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Modelling of indoor air humidity: the dynamic behaviour within an enclosure

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Abstract

Indoor air humidity behaviour within an enclosure has been mathematically modelled. A linear differential equation is used to describe the response of indoor air humidity to different moisture transport processes within the space. The modelled processes include: moisture absorption/desorption, surface condensation, air movement across enclosure boundaries, indoor evaporation, and indoor moisture generation. By using a discrete time step, nonlinear processes, such as surface condensation, can be assumed linear within the time interval. The differential equation is solved as part of a comprehensive numerical formulation through which the behaviour of moisture transport process and its contribution to indoor humidity dynamics are modelled. A theoretical study of indoor air humidity response to different moisture transport processes has revealed the importance of the involved processes in determining indoor air humidity behaviour. The relative influence of a particular process depends on its time constant, its interaction with other processes, and the continuity of the process. In addition, the impact of a given moisture transport process will also depend on the building physical and functional characteristics, as well as the prevailing outdoor environmental conditions.

Introduction

Indoor air humidity is an extremely important environmental parameter which can greatly affect building functional performance. Many related studies [1–6] have emphasized the important role of indoor humidity in determining the overall building performance. Indoor humidity can affect a building's energy consumption and its structure, as well as occupants' comfort and health [4]. Very low indoor humidity may cause damage to interior furnishings due to shrinkage of moisture-sensitive materials. In cold climates, high indoor humidity could cause condensation on windows and within the building envelope components causing occupants dissatisfaction, material deterioration and in some cases mould and mildew growth, a possible by-product of condensation, which affects the indoor environmental quality and occupants' health.

Indoor air humidity is greatly variable and dependent on many physical and time-dependent environmental parameters. In buildings where there is no humidity control, the air humidity level depends on the natural balance between moisture gains and losses. Moisture can be gained into the space by the occupants and their household operations, infiltration, mechanical ventilation, evaporating damp surfaces, interior surfaces and material through desorption and by indoor moisture-generating operations. On the other hand, moisture can be lost from the space by exfiltration, surface condensation, mechanically exhaust air, and absorption by interior hygroscopic materials. Moisture can also be transferred in relatively negligible amounts by diffusion through the building envelope components.

A study of humidity in Canadian houses [7] has indicated that indoor humidity is subjected to considerable seasonal variations while the temperature remains almost constant. It mainly follows the trend of outdoor humidity, especially in summer because of increased ventilation, but also is influenced by the seasonal ventilation habits of occupants and the moisture storage of hygroscopic material in the

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space. In winter, the difference between indoor and outdoor humidity reaches a maximum due to increased activities and reduced ventilation. The relationship between indoor humidity and air leakage and ventilation has been modelled [8, 9] to determine the effect of air exchange rate on indoor moisture level and to examine the effect of other parameters, such as occupancy, building characteristics and external environmental parameters. Most of these parameters can have a significant impact on indoor humidity, especially at a lower air exchange rate.

Condensation on interior surfaces and moisture absorption by building construction and furnishing materials inside the space can have a considerable effect on the variational behaviour of indoor humidity. In winter, the interior surface of a window could act as a dehumidifier removing a lot of water vapour from the space; methods for calculating surface condensation have been discussed [10–12], based on the well-known mass transfer equations under steady-state conditions. Considerable amounts of moisture can also be removed or delivered into the space by absorption and desorption processes of interior building materials. It is estimated that approximately one third of the moisture generated in a room can be absorbed by room surfaces [13]. Moisture absorption and desorption by building materials have been experimentally and theoretically investigated [13–18]. Different building materials have been found to respond differently to changes in ambient relative humidity. However, most of them follow a rising exponential function in the case of absorption and a falling exponential function in the case of desorption [14]. In some studies the processes of absorption and desorption have been theoretically approximated by a capacitor–resistor electrical circuit in which the voltage represents the vapour pressure and the capacitance represents the material moisture capacity [16]. In other studies [13, 18], the absorption/desorption processes have been modelled using lumped parameter analysis. In these models, experimental and theoretical results are matched through one or more parameters, such as the convective mass transfer coefficient and the effective moisture capacity of the material surface.

Models for indoor humidity calculations have been developed [13, 19, 20]; however, a detailed humidity calculation model has not been available with the same degree of sophistication as that for the heat transfer processes [13]. All available models are based on the mass balance between gained and lost moisture. The simplest model [20] predicts the indoor humidity as a function of occupancy and ventilation rate with a simple treatment of the effect of moisture storage in hygroscopic materials. Other models [13, 19] have considered more time-dependent mechanisms of moisture transport in an enclosure. In addition, moisture absorption has been treated in more detail. However, for these models to be used, material properties which are generally not available or difficult to determine must be known. In a related situation, the dynamic long-term behaviour of moisture conditions in building cavities has been modelled, both without condensation [21] and with condensation [22]. The effect of hygroscopic materials in the cavity under non-steady conditions is considered in both cases. The result is a pair of linear coupled differential equations which can be solved easily to predict long-term cavity moisture performance based on two primary time constants. The model was then modified [23] to include the presence of evaporating surfaces within the wall cavity.

