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Superposition in Infiltration Modeling

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Abstract

Simplified, physical models for calculating infiltration and ventilation in a single zone usually calculate the airflows from the two natural driving forces (i.e., wind and stack effects) separately, and then use a superposition rule to combine them. Similarly, superposition rules may be used to ascertain the effects of mechanical systems on infiltration. In this report a general superposition rule will be derived for combining wind, stack, and mechanical ventilation systems together. The superposition rule will be derived using general principles of leakage distribution and airflow and will not depend on the details of a particular infiltration model. In the process of generating this rule, a quantity called leakage distribution angle is developed to quantify the separation of areas of the building envelope which are subject to infiltration and exfiltration. The general superposition rule is compared to other proposed superposition rules including those based on measured data, and the general rule is shown to have strong explanatory power. Results are generated for typical buildings. The concept of fan addition efficiency is developed to determine the effectiveness of unbalanced (mechanical) ventilation systems at augmenting infiltration.

KEY WORDS:
Infiltration, Modeling, Superposition, Stack, Wind, Fan.

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Introduction

Infiltration is the dominant mechanism for providing ventilation and thus adequate indoor air quality to residential buildings. While most residences have small local exhaust fans for spot ventilation of wet rooms, few – especially in the United States – have a mechanical ventilation system. The estimation of ventilation rates in dwellings, then, becomes the estimation of infiltration-dominated effects.

The calculation of infiltration-dominated ventilation usually requires the combination of wind-induced, temperature-induced, and mechanically-induced airflows. Complex models solve the problem by finding the pressure at each point on the envelope and then solving for the flow – modifying the internal pressure in order to satisfy the continuity equation (Feustel and Raynor Hoosen, 1990). Such an approach is very powerful, but may require inputs and computational requirements that may make it impractical. Furthermore, the structure of the model is often too detailed for the user to understand the physical relationships between parameters. Parametric studies using detailed models (Etheridge and Stanway, 1988), can recover some of the physical relationships, but for many applications simpler physical models are desirable, even if less accurate.

Simplified models have the benefit that they can be readily applied and require far less input data. The price for this ease is that specific details may be lost. It is important to have a physical basis for the assumptions used in deriving simplified models. Although general properties of the three driving mechanisms will be discussed, details such as fan curves, pressure coefficients, leakage distributions, etc. will not. When it is important to understand all such details, simplified models are inappropriate, but for many types of work (such as analysis of large datasets), the level of accuracy of simplified models is sufficient.

In most simple (single-zone) models it is a relatively straightforward problem to calculate the pressure-induced flow for one of the driving forces. Each of these three mechanisms induces pressures across the envelope to drive the flow, but the spatial distribution of the pressure is different for each of them. Although these pressures are additive on a point-by-point basis, the flows induced by those pressures are not. Combining these flows together in a similarly simple way is the topic of this report.

Nomenclature

B	AIM Superposition constant [–]
C'	Shielding coefficient [–]
f_θ	Leakage distribution factor [–]
f	Airflow ratio [–]
g	Acceleration of gravity [m/s^2]
H	(Stack) Height of envelope [m]
$\mathbf{K}$	Leakage coefficient
n	Leakage exponent [–]
P	(Air) pressure [Pa]
ΔP	Driving pressure across the envelope (outside-inside) [Pa]
Q	Airflow [m^3/hr]
v	(Local) wind speed [m/s]
α	Quadrature constant [–]
β	Neutral level (when only stack effect operates) [–]
ε_+	Addition efficiency [–]
$\dot{m}$	Mass flow rate of air [kg/s]
ρ	Density (of air) kg/m^3
θ	Leakage distribution angle [–]

Subscripts indicate values associated with:

+	infiltrating (outside) air
–	exfiltrating (inside) air
i	internal conditions
1,2	any driving force
n	a natural driving force
w	wind effect
≡	wind striking a building face
≢	wind striking diagonally
s	stack effect
f	(unbalanced) fan

Air Leakage

Infiltration is pressure-driven airflow through the envelope of the building, so it is important to understand the leakage properties in order to understand the infiltration. The leakage of the envelope is conventionally treated as a power law (Sherman, 1991). The measurement of leakage is usually performed with a technique called fan pressurization (ASTM, 1987) where the fan flow induces a shift in the internal pressure:

$$Q_f = \mathbf{K}\Delta P_i^n \tag{1}$$

where the exponent ½ $\leq n \leq 1$ depending on the hydrodynamics of the leaks.

As can be seen from the appendix, the fan pressurization flow is the large fan limit of an unbalanced fan. **K** and the exponent, n, characterize the leakage. In addition to being measured from a fan pressurization test, they can be found from more advanced techniques (Sherman and Modera, 1986; Sherman and Modera, 1988). Although the envelope could have different exponents in different areas, we will assume that the exponent does not vary.

The exponent is a particularly important characteristic of the flow for both understanding the behavior and modeling it. If the exponent were unity, the modeling would be linear and relatively simple. For most buildings, however, the exponent is in the range $0.55 \leq n \leq 0.75$ with n = ⅔ being a typical value (Sherman et al., 1986), and the modeling becomes more complex. For this typical value of exponent, however, **K** becomes independent of temperature in normal situations (Sherman, 1991).

Combining Driving Forces

Each one of the three driving forces has a particular pattern of pressures induced across the envelope. From this pattern of pressures and the flow characteristics of Equation 1, the airflow from that driving force can be calculated, albeit differently for different models. Since combining the different driving forces on a point-by-point basis is more complex than the simple models can deal with, a superposition rule that combines the individually calculated flows in a simple but robust way is needed.

When combining flows from different driving forces it is important to understand how the pattern of pressures interact. The external pressure pattern may vary only with height (as in the stack effect) only with orientation (as in the wind effect) or not at all (as in a mechanical system); the internal pressure may or may not need to change in response to these external patterns. The correlation of these external patterns and internal pressure changes will determine how the driving pressures superpose.

If the pressure patterns did not interact, superposition would simply be to add the flow from the natural sources to either the supply or the exhaust flow, algebraically. This pressure-independent situation occurs only for the case of balanced supply and exhaust fans (e.g., an air-to-air heat exchanger).

$$Q = Q_{balanced} + f(Q_s, Q_w, Q_f) \tag{2}$$

If there are both supply and exhaust fans operating simultaneously, the balanced part of the flow can be expressed as follows:

$$Q_{balanced} \equiv MIN\ (Q_{supply}, Q_{exhaust}) \tag{3}$$

For any unbalanced combination of supply and exhaust fans,

$$Q_f \equiv |Q_{exhaust} - Q_{supply}| \tag{4}$$

there will be a shift in the internal pressure and hence the pressure pattern (as is explained in the appendix).

