $t$ = temperature at $r$
- $\Delta t$ = temperature difference between the point at $r$ and surrounding ($t = t_m$)
- $t_s$ = surrounding air temperature
- $\beta$ = similitude ratio of temperature difference profile to velocity profile (dependent on turbulent Prandtl number, adopting constant value of 0.65)
- $\xi$ = coefficient of cubic expansion (defined to the average of mean jet temperature and surrounding air temperature)
- $g$ = gravitational acceleration

Figure 25 Influences of $C_a$ values on vertical temperature distribution $(H-100)$.

Figure 26 Primary airstream evaluation model.
\[ T_{0}(I,K) = \text{interior side surface temperature of vertical wall } K \]

\[ q_{w}(I,K) = \text{convective heat flow from wall } K \text{ to zone } I \]

\[ \alpha(I,K) = \text{convective heat transfer coefficient} \]

\[ C' \gamma \]

**Wall Surface Current Model (Described in Paragraph on Descending Current)**

\[ V_{out}(I,K) = \text{air volume flowing out of zone } I \text{ toward the wall } K \]

\[ T_{D}(I,K) = \text{average temperature of the descending current generated in zone } I \text{ on wall surface } K \text{ (see Figure } 2a) \]

\[ V_{in}(I,K) = \text{air volume flowing into zone } I \text{ from the surface current along the wall } K \]

\[ V_{up}(I,K) = \text{air volume of descending current moving down from zone } I \text{ to zone } (I+1) \text{ along the wall } K \]

\[ V_{h}(I,K) = \text{air volume of compound surface current (see Figure } 2b, \text{ Equation } 4) \]

\[ T_{m}(I,K) = \text{average temperature of compound descending current (Equation } 5) \]

**Air-Conditioning Air Current and Others**

\[ V_{a} = \text{supply air temperature} \]

\[ V_{s}(L) = \text{supply air volume} \]

\[ V_{e}(L) = \text{return air volume} \]

\[ V_{c}(L) = \text{entrained air volume from zone } I \text{ to primary air, and flowing into zone } (L) \text{ (distance } y \text{ from wall)} \]

\[ V_{c}(L) = \text{air volume transferred from zone } (L) \text{ to zone } (L-1) \text{ through zone boundary} \]

\[ C_{h}(I) = \text{heat transfer factor by temperature difference between adjacent zones} \]

**REFERENCES**


**APPENDIX A**

Regarding flow along a vertical wall surface in a large space, we assume that the following Equations 9 and 10 (Eckert and Jackson 1951) for representing the turbulent boundary layer (natural air convection) on a vertical flat plate will hold approximately true:

\[
\frac{u}{u_{i}} = \left( \frac{y}{\delta_{l}} \right)^{1/7} \left( 1 - \frac{y}{\delta_{l}} \right)^{4}
\]

\[
\frac{\theta - \theta_{w}}{\theta_{w} - \theta_{\infty}} = 1 - \left( \frac{y}{\delta_{l}} \right)^{6/7}
\]

where

\[ y = \text{vertical distance from wall}, \]

\[ u, \theta = \text{air velocity and temperature of boundary layer (distance } y \text{ from wall)} , \]

\[ \delta_{l} = \text{boundary layer thickness}, \]

\[ u_{i}, \theta_{l} = \text{standard velocity in the boundary layer}, \]

\[ \theta_{w}, \theta_{\infty} = \text{wall surface temperature and air temperature outside of boundary layer}. \]

Average temperature, \( T_{D} \), of the flow along the wall is represented with the following equation, with the assumption that \( C' \gamma \) is nearly uniform in that flow:

\[
T_{D} = \delta_{l} / \int_{0}^{\delta_{l}} u \theta \, dy / \int_{0}^{\delta_{l}} u \, dy
\]
Substituting Equations 9 and 10 into Equation 11, the following relation is obtained regardless of the distance in the direction of a natural convection current along the wall:

\[ T_0 \approx 0.75 \cdot \theta_m + 0.25 \cdot \theta_w \]

That is, the descending current temperature \( T_d(I,K) \) generated in zone \( I \) can be obtained using zone temperature \( T(I) \) and wall surface temperature \( T_w(I,K) \), as follows:

\[ T_0(I,K) \approx 0.75 \cdot T(I) + 0.25 \cdot T_w(I,K) \]

**APPENDIX B**

**Primary Airstream Evaluation Model**

We assume that, in a large space, supplied airflow behaves nearly as a free air jet and use the analytical method for non-isothermal jets presented by Koestel (1955).

Our model has several features, as follows (Figure 26):

a) Vertically divided zone temperatures are used as temperature boundary conditions.

b) An airstream supplied at an arbitrary angle can be handled.

c) This method aims at calculating the path and the total air volume of the primary airstream.

1. **Assumption**

The method uses the following assumptions, as did Koestel (1955):

a) The air velocity distribution in the section vertical to the axis and the temperature difference distribution between the jet and the surrounding air can be approximated with error functions, and these two distributions become similar in shape to each other (see Equations 14 and 15).

b) The expansion angle of jet is constant along the path.

c) Air density is assumed constant, except that buoyancy is taken into account.

d) Buoyancy acts on the section vertical to the axis.

e) The quantity of heat that a jet holds varies only with entrained air.

2. **Basic Equation and Boundary Conditions**

a) Distribution in jet

Air velocity distribution:

\[ u(r) = \exp \left\{ -2K \frac{r^2}{s} \right\} \]

Temperature velocity distribution:

\[ \Delta^t = \exp \left\{ -b \cdot 2K \frac{r^2}{s} \right\} \]

b) Conserved quantity

Momentum in \( y \) direction:

\[ d \frac{d}{ds} \left( \frac{u_m^2 \cdot s^2 \cdot \sin \theta}{\Delta^t} \right) = \frac{d}{ds} \left( \frac{g \cdot \beta \cdot (t_m - t_e)}{b} \right) \cdot s^2 \]

Conserved quantity of heat:

\[ \frac{dt_m}{dy} = (1+b) \cdot u_m \cdot s^2 \sin \theta \cdot \frac{dx}{dy} \]

where \( dx/dy \) in the basic equations are boundary conditions.

3. **Solution**

A simultaneous equation system is set up with respect to conserved quantity, and the Runge-Kutta-Gill method is used to numerically solve the simultaneous equations. In the calculation used here, the width of steps was set at 0.05 m. Further, the boundary condition used the value of the height of the center of the jet.

The air volume passing through the section vertical to the axis is obtained by integrating an air velocity range greater than 5% of the central velocity of jet.

In this analysis, the calculation is truncated when the jet reaches the interior surface of the test room or when the central velocity of the jet becomes less than a set value (e.g., 0.25 m/s).

4. **Output**

Final entering zone numbers of the primary airstream and air volume \( V_e(I,L) \) entrained from zone \( I \) to primary airstream and flowing into zone \( L \) are given from the path and total air volume calculated by non-isothermal jet analysis.