The objective of this study is to develop a mathematical model for predicting indoor humidity dynamics in response to the different moisture transport processes within the space and across its boundaries. Air humidity behaviour within an enclosure is described by a linear differential equation, which includes moisture absorption/desorption processes, air movement, surface condensation, indoor evaporation, and moisture generation processes. All the processes involved are nonlinear in behaviour, and hence the differential equation must be solved as part of a numerical formulation by using a discrete time step during which linear behaviour can be assumed for all processes. Numerical solution of the differential equation is obtained by the Runge–Kutta method.

Model logic and development

Conceptual approach

The moisture content of the air inside occupied buildings is typically in a dynamic state. Many time-dependent processes, such as surface condensation, ventilation and absorption by interior materials, will determine the moisture content of indoor air and contribute to its dynamic behaviour as illustrated by the schematic shown in Fig. 1. When indoor humidity is not mechanically controlled, its level is determined by the natural balance between moisture gains and losses within the space.

Based on the moisture mass balance concept, the transient response of air moisture content inside a room can be generally modelled via the following linear differential equation:

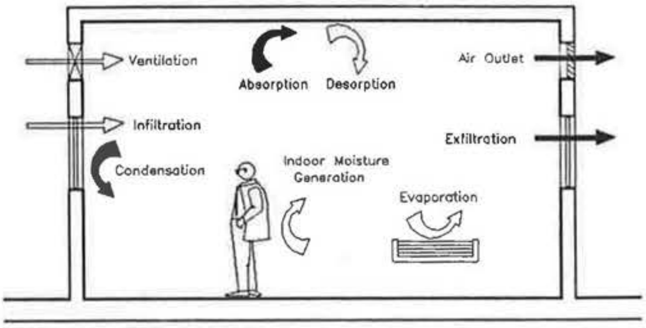

Fig. 1. A schematic for moisture transport processes within an enclosure.

$$\frac{dW_r}{dt} = \frac{1}{\rho_a V}\left[\sum_{i=1}^{ns} \frac{dms_i}{dt} + \sum_{i=1}^{na} \frac{dma_i}{dt} + \sum_{i=1}^{nc} \frac{dmc_i}{dt} + \sum_{i=1}^{nv} \frac{dme_i}{dt} + \sum_{i=1}^{nq} mg_i\right] \quad (1)$$

where

dW_r/dt = rate of change in indoor air moisture content, kg/kg s

dms/dt = rate of moisture absorption or desorption by interior materials, kg/s

dma/dt = rate of moisture added or removed due to air movement across room boundaries, kg/s

dmc/dt = rate of moisture removed by condensation, kg/s

dme/dt = rate of moisture added by evaporation, kg/s

mg = rate of moisture generation from indoor sources (e.g., people, indoor operations, etc.), kg/s

For eqn. (1) to be solved, several time-dependent parameters, which are needed to model the processes involved, must be expressed in terms of time. However, because of the transient nature of boundary conditions and the complexity of some processes, the variations of corresponding parameters cannot be expressed in terms of time in a continuous function. Moreover, eqn. (1) involves many transient nonlinear moisture transport processes which are interrelated in a complicated manner. Therefore, eqn. (1) can only be solved as an integral part of a comprehensive numerical technique through which the interrelation between the processes involved can be modelled. Depending on the room physical and functional characteristics, its air moisture content behaviour can be determined by one or more of the processes described by eqn. (1). In a real situation, a combination of these processes will normally determine the state of the indoor air moisture content.

The time-dependent processes affecting indoor air humidity

Moisture absorption and desorption

When moisture absorption and desorption by interior materials are the only determining processes the room air humidity response can be given by:

$$\frac{dw_r}{dt} = \frac{A_m h_{ms}}{\rho_a V}(W_m - W_r) \quad (2)$$

The material moisture conditions W_m and room air conditions W_r in eqn. (2) are interdependent parameters. For materials where lumped-parameter analysis can be used (i.e., $Bi \leqslant 0.1$) to describe their dynamic moisture interaction, the surface air moisture content W_m can be given by eqn. (3):

$$\frac{dW_m}{dt} = \left(\frac{A_m h_{ms}}{R_v \rho_a V_m C_m T_m}\right)(W_r - W_m) \quad (3)$$

Equations (2) and (3) represent a pair of coupled differential equations that describe the dynamic moisture interaction between room air humidity and a given material within the space. Utilizing Laplace transformation, this system of differential equations can be solved to give:

$$W_r = W_{ro}\exp[-(B+C)t] + \left(\frac{CW_{mo} + BW_{ro}}{C+B}\right) \times [1 - \exp(-(B+C)t)] \quad (4)$$

where

$$B = \frac{A_m h_{ms}}{R_v \rho_a V_m C_m T_m}$$

$$C = \frac{A_m h_{ms}}{\rho_a V}$$

Equation (4) describes the air humidity response to moisture absorption/desorption by one single material which can be modelled by the lumped-parameter analysis. For each material within the space a separate differential equation must be written. To determine the air humidity response, the resulting differential equations must be solved simultaneously with eqn. (2). As the number of differential equations increases, an analytical solution becomes more complicated and more difficult to obtain. For most interior materials a lumped-parameter analysis is not possible, hence their moisture behaviour cannot be directly modelled as in eqn. (3). In addition, other moisture transport processes within the space, which cannot be accounted for analytically, are indirectly affecting a material's moisture conditions.