If the pressure patterns were exactly correlated (e.g., two exhaust fans or a doubling of the wind speed), all driving pressures would increase in proportion. In this case the driving pressures are additive, but because the exponent is less than unity, the flows are sub-additive. Any time the pressure patterns are independent there will be locations in which positive and negative pressures mitigate one another, thus reducing the combined flow from the algebraic sum. Accordingly, a sub-additive superposition rule is required to account for these physical effects.

General Quadrature

For naturally-induced infiltration (i.e., stack and wind effects) different areas of the envelope will see positive pressures and negative pressures. Since the exponent is never greater than unity, we can be assured that any combination of natural and (unbalanced) fan flows will be subadditive. Thus, an expression (analogous to the law of cosines) can be used to combine two flows:

$$Q^2 = Q_1^2 + Q_2^2 - \alpha\, Q_1 Q_2 \tag{5}$$

where $|\alpha| \leq 2$. We call this rule *general quadrature.*

This rule encompasses all the physical situations, but the quadrature constant, α is not a universal constant; it depends on many of the details of the individual flows – specifically on the patterns. For specific physical realizations it is possible to develop an exact superposition formula. This level of detail, however, is not in keeping with the simplified models used to calculate the individual term. Although Equation 5 does not necessarily represent any particular physical realization, it has the advantage of simplicity, robustness, and symmetry.

It is instructive to examine some special cases. The limits of this expression are to be found when combining flows that do not change the pressure pattern:

- combining two (constant flow) exhaust fans yields $\alpha = -2$.
- combining a supply and exhaust fans yields $\alpha = 2$.
- combining a balanced supply/exhaust system with a natural driving force yields $\alpha = -2$.

Combinations that affect the pressure pattern will yield intermediate results. Although for simplistic cases (e.g., two leaks in various configurations) the quadrature constant can be solved analytically, simplified methods (see appendix) will be required for realistic cases. Before deriving specific values for the quadrature constant, a review of previous work is in order.

Review of Superpostion Methods

One of the first simplified physical models of infiltration, the LBL model, (Sherman and Grimsrud, 1980; Sherman, 1980), used the following superposition rule:

$$Q^2 = Q_1^2 + Q_2^2 \tag{6}$$

This superposition rule is called *"simple quadrature"* or *"LBL superposition"*.

Using measured data, Wilson and Pittman (1983) have shown that this type of model captures much of the physical behavior. Using measurement and simulation for full-scale test structure, Mobile Infiltration Test Unit (**MITU**), Modera et al. (1983) have shown that there can be an overprediction error on the order of 25% when the wind and stack effects are equal.

The LBL infiltration model assumes orifice flow and thus fixes the leakage exponent at one half. Various other efforts have attempted to generalize the model by using the measured exponent (Liddament and Allen, 1983). In some of these Variable Flow Exponent (**VFE**) models an "exponified" version of simple quadrature is used to generalize the superposition rule (Reardon, 1989):

$$Q^{1/n} = Q_1^{1/n} + Q_2^{1/n} \tag{7}$$

We call this superposition rule *"VFE superposition"*.

Using a detailed simulation, Modera and Peterson (1985) have investigated both LBL and VFE superposition for the combination of wind effects and stack effects with and without the operation of mechanical exhaust. The specific example cited uses the configuration of MITU. They found that, in general, simple quadrature works better than the exponified version and both may overpredict the total when there is no fan operation. Further, the deviation in simple quadrature is found to be a strong function of leakage distribution.

In order to mitigate the overprediction of the VFE superposition rule, the NRC model (Reardon, 1989; Shaw, 1981) uses an ad-hoc correction factor:

$$Q = [0.8f_n^{-0.1}]\,(Q_s^{1/n} + Q_w^{1/n})^n \tag{8}$$

where:

$$f_n \equiv \frac{Q_{smaller}}{Q_{larger}} \leq 1 \tag{9}$$

When f_n becomes small enough ($\approx$ 0.1) the term in brackets is replaced by unity. This correction, therefore, always reduces the value relative to VFE superposition and has the greatest effect ($\approx$ 20%) when the two flows are equal (i.e., $f_n \approx 1$). We call this rule *"NRC superposition"*.

Walker and Wilson (1990) modify VFE superposition in an algebraically simpler method:

$$Q^{1/n} = Q_s^{1/n} + Q_w^{1/n} + B\,Q_w^{1/2n}Q_s^{1/2n} \tag{10.1}$$

From their data they have found that

$$B \approx -\frac{1}{3} \tag{10.2}$$

We call this rule *"AIM superposition"*.

Figure 1 summarizes all of these superposition methods by comparing them to simple quadrature (i.e., $\alpha = 0$) as a function of the airflow ratio, f. Simple quadrature is not necessarily the correct rule to use but provides a reference point to compare trends. For example, it can be noted that the AIM superposition rule does not deviate very much from simple quadrature, but the VFE rule always predicts a larger flow. The physical limits of $\alpha = \pm 2$ as well as $\alpha = \pm 1$ are plotted for reference.

For each of the superposition rules described above, Equation 5 can be used to derive a value of the quadrature constant. Because these rules are all symmetric with respect to wind and stack, the quadrature constant will depend only on the ratio of the smaller flow to the larger one. Table 1 displays these data for two values of the airflow ratio.

Table 1 Equivalent values of quadrature constant (α).

(Combined stack and wind at $n = 2/3$)					
f_n	*VFE*	*LBL*	*AIM*	*NRC*	*MITU*
1	-0.52	0	0.02	0.39	0.61
½	-0.49	0	0.08	0.31	0.43

In the appendix we derive some simplified expressions for the quadrature constant. The section below summarizes these results and allows us to estimate numerical values as well as the strongest functional dependencies.

Infiltration-dominated Ventilation

The process of infiltration derives from pressure interactions across the building envelope. The distribution of these pressures will depend on the specifics of the driving forces. Three quantities characterize the infiltration from a single driving force: the driving pressure, the total envelope leakage, and the leakage distribution relative to the driving pressure. In order to describe the effects of the leakage distribution, we have introduced the *leakage distribution angle*. The leakage distribution angle quantifies the partitioning between the areas of the envelope that infiltrate and exfiltrate. The importance of the leakage distribution angle is that it is a single quantity that (non-dimensionally) quantifies the pattern of the driving pressure in a way where trigonometric identities are useful.

The mathematical derivation of a generic simplified physical model for infiltration, leakage distribution, and addition efficiencies are all contained in the appendix and will not be repeated in the body of this report. The first part of the appendix leads to the definition of the leakage distribution angle and calculates it for the three driving forces in typical conditions. The results are summarized in Table 2.

Table 2 Properties of driving forces.