When eqn. (2) is used as a part of a numerical formulation, it can be directly solved by assuming that W_m is constant during the time step used in the modelling. However, for most building materials, in order to assume a constant surface moisture condition, the time step must be taken so small that the solution becomes impractical and the accumulative error becomes unacceptable for large modelling periods. When a practically large time step is used, the variability of W_m during the time step must be modelled. Assuming constant W_r during the time step, then for a material where lumped-parameter analysis can be used, the material surface air moisture content can be given by eqn. (5).

$$W_m(t) = W_r - (W_r - W_{mo}) \exp(-Bt) \tag{5}$$

Substituting eqn. (5) into eqn. (2) and solving the resulting equation yields:

$$W_r(t) = W_{mo} + (W_{ro} - W_{mo}) \exp\left[-\frac{C}{B}(1 - \exp(-Bt))\right] \tag{6}$$

Equation (6) describes the dynamic behaviour of indoor air humidity during the time step used in the numerical formulation. Both the material moisture condition, W_{mo} and the indoor air humidity condition, W_{ro} have to be updated at the end of each time interval. Equation (5) is used to evaluate material moisture conditions at the end of each time interval by using the time-step average air moisture content, W_{ra}.

When moisture interaction occurs at the material surface (i.e., $Bi \gg 0$), a lumped-parameter analysis cannot be used because of the non-uniformity of moisture conditions across the material thickness. Instead, the following expression is used:

$$\frac{\partial W_m}{\partial t} = \alpha_m \frac{\partial^2 W_m}{\partial x^2} \tag{7}$$

$$\frac{\partial W_m}{\partial x} = 0 \qquad \text{at } x = L_m \tag{7a}$$

$$-D_v \frac{\partial W_m}{\partial x} = h_{ms}(W_r - W_m) \qquad \text{at } x = 0 \tag{7b}$$

$$W_m = F(x) \qquad \text{at } t = 0 \tag{7c}$$

Equation (7), which describes the transient moisture transfer through the material, is solved for the boundary and initial conditions given by eqns. 7(a)–7(c).

Using the moment method described in ref. 24, eqn. (7) can be approximately solved as part of a numerical formulation using a discrete time step t. By assuming a linear behaviour of the surface moisture conditions during this time interval, then for a fourth-degree polynomial representation, the surface air moisture content at the end of t is determined by:

$$W_{ms}(t) = \mathbf{A}/\mathbf{Z} \tag{8}$$

where

$$\begin{aligned}\mathbf{A} = {} & [(-217b_1 + 1302b_2 - 1302b_3)\exp(-42\tau) \\ & + (168b_1 - 1008b_2 + 1008b_3)\exp(-52\tau) \\ & + (55b_1 - 320b_2 + 315b_3)\exp(-10\tau) + b_1] \\ & + \left[\left(-\frac{217}{2520\tau} - \frac{1519}{420}\right)\exp(-42\tau)\right. \\ & + \left(\frac{14}{5} + \frac{1}{15\tau}\right)\exp(-52\tau) \\ & + \left(\frac{1}{24\tau} + \frac{13}{12}\right)\exp(-10\tau) \\ & \left. - \left(\frac{\tau}{2} + \frac{1}{45\tau}\right)\right]\frac{h_{ms}L_m}{D_v} W_{mo} \\ & + \left[\frac{1519}{420}\exp(-42\tau) - \frac{14}{5}\exp(-52\tau)\right. \\ & \left. - \frac{13}{12}\exp(-10\tau) + \tau + \frac{1}{3}\right]\frac{h_{ms}L_m}{D_v} W_r\end{aligned}$$

$$\begin{aligned}\mathbf{Z} = {} & 1 + \frac{h_{ms}L_m}{D_v}\left(\frac{\tau}{2} - \frac{1}{45\tau} + \frac{1}{3}\right) + \frac{h_{ms}L_m}{D_v\tau} \\ & \times \left[\frac{1}{15}\exp(-52\tau) + \frac{1}{24}\exp(-10\tau)\right. \\ & \left. - \frac{217}{2520}\exp(-42\tau)\right]\end{aligned}$$

$$\tau = \frac{\alpha_m t}{L_m{}^2}$$

and b_1, b_2, b_3 are the initial conditions determining parameters which depend on the initial moisture distribution. For a uniform initial moisture distribution W_{mi};

$$b_1 = W_{mi},\ b_2 = W_{mi}/2,\ b_3 = W_{mi}/3$$

Indoor air moisture content can be expressed by combining eqns. (2) and (8) yielding:

$$\frac{dW_r(t)}{dt} = \frac{A_m h_{ms}}{\rho_a V}[W_{ms}(t) - W_r] \tag{9}$$

Equation (9) can be solved numerically to evaluate room air humidity response to moisture absorption and desorption. In reality, many materials with different moisture characteristics could exist inside a space within a building. The combined moisture interaction of these materials with indoor air humidity will determine its dynamic behaviour as given by:

$$\frac{\mathrm{d}W_r}{\mathrm{d}t} = \sum_{i=1}^{ns1} C_i(W_{moi} - W_r)\exp(-B_i t) + \sum_{j=1}^{ns2} C_j(W_{ms}(t) - W_r) \tag{10}$$

Air movement across enclosure boundaries

Moisture transport associated with air movement is the fastest, and thus the most important mode of moisture transport in buildings. Depending on the flow rate and the humidity conditions of the outdoor air, indoor air humidity could instantly and dramatically respond to air movement across enclosure boundaries. The response of indoor air humidity to air movement across enclosure boundaries (e.g., infiltration) can be expressed by:

$$\frac{\mathrm{d}W_r(t)}{\mathrm{d}t} = \frac{Q_a}{V}\,[W_o - W_r(t)] \tag{11}$$

For constant external conditions, eqn. (11) can be solved to give:

$$W_r(t) = W_o + (W_{ro} - W_o)\exp\left(-\frac{Q_a}{V}t\right) \tag{12}$$

In practice, airflow across enclosure boundaries could occur through several different flow paths under different driving forces at the same time. For a given space within a building, airflow could occur through its exterior boundaries as well as through inter-space flow paths connecting it with other spaces (i.e., a doorway). Airflow across exterior boundaries may occur due to a natural driving force through an arbitrary flow path (i.e., infiltration/exfiltration), or due to a controlled driving force (i.e., ventilation). Each airflow process across the enclosure boundaries will have a unique effect on the behaviour of air humidity inside the enclosure. For a combination of airflow processes, the dynamic behaviour of indoor air moisture content can be given by:

$$W_r(t) = \sum_{i=1}^{na} \frac{Q_{ai}W_{oi}}{\sum_{i=1}^{na} Q_{ai}} + \left[W_{ro} - \sum_{i=1}^{na} \frac{Q_{ai}W_{oi}}{\sum_{i=1}^{na} Q_{ai}}\right] \times \exp\left[-\left(\sum_{i=1}^{na} \frac{Q_{ai}}{V}\right)t\right] \tag{13}$$

Surface condensation

Water vapour in an enclosure will condense on interior surfaces having a temperature less than the saturation temperature of the surrounding air. A cold surface, such as a window, could act as a major moisture sink removing a lot of moisture from the indoor air. The dynamic response of indoor air moisture content to surface condensation is given by:

$$\frac{\mathrm{d}W_r(t)}{\mathrm{d}t} = \frac{h_{mc}A_c}{\rho_a V}\,[W_{sc} - W_r(t)] \tag{14}$$

As condensed water is deposited on a cold surface, the released latent heat will raise its temperature, and consequently, the saturation moisture content, W_{sc} which is a function of the surface temperature will increase. In order to solve eqn. (14), variations of W_{sc} with time must be known. It is difficult to express the surface temperature as a function of time because of the thermal characteristics of windows (the most likely condensation surface in buildings) and the type of boundary conditions which are transient in nature. However, for short periods of time or when used as a part of a numerical technique, eqn. (14) can be solved by assuming constant saturation moisture content during the time interval to give:

$$W_r(t) = W_{sc} + (W_{ro} - W_{sc})\exp\left(-\frac{h_{mc}A_c}{\rho_a V}t\right) \tag{15}$$

In practice, there are likely to be several condensation surfaces in the building enclosure. The condensation behaviour of these surfaces could be different either due to different thermal characteristics or different exterior environmental conditions. For a number of condensation surfaces in an enclosure, indoor air humidity response can be modelled by eqn. (16).

$$W_r(t) = \sum_{i=1}^{nc} \frac{A_{ci}h_{mc}}{\sum_{i=1}^{nc} A_{ci}h_{mc}} W_{sci} + \left[W_{ro} - \sum_{i=1}^{nc} \frac{A_{ci}h_{mc}}{\sum_{i=1}^{nc} A_{ci}h_{mc}} W_{sci}\right] \times \exp\left(-\sum_{i=1}^{nc} \frac{h_{mc}A_{ci}}{\rho_a V}t\right) \tag{16}$$

Indoor evaporation

Water vapour can be added to indoor air by the evaporation process. Evaporating surfaces could be a free water surface or the surface of a wet material or soil. The dynamic response of indoor air humidity to surface evaporation can be expressed as:

$$W_r(t) = W_{se} + (W_{ro} - W_{se}) \exp\left(-\frac{h_{me} A_e}{\rho_a V} t \right) \quad (17)$$

For a number of evaporation surfaces, nv, the dynamic response is given by:

$$\begin{aligned} W_r(t) = & \sum_{i=1}^{nv} \frac{A_{ei} h_{me}}{\sum_{i=1}^{nv} A_{ei} h_{me}} W_{sei} \\ & + \left[W_{ro} - \sum_{i=1}^{nv} \frac{A_{ei} h_{me}}{\sum_{i=1}^{nv} A_{ei} h_{me}} W_{sei} \right] \\ & \times \exp\left(-\sum_{i=1}^{nv} \frac{h_{me} A_{ei}}{\rho_a V} t \right) \quad (18) \end{aligned}$$

Indoor moisture generation

Indoor moisture-generation processes differ from the indoor evaporation processes by being independent of indoor humidity conditions. Substantial amounts of moisture could be released indoors from occupants and moisture-generating operations. The amount and the variational behaviour of indoor moisture generation are dependent on many factors related to the type of indoor operations, the number of occupants, and their activities and habits. Practically, it is difficult to express the rate of indoor moisture generation in terms of time by a continuous function. Instead, a discrete variational profile can be used to describe its variations for each indoor operation and activity.

Assuming that perfect and instantaneous mixing occurs, then for a short period of time, where the moisture generation rate Mg can be assumed constant, the response of indoor humidity to moisture generation is given as:

$$W_r(t) = W_{ro} + \frac{Mg}{\rho_a V} t \quad (19)$$

For a number of moisture-generation operations, the response of indoor air humidity will follow eqn. (20).

$$W_r(t) = W_{ro} + \sum_{i=1}^{ng} \frac{Mg_i}{\rho_a V} t \quad (20)$$

Air humidity dynamic response inside a room

In practice, different combinations of moisture sources and sinks could contribute to air humidity behaviour inside a building enclosure as described by eqn. (1). Two possible general cases will be discussed below.