Driving force	*Distribution angle*	*Typical pressure*
Stack (winter)	$\cos^2\theta_s = \beta$	$\lvert\Delta\rho\rvert gH/2$
Stack (summer)	$\sin^2\theta_s = \beta$	$\lvert\Delta\rho\rvert gH/2$
Wind (head-on)	$\tan\theta_= = .44$	$C'\frac{1}{2}\rho v^2$
Wind (diagonal)	$\tan\theta_{\neq} = 1$	$C'\frac{1}{2}\rho v^2$
Fan (supply)	$\cos 2\theta_f = 1$	$(Q_f/K)^{1/n}$
Fan (exhaust)	$\cos 2\theta_f = -1$	$(Q_f/K)^{1/n}$

The pressures are shown for information only and represent the typical pressure drop across a leakage site. Their exact calculation depends on the details of the infiltration model used and does not materially affect the superposition. Care was taken to carry the non-linearities (associated with the exponent) through the formulation. Although it appears in expressions for flows and leakage distribution angles, the exponent has very little effect on the value of the addition efficiencies and hence the superposition.

When two forces are acting together, a perturbation analysis can be used to estimate the interaction and derive an addition efficiency for the effect of the smaller force:

$$Q = Q_1 + \varepsilon_+ Q_2 \qquad (11)$$

where

$$\varepsilon_+ \leq 1 \text{ and } Q_2 < Q_1 \qquad (12)$$

The last parts of the appendix deal with the flow addition of two driving forces. The result of this derivation is an expression for the addition efficiency. As shown in Equation A29, the addition efficiency can be expressed in terms of the leakage distribution angles of the two driving forces:

$$\varepsilon_+ = \left| \frac{\cos 2\theta_1 + \cos 2\theta_2}{2} \right| \qquad (13)$$

These factors can be used for fan flows larger than the naturally-induced flow by applying a minimum value of the efficiency to convert ε_+ to ε_f.

Table 3 lists the addition efficiencies for different combinations of wind, stack, and (unbalanced) fan flow.

Unfortunately, these addition efficiencies are least precise when the two flows are of the same size (i.e., $f_n \approx 1$), because this is the regime in which the flow is most sensitive to the details of the leakage. Within the context of simple models, however, such uncertainty must be accepted. Since typical applications of simple models involve large datasets, the overall prediction using this superposition rule will be robust.

Quadrature Constant

In the form of Equation 13 the expressions do not directly relate to quadrature, but we can put them into such a form by squaring the expression:

Table 3 Addition efficiencies.

$\text{Wind}_{\neq}$Effect + fan:	$\varepsilon_+ = 0.5$
$\text{Wind}_{\equiv}$Effect + Exhaust fan:	$\varepsilon_+ = 0.16$
$\text{Wind}_{\equiv}$Effect + Supply fan:	$\varepsilon_+ = 0.84$
Stack effect(winter) + Exhaust fan:	$\varepsilon_+ = 1\text{-}\beta$
Stack effect(winter) + Supply fan:	$\varepsilon_+ = \beta$
Stack effect(summer) + Exhaust fan:	$\varepsilon_+ = \beta$
Stack effect(summer) + Supply fan:	$\varepsilon_+ = 1\text{-}\beta$
Stack + $\text{Wind}_{\neq}$Effects:	$\varepsilon_+ = \lvert\beta\text{-}.5\rvert$
Stack(winter) + $\text{Wind}_{\equiv}$Effects:	$\varepsilon_+ = \lvert\beta\text{-}.16\rvert$
Stack(summer) + $\text{Wind}_{\equiv}$Effects:	$\varepsilon_+ = \lvert.84\text{-}\beta\rvert$

$n = 2/3$ was used for the head-on wind effect, $\text{Wind}_{\equiv}$.

$$Q^2 = Q_1^2 + 2\varepsilon_+ Q_2 Q_1 + \varepsilon_+^2 Q_2^2 \qquad (14)$$

or, equivalently,

$$Q^2 = Q_1^2 + Q_2^2 + (2\varepsilon_+ + (\varepsilon_+^2 - 1)f_n)\, Q_1 Q_2 \qquad (15)$$

Comparing this to Equation 5 and solving for α we obtain the following:

$$\alpha = f_n (1 - \varepsilon_+^2) - 2\varepsilon_+ \qquad (16)$$

for a combination of stack and wind.

These expressions also apply to natural flows plus a small fan. If the fan becomes larger than the natural flow, however, f_n must be replaced by f_f and ε_+ by ε_f, in order to account for fan domination (see Equations A34-A36 in the appendix).

We can use Equation 16 to derive numerical values of α for a few combinations of driving forces in Table 4. Table 4a combines the wind and the winter stack effect. Because Equation 13 is symmetric with respect to the two leakage distribution angles, Table 4a (like Table 1) does not differ depending on which natural driving force is greater, where β is the (dimensionless) neutral level.

In these tables we have assumed that $n = 2/3$, (which is important only for the head-on wind effect), and that winter conditions prevail. If the outside temperature is higher than the inside, the $\beta = 3/4$ and $\beta = 1/4$ columns should be interchanged. Table 4b contains the combination of the winter stack effect with fans. Note that for large fan flows the quadrature constant becomes equal to the inverse of the the fan efficiency factor (i.e., $\alpha \rightarrow 1/f_f$). Table 4c combines the wind with fans. The combination of all three driving forces can be done using pairwise combinations, for a restricted set of assumptions.

Table 4a Typical values of α.

Combined wind and (winter) stack effects				
	f_n	$\beta = 3/4$	$\beta = 1/2$	$\beta = 1/4$
$\text{Wind}_{=}$ + Stack	1	-0.53	0.20	0.81
	½	-0.85	-0.24	0.32
$\text{Wind}_{\neq}$ + Stack	1	0.44	1.00	0.44
	½	-0.33	0.50	-0.03

Table 4b Typical values of α.

Combined (winter) stack and fan effects			
		SUPPLY + STACK	
f_f	$\beta = 3/4$	$\beta = 1/2$	$\beta = 1/4$
2	-0.63	0.5	0.5
1	-1.06	-0.25	0.44
½	-1.28	-0.63	-0.03
f_f	$\beta = 1/4$	$\beta = 1/2$	$\beta = 3/4$
		EXHAUST + STACK	

Table 4c Typical values of α.

Combined wind and fan effects			
	$\text{WIND}_{=}$		*$\text{WIND}_{\neq}$*
f_f	*Exhaust fans*	*Supply fans*	
2	0.5	-1.09	0.5
1	0.65	-1.38	-0.25
½	0.16	-1.53	-0.63

Discussion

It is instructive to compare the values of the quadrature constant calculated here, with the equivalent values of α from the literature, which predominantly focus on combining wind and stack effects. Thus we can compare the entries in Table 4a with the reported values from Table 1. There are two entries in Table 1 that are based on detailed measurements, the AIM dataset (Walker and Wilson, 1990), or detailed simulations, the MITU dataset (Modera et al., 1983 and Modera and Peterson, 1985), of specific buildings. Our derivation may have some explanatory power for these entries.

The test houses in the AIM dataset are closely set in a row; thus, the wind effect could contribute only when the wind impacted directly (i.e., only the $\text{wind}_{=}$ entries apply). Further, these houses had little low leakage, but did have some high leakage; thus we would expect the neutral level to be in the range $1/2 \leq \beta < 3/4$. Thus the first two entries on the first line of Table 4a would be expected to (and do) bracket the AIM entry in Table 1. For this configuration the quadrature constant is consistent with zero.

The configuration for the MITU dataset was somewhat different. MITU was unshielded and completely exposed to the wind, whose speed and direction varied (i.e., both the $\text{wind}_{=}$ and $\text{wind}_{\neq}$ entries apply). Further, MITU had no high leakage, but had significant floor leakage into a crawlspace; thus we would expect the neutral level to be in the range $1/4 < \beta < 1/2$. Thus the right two columns of Table 4a should best bracket the results for this dataset in Table 1. Both the calculations and data are consistent with a value of α for the MITU dataset of approximately one half.

Figure 2 displays our predictions (vs. simple quadrature) along with the other superposition methods and the MITU data points in the same manner as Figure 1. Although the curves bend in different directions, it can be seen that there is general agreement between the MITU dataset and a low neutral level prediction, and that the AIM model (and simple quadrature) is consistent with a higher neutral level. It is important in examining Figures 1 and 2 (and 3) to remember that the modeling assumptions break down in the vicinity of $f = 1$ and the truth becomes highly dependent on the details of the structure. While the model may be expected to work in a general way in this regime, large variability should be expected.

Comparing Tables 1 and 4a again, as well as Figure 2, it is clear that VFE superposition is consistent with our model only if the neutral level is quite high and the wind is head on. As these conditions are not typical, it is not surprising that the literature finds that such a superposition model overpredicts. Similarly, NRC superposition is consistent with either wind that strikes primarily on the diagonal or a low neutral level. The literature, however, does not contain enough details to carry this comparison further.

It is clear that an optimum value for the quadrature constant depends on the distribution of leakage and wind angle. Values in the range $-1 < \alpha < 1$ are not unreasonable. Often we do not have enough specific information about a structure to estimate the quadrature constant and it would be useful therefore to have a default value. If we assume that the default house has a slightly high neutral level, that we are interested in non-summer conditions and a majority of the wind effect comes from wind impinging directly on a surface, then simple quadrature (i.e., $\alpha = 0$) is a good default. (For summer conditions $\alpha = 1/2$ might be a better assumption.)

Addition of Fans

The discussion has focused so far on the combination of wind and stack effects. Indeed, this has been the area of most interest over the last decade. As mechanical ventilation becomes a more important component in residential buildings, the need to accurately include the effects of fans increases.

Figure 3 displays the results of our fan addition modeling referenced against simple quadrature. Because fan addition efficiencies allow for fan flows larger than the natural infiltration, the airflow ratio extends to higher values. Each curve has a cusp at which point the fan completely dominates the ventilation and the curve changes shape. It is clear from the shape of the curves that quadrature is not a particularly good method of representing the effect of fans. Tables 4b and 4c contain the calculated values of the quadrature constant for the case when a (supply or exhaust) fan is added to either wind or stack flow. Since, however, quadrature does not capture the physical dependencies well, it is better to use the fan efficiencies directly.

An examination of the fan addition efficiencies in Table 3 leads one to the conclusion that in general (small, unbalanced) fans contribute approximately 50% of their actual flow rate towards increasing the total ventilation. Such efficiencies must be considered when making either energy or indoor air quality calculations. Furthermore, if we can assume this 50% rule in general, then we can combine all three driving forces easily.

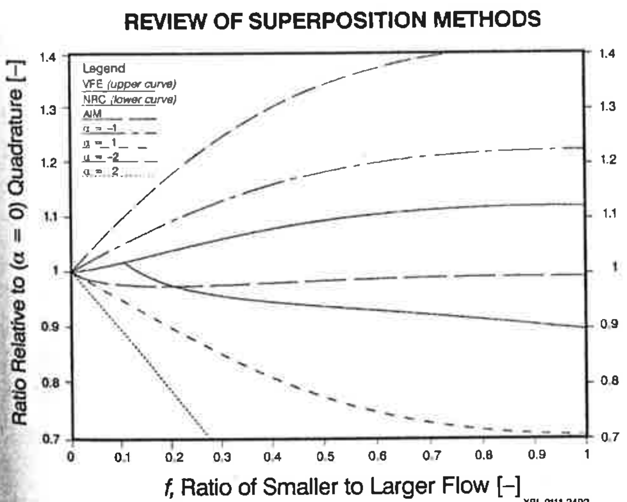

Fig. 1. A plot of the superposition rules from the literature against simple quadrature as a function of airflow ratio. Curves for values of the quadrature constant of -2,-1,1,2 are also plotted. (α = 0 would be a horizontal line at unity.)

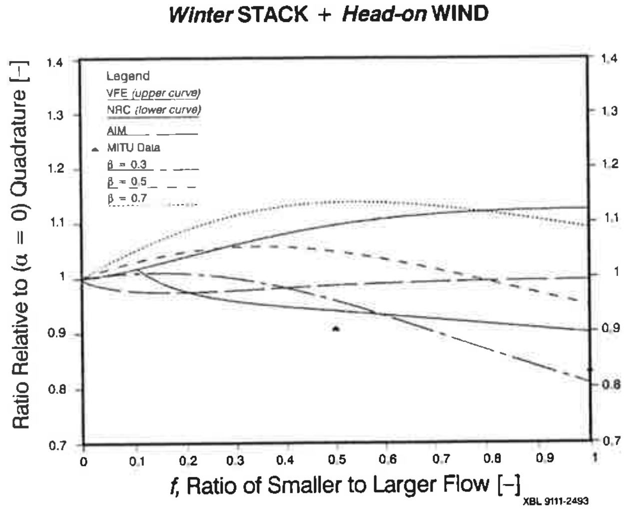

Fig. 2. A plot of the superposition rules against simple quadrature, including curves generated for three different envelope conditions. Two measured data points from the Mobile Infiltration Test Unit are also plotted. (Exchange .3 and .7 curves for summer conditions.)