No absorption or desorption by interior surfaces

When moisture absorption and desorption by interior materials can be neglected, eqn. (1) can be written as:

$$\begin{aligned} \rho_a V \frac{dW_r}{dt} = & \sum_{i=1}^{na} \rho_a Q_{ai} (W_{oi} - W_r) \\ & + \sum_{i=1}^{nc} A_{ci} h_{mc} (W_{sci} - W_r) \\ & + \sum_{i=1}^{nv} A_{ei} h_{me} (W_{sei} - W_r) \\ & + \sum_{i=1}^{ng} Mg_i \quad (21) \end{aligned}$$

The time-dependent parameters describing the moisture transport process in eqn. (21) (i.e., W_o, W_{sc}, W_{se}) experience negligible changes during a relatively small time interval. Therefore, these parameters can be assumed constant during the time interval used in the numerical solution. Equation (21) can be solved to give:

$$W_r(t) = \alpha + (W_{ro} - \alpha) \exp\left(-\frac{t}{t_r} \right) \quad (22)$$

where

$$\begin{aligned} \alpha = \frac{1}{\beta} \Bigg(& \sum_{i=1}^{na} \rho_a Q_{ai} W_{oi} + \sum_{i=1}^{nc} A_{ci} h_{mc} W_{sci} \\ & + \sum_{i=1}^{nv} A_{ei} h_{me} W_{sei} + \sum_{i=1}^{ng} Mg_i \Bigg) \end{aligned}$$

and

$$\beta = \sum_{i=1}^{na} \rho_a Q_{ai} + \sum_{i=1}^{nc} A_{ci} h_{mc} + \sum_{i=1}^{nv} A_{ei} h_{me}$$

Equation (22) describes the air humidity response to different moisture gain and loss processes within an enclosure. The parameter, t_r, is defined as the room air humidity response time. Physically, it describes the rate of response of air humidity to moisture transport by condensation, evaporation, and airflow processes. Each process has a time constant associated with it that defines t_r according to:

$$\frac{1}{t_r} = \frac{1}{t_a} + \frac{1}{t_c} + \frac{1}{t_e} \quad (23)$$

In terms of the influencing parameters of each process, t_r can be written as:

$$\frac{1}{t_r} = \sum_{i=1}^{na} \frac{Q_{ai}}{V} + \sum_{i=1}^{nc} \frac{A_{ci} h_{mc}}{\rho_a V} + \sum_{i=1}^{nv} \frac{A_{ei} h_{me}}{\rho_a V} \tag{24}$$

With absorption and desorption by interior materials

In most cases, moisture absorption and desorption processes within buildings substantially contribute to the behaviour of indoor air humidity. Considering the two types of material–moisture interaction (i.e., $Bi \leqslant 0.1$ and $Bi \gg 0.1$), eqn. (1) can be rewritten as eqn. (25).

$$\begin{aligned} \frac{\mathrm{d}W_r}{\mathrm{d}t} &= \sum_{i=1} [C_i(W_{moi} - W_r)\exp(-B_i t) \\ &\quad + C_i(W_{ms}(t) - W_r)] \\ &\quad + \frac{1}{\rho_a V} \sum_{i=1} [\rho_a Q_{ai}(W_{oi} - W_r) \\ &\quad + A_{ci} h_{mc}(W_{sci} - W_r) \\ &\quad + A_{ei} h_{me}(W_{sei} - W_r) + Mg_i] \end{aligned} \tag{25}$$

The solution of eqn. (25) can be found through numerical techniques. One accurate and practical technique for solving first-order differential equations is the Runge–Kutta method [24]. Since eqn. (25) involves many time-dependent parameters, it can be solved only as a part of a numerical formulation through which the variability of these parameters can be modelled. The material surface moisture conditions, W_{mo} can be calculated at the end of each time step from eqn. (5) or eqn. (8) using the time-step average indoor air humidity W_{ra}, where

$$W_{ra} = \frac{(W_r^{\tau} + W_r^{\tau+1})}{2} \tag{26}$$

Values of the other parameters can also be evaluated at the end of each time interval using the appropriate mathematical modules. For example, when evaluating the amount of surface condensation, the variability of the saturation moisture content, W_{sc}, has to be considered by evaluating the rise in the condensation surface temperature resulting from the released latent heat of water vapour. In order to evaluate the rise in surface temperature, the mass condensation rate and the condensation surface thermal characteristics have to be known. Assuming constant W_{sc}, the amount of mass condensation during a time interval Δt can be evaluated as:

$$M_c = A_c h_{mc}(W_{ra} - W_{sc}) \cdot \Delta t \tag{27}$$

By using numerical techniques, such as the implicit finite-difference formulations, the transient heat transfer process due to surface condensation can be modelled and the surface temperature is evaluated.

Applications and discussions

The response of air humidity in a given enclosure to moisture transport processes can be modelled using the above equations. In order to illustrate the applicability of the proposed model, a case study is used. For a 600 m^3 enclosure, the exact air humidity response to moisture absorption by building materials, eqn. (4), is compared with the solution of eqn. (6) when used as a part of a numerical formulation. Figure 2 shows very good agreement between the two equations when a time step of one hour is used. Full agreement is obtained when the time step is reduced to six minutes as shown in Fig. 3. For practical considerations a time step of one hour is considered satisfactory.