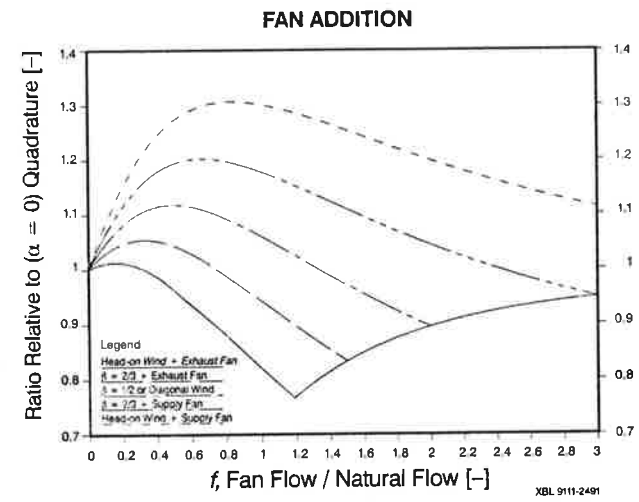

Fig. 3. A plot of fan addition vs. simple quadrature for different wind, stack and fan situations. For the stack effect it is assumed to be winter. (Change 2/3 to 1/3 in label for summer conditions or to interchange exhaust and supply.)

A comparison of simple quadrature and the fan efficiency shows that they never deviate by more than 10% of the total infiltration. Therefore, when combining natural (i.e., any combination of wind and stack) and fan-induced infiltration, simple quadrature is a good approximation, for the special case of the 50% rule.

The 50% rule is good overall, but there are deviations depending on the distributions and which natural force dominates. Some of the important deviations can be summarized as follows:

- *When the wind dominates, supply fans have a larger addition efficiency than exhaust fans.* The differential becomes larger as more of the wind strikes directly on a face. This effect may be especially important during shoulder seasons in which a small ventilation fan is being utilized for indoor air quality purposes.
- *For high neutral level houses, when the winter stack effect dominates, supply fans have a larger addition efficiency than exhaust fans.* This effect implies that for a house with many ceiling penetrations such as kitchen and bathroom exhausts, there may be less impact on total ventilation from running these fans during the winter than was thought; thus they do not work well as whole-house ventilators. It is interesting to note, however, that as spot extract ventilators (e.g., bathrooms and stoves), they are effective and particularly energy-efficient.
- *For high neutral level houses, when the summer stack effect dominates, supply fans have a smaller addition efficiency than exhaust fans.* This effect may be important for the slab-on-grade house typical of the sun-belt of the United States and is the converse of the previous one.
- *For large fan flows the total airflow through the envelope becomes equal to the fan flow.* As the fan dominates the infiltration, the fan addition efficiency increases asymptotically to unity.

Conclusions

The concept of leakage distribution angle as developed in this report is important to the understanding of how the forces that drive infiltration interact. Although there are many details about the leakage distribution that can impact the resulting airflows, the majority of effects can be described by this single parameter. The leakage distribution angle quantifies the partitioning between the areas of the envelope that infiltrate and exfiltrate. Since the key factor in superposition is the separation of canceling and augmenting pressures, this partitioning allows a more fundamental description of the superposition process.

An examination of how pressure and leakage distributions interact across the envelope of a building has allowed us to develop some general guidelines for the superposition of stack, wind and fan effects without detailed modeling assumptions. We can summarize the work of the report in a single superposition equation which takes into account all three of these forces:

$$Q = \sqrt{Q_s^2 + Q_w^2 - \alpha Q_s Q_w} + Q_{balanced} + \varepsilon_f Q_f \qquad (17)$$

The quadrature constant, α, depends on the leakage and wind angle distributions as well as the sign of the temperature difference. For any reasonable configuration

$$-1 < \alpha < 1. \qquad (18)$$

When little is known about the details of the building, a default value of zero can be used; if some information is known, however, the estimate can be improved using the methods developed herein.

The fan addition efficiency, ε_f, indicates the contribution an unbalanced mechanical ventilation system has on the total ventilation:

$$0 < \varepsilon_f \leq 1 \qquad (19)$$

When little is known about the details of the system, the 50% rule (i.e., a value of one half subject to an overall minimum of the fan flow) can be used as default. As this effect may depend strongly on the season and whether it is a supply or exhaust fan, care should be taken when estimating the impact of a particular mechanical system on the ventilation rate.

When both default values are used the superposition rule becomes the following[a]:

$$Q = \frac{Q_{balanced}}{2} + \frac{Q_{exhaust}}{2} + \sqrt{Q_s^2 + Q_w^2} \qquad (20.1)$$

alternatively, the following may also be used[1]:

$$Q = Q_{balanced} + \sqrt{Q_s^2 + Q_w^2 + Q_f^2} \qquad (20.2)$$

Without information on the neutral level, the wind direction and the relative dominance of wind and stack effects, this expression is the best general rule of superposition for infiltration-dominated ventilation.

Although these expressions do not explicitly contain the exponent, the exponent was considered in their derivation. The value of the exponent has little to do with the form or result of the superposition

[1]When Modera and Peterson simulated the MITU configuration, they found the same result. As can be seen from Table 4, this was caused by a cancellation of errors for low neutral level in the winter with an exhaust fan. Their result does, however, indicate that this expression may be useful in some circumstances when the default assumptions are not met.

equation, save through its effect on the leakage distribution (angle). Under special circumstances the effect of the exponent on the superposition can be quite significant, but as the numerical impact of this effect is in general small, the exponent is not of critical importance to the issue of superposition. Superposition notwithstanding, the exponent has an appreciable impact on the individual flows. Future work will investigate the extension of the leakage-distribution-angle concept for the calculation of the individual flows and the change in the leakage distribution angle as two forces operate.

Appendix

Generalized Calculation of Infiltration

For natural driving pressures such as wind and stack some of the building envelope will be under negative pressure and some of the building envelope will be under positive pressure. We can conceptually simplify the formulation by combining together all of the positive pressures into a single value (and similarly for the negative ones) without having to know the details. This assumption is justifiable as long as the variation in leakage does not correlate with the pressures driving it. Violations of this assumption include leaks which themselves modify the pressures or leaks which change behavior because of the induced pressure. Incidental correlations can affect the result for a particular site at a particular time, but are unlikely to invalidate this assumption in the typical case.

This assumption is conceptually equivalent to having two holes (one for infiltration and one for exfiltration) whose combined leakage is equal to the actual leakage, and whose external pressures are the positive and negative ones accordingly. The simplified formulations can be written as follows:

$$Q_{\pm} = \mathbf{K}_{\pm}(\Delta P_{\pm})^{n} \quad \text{(A1)}$$

where the subscripts refer either to infiltration (positive pressures) or exfiltration (negative pressures), and

$$\mathbf{K} = \mathbf{K}_{+} + \mathbf{K}_{-} \quad \text{(A2.1)}$$

$$\Delta P_{+} = \mathrm{P}_{+} - \mathrm{P}_{i} \quad \text{(A2.2)}$$

$$\Delta P_{-} = \mathrm{P}_{i} - \mathrm{P}_{-} \quad \text{(A2.3)}$$

To preserve the mass balance the mass flow of exfiltrating air must be the same as that for infiltrating air

$$\dot{m} = \rho_{+}Q_{+} = \rho_{-}Q_{-} \quad \text{(A3)}$$

so that the internal pressure must be

$$P_{i} = \frac{(\rho_{+}\mathbf{K}_{+})^{1/n}P_{+} + (\rho_{-}\mathbf{K}_{-})^{1/n}P_{-}}{(\rho_{+}\mathbf{K}_{+})^{1/n} + (\rho_{-}\mathbf{K}_{-})^{1/n}} \quad \text{(A4)}$$

The infiltration can be rewritten as

$$\dot{m} = \frac{\rho_{+}\mathbf{K}_{+}\,\rho_{-}\mathbf{K}_{-}}{(\rho_{+}^{1/n}\mathbf{K}_{+}^{1/n} + \rho_{-}^{1/n}\mathbf{K}_{-}^{1/n})^{n}}\Delta P^{n} \quad \text{(A5)}$$

where

$$\Delta P \equiv P_{+} - P_{-} \quad \text{(A6)}$$

Leakage Distribution Angle

The equations in the previous section are suggestive of trigonometric identities and can be simplified by the introduction of the leakage distribution angle, which is defined as follows:

$$\tan\theta \equiv \left(\frac{\rho_{+}\mathbf{K}_{+}}{\rho_{-}\mathbf{K}_{-}}\right)^{1/2n} \quad \text{(A7)}$$

The leakage distribution angle is defined only for the first quadrant.

We can now rewrite the equations from the previous section using the leakage distribution angle to eliminate $\mathbf{K}_{\pm}$ in the pressure relationships. The internal pressure is the weighted sum of the driving pressures:

$$P_{i} = \sin^{2}\theta P_{+} + \cos^{2}\theta P_{-} \quad \text{(A8)}$$

(which is equal to ΔP_i if we set the reference of pressure to zero) and the pressure drops across the envelope are

$$\Delta P_{+} = \cos^{2}\theta\Delta P \quad \text{(A9.1)}$$

$$\Delta P_{-} = \sin^{2}\theta\Delta P \quad \text{(A9.2)}$$

We can use Equation A7 to eliminate $\mathbf{K}_{\pm}$ from Equation A5 in favor of the total leakage and the leakage distribution angle:

$$\dot{m} = \mathbf{K}\,\frac{\rho_+\rho_-(\Delta P \sin^2\theta \cos^2\theta)^n}{\rho_+ \cos^{2n}\theta + \rho_- \sin^{2n}\theta} \qquad \text{(A10)}$$

Since it is conventional to express infiltration in volumetric terms (and since Sherman (1991) demonstrates that the volumetric leakage is insensitive to the density in the leaks), we seek to separate the mass flow into an effective density and a volumetric flow so that

$$\dot{m} = \rho_0\, Q \qquad \text{(A11)}$$

If we assume that this volumetric flow would occur if there were no expansion or contraction of the air when crossing the envelope, then

$$Q = \mathbf{K}\,\frac{\sin^{2n}\theta \cos^{2n}\theta}{\sin^{2n}\theta + \cos^{2n}\theta}\,\Delta P^n \qquad \text{(A12)}$$

and

$$\rho_0 = \frac{\rho_+\rho_-(\sin^{2n}\theta + \cos^{2n}\theta)}{\rho_+\cos^{2n}\theta + \rho_-\sin^{2n}\theta} \qquad \text{(A13)}$$

For the remainder of this appendix and in the body of the paper we have used this volumetric flow formulation. It is important, however, to remember to use the proper air density (Equation A13) to calculate the mass flow. The errors in not doing so can be important if the density (i.e., temperature) difference is significant between inside and outside.

We began this derivation by treating the process as equivalent to a two leak envelope. We can put Equation A12 in a form that preserves this sense as follows:

$$Q = \frac{\mathbf{K}}{2}\, f_\theta \left(\frac{\Delta P}{2}\right)^n \qquad \text{(A14)}$$

where

$$f_\theta \equiv \frac{2^{1+n}\sin^{2n}\theta\cos^{2n}\theta}{\sin^{2n}\theta + \cos^{2n}\theta} \qquad \text{(A15)}$$

f_θ is a function that incorporates all of the leakage distribution effects for a single driving force and goes from a maximum of unity (when the leakage is completely balanced) towards a minimum of zero (when the leakage is completely unbalanced). When the leakage is balanced and the factor is unity, this equation is just that for two leaks (of leakage $\mathbf{K}/2$) being driven by a total pressure drop of ΔP (i.e., $\Delta P/2$ across each one).

Individual Driving Forces

These expressions are applicable to any set of driving forces that operate on the pressure fields. This section solves these equations, in a simplified manner, for typical cases of the driving forces operating alone. The intent of this section is not to solve each case in detail, but rather to determine their leakage distribution angles for use in superposition.

Stack Effect

If the densities of two bodies of air are different, there will be a gravity-induced pressure gradient between them. In buildings this density difference is caused by temperature differences and is known as the *stack effect.* We can approximate the pressure drop and leakage distribution angle for the stack effect as follows:

$$\Delta P_s \approx |\Delta\rho|\, gH \qquad \text{(A16)}$$

where H is the stack height of the building. The exact formulation of this pressure difference depends on leakage distribution and is beyond the scope of this report; the stack height, however, is on the order of the height of the building.

One simple approximation uses the (dimensionless) neutral level of building:

If the inside temperature is greater than outside:

$$\cos^2\theta_s \approx \beta \qquad \text{(A17.1)}$$

If the inside temperature is less than outside:

$$\sin^2\theta_s \approx \beta \qquad \text{(A17.2)}$$

The neutral level, β, is that height (divided by the height of the building) at which the inside and outside pressures are equal when only the stack effect is in operation.

EXAMPLE: As an example, assume that all of the leakage is at or near the floor and ceiling and that there is twice as much high leakage as low leakage. If the floor-ceiling height is H and the inside is warmer than the outside,

$$\cos^2\theta_s = \beta = \frac{2^{1/n}}{1+2^{1/n}} \tag{A18.1}$$

$$Q_s = \mathbf{K}\left\{\frac{1}{3}\ (1 + \frac{1}{2}^{1/n})^{-n}\right\}\left|\Delta\rho g H\right|^n \tag{A18.2}$$

where for typical values of exponent (i.e., $n = 2/3$) the term in curly brackets is approximately 0.27.

Wind Effect

The wind effect acts by causing different pressure shifts on the faces of the structure. We can approximate the pressure drop and leakage distribution angle for the wind effect as follows:

$$\Delta P_w = C'\rho + v^2 \tag{A19}$$

where C' is the shielding coefficient of the structure. This coefficient is less than unity and becomes smaller as the local shielding increases.