The effect of material area (i.e., material moisture storage capacity) on indoor humidity response is shown in Fig. 4. It can be seen, for this particular material, that increasing the exposed material area by 50 m^2 has resulted in a drop of the space air moisture content of 0.0005 kg/kg, which corresponds to a water vapour loss of about 0.36 kg. For all cases, the major change of air humidity occurs during the first hour. This behaviour is attributed to the material geometrical and moisture characteristics. In the present situation, the material thickness is taken equal to 0.5 mm so as to be modelled using the lumped-parameter analysis. For

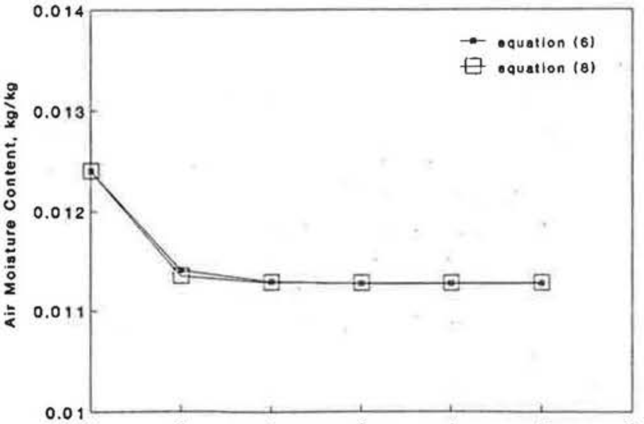

Fig. 2. Response of indoor air humidity to moisture absorption by interior materials using exact and proposed numerical solutions for a time step of one hour.

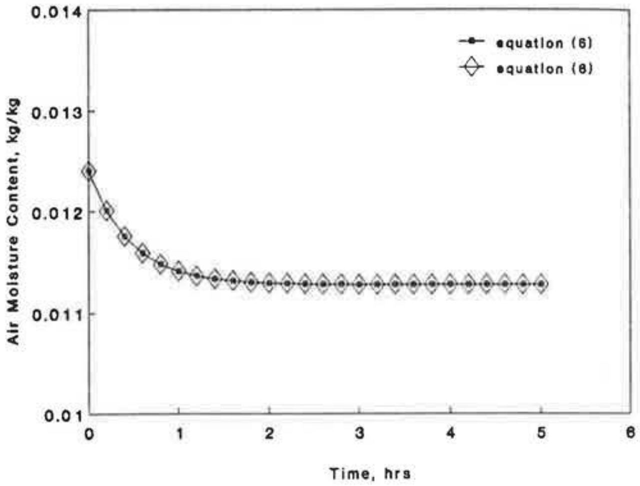

Fig. 3. Response of indoor air humidity to moisture absorption by interior materials using exact and proposed numerical solutions for a time step of 1/10 hour.

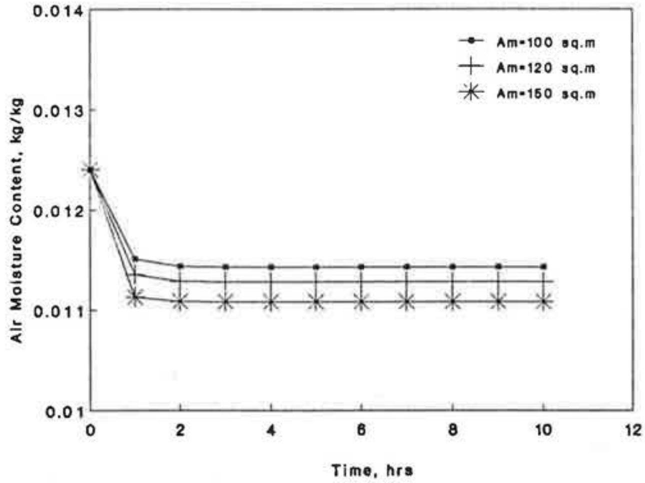

Fig. 4. Response of indoor air humidity to moisture absorption by interior materials at different material surface areas.

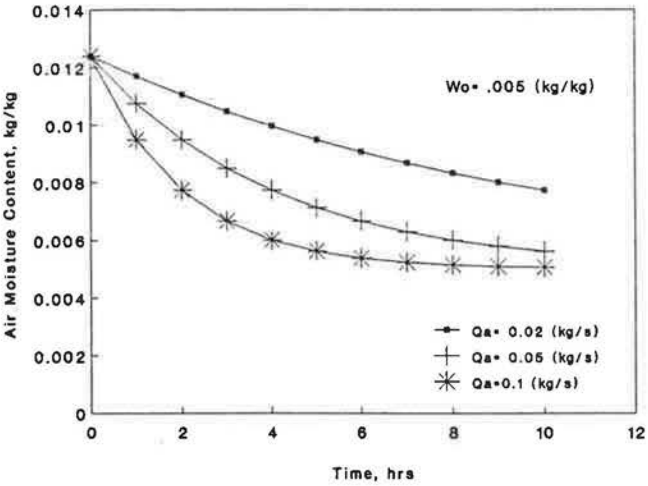

Fig. 5. Response of indoor air humidity to air infiltration at different airflow rates and an outdoor air moisture content of 0.005 kg/kg.

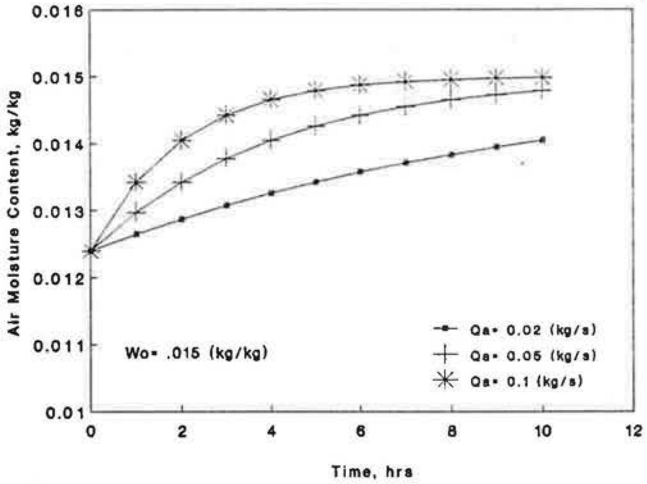

Fig. 6. Response of indoor air humidity to air infiltration at different airflow rates and an outdoor air moisture content of 0.015 kg/kg.

other materials with larger time constants the response is expected to be much slower.

Depending on airflow rate, air infiltration can substantially modify indoor air humidity. Figures 5 and 6 show the air humidity response to different air infiltration rates at different outdoor conditions. Regardless of the outdoor conditions, the response of indoor air is determined by the airflow rate since the time constant of the process is only a function of the flow rate. Decreasing the flow rate from 0.1 to 0.02 kg/s delays the air humidity response fivefold.

The response of air humidity to moisture loss due to condensation on a single glazed window for different surface areas at 0 °C outdoor temperature and 21 °C indoor temperature is shown in Fig. 7. Increasing the condensation area will substantially modify the air humidity behaviour. However, in contrast with the absorption process, shown in Fig. 4, increasing the surface area does not affect the final air humidity conditions, because the material moisture conditions are not affected by the amount of condensed water (i.e., no moisture storage capacity). Instead its moisture interaction is modified by the increase in surface temperature due to latent heat release by condensation. The effect of increased surface temperature on air humidity behaviour is shown in Fig. 8. For this particular case, the increase in surface temperature has little effect on the air humidity behaviour within the space, and the final conditions are unaffected. This is because of the small surface temperature rise experienced, partly due to the falling mass condensation rate and due to the thermal characteristics of the window. How-

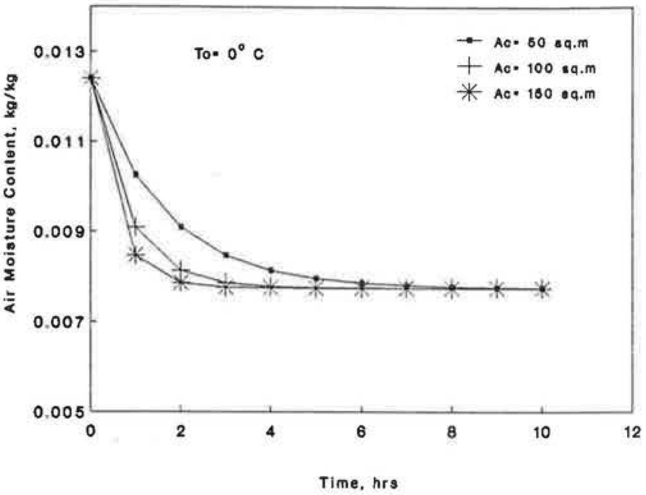

Fig. 7. Response of indoor humidity to surface condensation for different glazing areas of 0 °C outdoor air temperature.

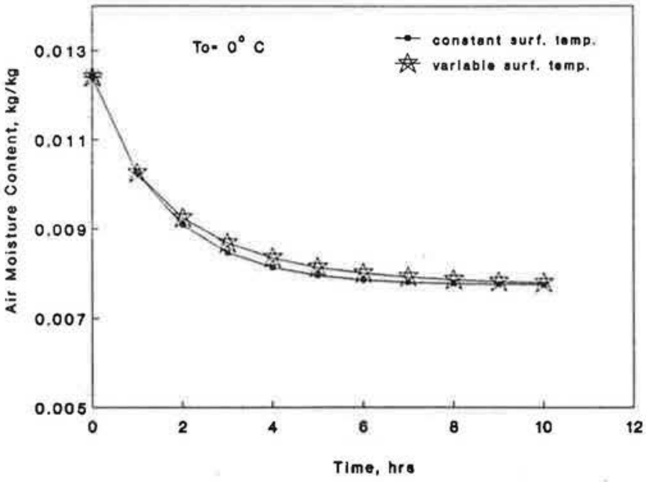

Fig. 8. Effect of temperature rise of the condensation surface on indoor humidity response due to surface condensation.