Both the shielding coefficient and the leakage distribution angle will be a function of wind direction, building aspect ratio, and leakage distribution, and will not be developed herein.

The inside pressure coefficient normally is about –0.2 for evenly distributed leakage (Allen, 1983), but may be anywhere between –1 and 1.

Depending on the exposure of the building, there can be a different number of faces (and hence leaks) seeing positive pressure. For an isolated structure the two prototypical orientations are for the wind to strike one wall (while the other three see suction) or for the wind to strike diagonally, pressurizing half of the walls. Most conditions will be bracketed by these two situations, but other geometries exist and the leakage distribution angle may have to be recalculated for them.

Head-On Wind

In the typical case in which the wind strikes one of the faces of the building head-on, we assume there is no airflow through the floor or ceiling, but all four walls have the same leakage.

$$\tan\theta_{\equiv} = \left(\frac{1}{3}\right)^{\frac{1}{2n}} \tag{A20}$$

which for the typical value of the exponent (i.e., n = 2/3)

$$\cos 2\theta_{\equiv} = 0.68 \tag{A21}$$

Diagonal Wind

If the wind comes from a diagonal rather than head-on, two sides of the building will have positive pressures and two will have negative. Although the total infiltration will not change much, the leakage distribution angle will; the tangent will become equal to unity or, equivalently,

$$\cos 2\theta_{\neq} = 0 \tag{A22}$$

EXAMPLE: As an example we take the head-on case and further assume that the wind pressure coefficient for the windward side is 0.7 and for all other sides it is –0.5. We can then solve for wind-induced airflow:

$$Q_{\equiv} = \frac{\mathbf{K}}{4}\ (0.45\rho + v^2)^n \tag{A23.1}$$

$$Q_{\equiv} = \mathbf{K}\left\{\frac{(1 + \frac{1}{3}^{1/n})^{-n}}{4}\right\}\left(.45(1+\left(\frac{1}{3}\right)^{1/n})\rho + v^2\right)^n \tag{A23.2}$$

where for this example $C'_{\equiv}$ = 0.54 and the factor in curly brackets is equal to 0.22.

Fans

If the fan is the only driving force, the flow will follow Equation 1. Using the development of this appendix, fans are a limiting case which can be described as a large driving force operating over a small area (i.e., the fan) and the rest of the envelope having no external pressure applied. The net effect is that the fan either completely pressurizes or completely depressurizes the (vast majority of the) envelope. Thus, as we can see from Equation A7, the leakage distribution angle takes only one of two values depending on the fan direction.

$$\cos 2\theta_f \rightarrow \begin{cases} 1 \text{ for supply fans} \\ -1 \text{ for exhaust fans} \end{cases} \tag{A24}$$

It is important to note that the pressure drop across the fan (ΔP) is very different from and larger than the change in internal pressure (ΔP_i). We have assumed that the fans are constant flow devices, which will be true when the pressure drop across the fan is much larger than the internal pressure change.

Combining Driving Forces

As stated in the text, we are only interested in adding driving forces that are independent of each other; that is, we assume that the pattern of pressurization and depressurization areas are uncorrelated. To the extent that they are not independent, they must be treated simultaneously.

Consider the case in which we start with one of the natural driving forces and then a second (independent) smaller driving force is added. We will seek an expression of the form

$$Q = Q_1 + \varepsilon_+ Q_2 \qquad \text{(A25)}$$

where the *addition efficiency* is a constant to be determined by considering the effect of the Q_2 as a perturbation.

In order for the total ventilation to change as a result of the combination, there must be some change in the pattern of leakage and pressures. When the two driving forces are of the same order, the change in leakage distribution will be significant and is beyond the scope of simplified modeling to predict in general. In this section we assume that the leakage distribution does not change significantly.

Addition of a Small Fan

Fans affect the total ventilation by changing the internal pressure. We can examine combining a fan with a natural driving force by analyzing the response of the envelope to a change in internal pressure.

A change in the internal pressure will either increase the infiltration and decrease the exfiltration or the converse, depending on the sign of the pressure change:

$$Q_\pm = \mathbf{K}_\pm(\Delta P_\pm \pm \delta P_i)^n \qquad \text{(A26.1)}$$

which, since the pressure is small, can be expanded to

$$Q_\pm \approx Q\left(1 \pm n \frac{\delta P_i}{\Delta P_\pm}\right) \qquad \text{(A26.2)}$$

The difference between the infiltration and exfiltration through the envelope is the fan flow. Taking the specific case of an exhaust fan (i.e., $\delta P_i > 0$),

$$Q_+ - Q_- = Q_f = Q_n \frac{n\,\delta P_i}{\Delta P \sin^2\theta \cos^2\theta} \qquad \text{(A27)}$$

and the total infiltration will be equal to Q_+:

$$Q = Q_n + \sin^2\theta_n Q_f \quad \textit{(for exhaust fan)} \qquad \text{(A28.1)}$$

or, an equivalent derivation for a supply fan would yield:

$$Q = Q_n + \cos^2\theta_n Q_f \quad \textit{(for supply fan)} \qquad \text{(A28.2)}$$

We can combine these two expressions into a single one by making use of the leakage distribution angle for fans. Doing so yields the following expression for the addition efficiency:

$$\varepsilon_+ = \left| \frac{\cos 2\theta_n + \cos 2\theta_f}{2} \right| \qquad \text{(A29)}$$

Combining Two Natural Driving Forces

The stack and wind effects operate by inducing a pressure on the outside surface of the envelope; the internal pressure responds to balance the flow. When two natural driving forces are operating simultaneously, there will be areas in which their positive pressures coincide, in which their negative pressures coincide, and in which their positive and negative pressures overlap, some of which may cause changes in flow direction.

This situation can be conceptualized by considering that part of the larger driving force was being affected by an exhaust fan while the remainder was being affected by a supply fan. In this way we can use the results of the previous section to combine two natural driving forces.

For clarity consider the specific example of the same wind effect being added to a larger (winter) stack effect. All of the windward faces have either increased infiltration or decreased exfiltration and, therefore, act as though they were being exposed to an exhaust fan equivalent to the wind-induced flow. Similarly, all of the leeward faces act as though they were being exposed to a supply fan.

The combined total infiltration will come from the lower parts of the envelope and consist of augmented infiltration on the windward sides and decreased infiltration on the leeward sides. This total, of course, will be the same as the exfiltration from the upper parts (decreased on the windward side and increased on the leeward side). The allocation of augmented versus decreased infiltration and exfiltration, however, may be different. If we assume that there is more exfiltrating area (for both stack and wind separately, then we can use Equation A28 to find the total exfiltration:

$$Q = (Q_1 + \cos^2\theta_1 Q_2)\left(\frac{\mathbf{K}_-}{\mathbf{K}}\right)_2 + (Q_1 - \sin^2\theta_1 Q_2)\left(\frac{\mathbf{K}_+}{\mathbf{K}}\right)_2 \quad \text{(A30.1)}$$

$$Q = Q_1 + \frac{1}{2}\left(\cos^2\theta_1 + \left[\frac{\mathbf{K}_- - \mathbf{K}_+}{\mathbf{K}}\right]_2\right)Q_2 \quad \text{(A30.2)}$$

where in our example the subscripts 1,2 specifically mean stack, wind effects.