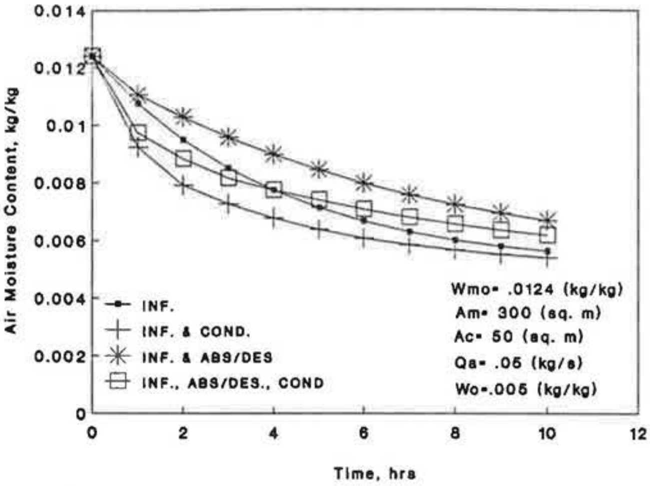

Fig. 9. Response of indoor air humidity to combined moisture transport processes.

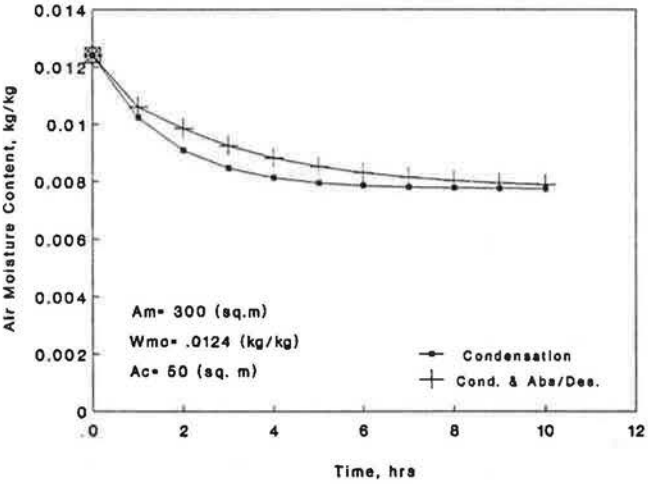

Fig. 10. Response of indoor air humidity to combined moisture transport processes.

ever, for combined moisture transport processes, the condensation process could last for a long time and consequently, the surface temperature rise could be a significant factor in modifying air humidity response. Moreover, as the condensation surface area increases, even a small rise in surface temperature could substantially affect indoor humidity behaviour.

The response of indoor air humidity to combined moisture transport processes is illustrated in Figs. 9 and 10. In the presence of air infiltration, the outdoor air moisture content will be the only determining factor of the final moisture conditions of the indoor air, provided that it is less than the saturation moisture content corresponding to the condensation surface temperature. Moisture loss by surface condensation in the presence of air infiltration leads to a considerable drop in indoor air moisture content in the first few hours. As air moisture content drops below the saturation moisture content corresponding to the condensation surface, air infiltration starts to dominate the moisture behaviour of the indoor air.

Depending on the initial moisture conditions of the interior materials and the influence of other moisture transport processes, moisture absorption and desorption can greatly modify the moisture conditions of the indoor air. From Figs. 9 and 10, it can be seen that moisture released by interior materials has regulated the negative effect of the air infiltration and the condensation processes on indoor air moisture content. Moisture absorption and desorption processes are dependent on indoor air moisture conditions. They become active whenever there is a difference in moisture conditions between the interior materials and the indoor air.

desorption process becomes active and dominates indoor humidity by releasing the stored moisture into the indoor air. The importance of air infiltration, surface condensation, moisture absorption/desorption and indoor moisture-generating processes in controlling the behaviour of indoor air humidity can be clearly appreciated from the above discussions.

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Nomenclature

A_c	condensation area (m^2)
A_e	evaporation area (m^2)
A_m	exposed area of material (m^2)
B_i	Biot number ($-$)
C_m	material moisture capacity (kg/m^2 pa)
D_v	material vapour diffusion coefficient (kg/Pa m s)
h_{mc}	surface mass transfer coefficient for condensation process (kg/m^2 s)
h_{ms}	surface mass transfer coefficient for absorption/desorption processes (kg/m^2 s)
h_{me}	surface mass transfer coefficient for evaporation process (kg/m^2 s)
L_m	moisture penetration depth (m)
ma	mass of water removed from or added to the space by airflow across enclosure boundaries (kg)
mc	mass of water removed by surface condensation (kg)
me	mass of water added by indoor evaporation (kg)
M_g	constant moisture generation rate (kg/s)
mg	mass of water added by indoor moisture generation (kg)
ms	mass of water absorbed or desorbed by interior materials (kg)
na	number of airflow processes
nc	number of condensation processes
ng	number of moisture generation processes
nv	number of evaporation processes
ns	number of absorption/desorption processes
Q_a	airflow rate (m^3/s)
R_v	water vapour characteristic constant (J/kg K)
t	time (s)
T_m	material temperature (K)
V	room volume (m^3)
V_m	material volume (m^3)
W_r	room air moisture content (kg/kg)
W_{ms}	material surface air moisture content (kg/kg)
W_{ro}	initial room air moisture content (kg/kg)
W_{mo}	initial material surface air moisture content (kg/kg)
W_o	outdoor air moisture content (kg/kg)
W_{sc}	saturation air moisture content at condensation surface (kg/kg)
W_{se}	saturation air moisture content at the evaporation surface (kg/kg)
W_{ra}	time-step average indoor air moisture content (kg/kg)
α_m	material vapour diffusivity (m^2/s)
ρ_a	air density (kg/m^3)
$\tau, \tau+1$	denote the start and end of any time interval

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