The quantities $\mathbf{K}_\pm/\mathbf{K}$ represent the fractions of the leakage under infiltration and exfiltration (for the second driving force). The quantity in brackets can nominally be found using Equation A7:

$$\left[\frac{\mathbf{K}_- - \mathbf{K}_+}{\mathbf{K}}\right]_2 = \frac{\cos^{2n}\theta_2 - \sin^{2n}\theta_2}{\cos^{2n}\theta_2 - \sin^{2n}\theta_2} \quad \text{(A31.1)}$$

Since we are treating the smaller driving force as a perturbation, there will be a change in the flow that is approximately linear due to the change in the pressure along the lines of Equations A26-A28. Therefore, this expression should be evaluated for $n = 1$ to yield:

$$\left[\frac{\mathbf{K}_- - \mathbf{K}_+}{\mathbf{K}}\right]_2 \approx \cos 2n\theta_2 \quad \text{(A31.2)}$$

Thus

$$Q = Q_1 + Q_2\left(\frac{\cos 2\theta_1 + \cos 2\theta_2}{2}\right) \quad \text{(A32)}$$

Care must be exercised in handling the signs of the intermediate terms. We chose our example to make all of the cosines positive and used the fan addition rules assuming Q_2 is positive. It turns out in general that the addition efficiency can be expressed as follows:

$$\varepsilon_+ = \left|\frac{\cos 2\theta_l + \cos 2\theta_2}{2}\right| \quad \text{(A33)}$$

Note that this expression reduces to the fan flow expression when a fan distribution angle is inserted.

Large Fan Flows

Because the addition efficiency for adding two natural driving forces is symmetric, it does not depend on whether the stack or wind effect is the larger. The same symmetry does not hold true for the additions of fans to natural driving forces.

If the fan flow is not small, the distribution of pressures across the envelope will change in a complex way as a result of the fan. For the example of an exhaust fan, the fan will eventually increase the leakage distribution angle until all of the flow is infiltration; at this point the total infiltration is equal to the fan flow and there is no exfiltration through the envelope. Once this point is reached, the total air change is just equal to the fan flow.

The behavior when the natural and fan flows are comparable depends strongly on leakage distribution and exponent. As a simplification we assume that the small fan flow expression is applicable until the fan dominates:

$$Q = MAX(Q_f, Q_n + \varepsilon_+ Q_f) \quad \text{(A34)}$$

Equivalently we could define a fan addition efficiency, ε_f to include this effect:

$$\varepsilon_f \equiv MAX\left(\varepsilon_+, 1 - \frac{1}{f_f}\right) \quad \text{(A35)}$$

where the fan factor is defined similarly to f_n:

$$f_f \equiv \frac{Q_f}{Q_n} \quad \text{(A36)}$$

In this discussion we have assumed that fans are constant flow devices, which they need not be. The change in internal pressure caused by the infiltration, δP_i, may cause a shift in the operating point of the fan. If it does, the techniques used in the appendix could be applied iteratively to find a more exact solution. For most fans a shift of a few Pascals in internal pressure is unlikely to cause a significant change. A larger effect, which is not considered herein, is the fact that the wind may have a much larger impact on the outlet.

Equivalent-sized Flows

When one attempts to combine two driving forces of equivalent size, the assumptions used in the sections above are quite likely not valid. Specifically, the leakage pattern (and hence distribution angle) may change significantly and thus the linearized result found in this appendix will begin to change. For example as an exhaust fan increases its effect, more of the envelope will become pressurized (and the internal pressure will decrease less than our linear prediction) until the entire envelope is pressurized.

These effects would cause the addition equation (Equation A25) to have higher order terms (i.e., curve) as Q_2 approaches Q_1. There are not, unfortunately, easy ways of combining the two forces without knowing many more details than are appropriate for simplified modeling.

Most simplified models are used either for averaging time-series data or for averages of large samples of different dwellings. In the former case it is highly unlikely that a large segment of the data would happen to be where two driving forces were comparable. In the latter case the variation in the details will tend to mitigate any individual biases. Thus in many cases it is not necessary to be able to predict accurately the case in which two flows are equivalent. For those cases in which it is, however, detailed network models should be utilized.

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References

Allen, C. (1983) *Wind Pressure Data Requirements for Air Infiltration Calculations*, Coventry, U.K. Air Infiltration and Ventilation Centre (Technical Note, 13).

ASTM (1987) "Standard test method for determining air leakage rate by fan pressurization," *ASTM E779-87*, American Society for Testing and Materials.

Feustel, H.E. and Raynor-Hoosen, A. (eds) (1990) *COMIS Fundamentals*, Coventry U.K., Air Infiltration and Ventilation Centre, (Lawrence Berkeley Laboratory Report No. LBL-28560).

Etheridge, D.W. and Stanway, R.J. (1988) "A parametric study of ventilation as a bases for design", *Building and Environment*, 23(2), 81-93.

Liddament, M. and Allen, C. (1983) *The Validation and Comparison of Mathematic Models of Air Infiltration*, Coventry U.K., Air Infiltration and Ventilation Centre, (Technical Note 11).

Modera, M.P., Sherman, M.H. and Levin, P.A. (1983) "A detailed examination of the LBL infiltration model using the mobile infiltration test unit", *ASHRAE Transactions*, **89**.

Modera, M.P. and Peterson, F. (1985) *Simplified Methods for Combining Mechanical Ventilation and Natural Infiltration*, Sweden, Department of Heating and Ventilating, Royal Institute of Technology (Lawrence Berkeley Laboratory Report LBL-18955).

Reardon, J. (1989) *Air Infiltration Modeling Study*, National Research Council of Canada (Report No. CR5446.3).

Shaw, C.Y. (1981) "A correlation between air infiltration and airtightness for houses in a developed residential area", *ASHRAE Transactions*, **87**, Pt II.

Sherman, M.H. (1991) *A Power Law Formulation of Laminar Flow in Short Pipes* (in Journal Review) (Lawrence Berkeley Laboratory Report LBL-28505).

Sherman, M.H. and Modera, M.P. (1986) "Low-frequency measurement of leakage in enclosures", *Review of Scientific Instruments*, 57(7), 1427.

Sherman, M.H. and Modera, M.P. (1988) "Signal attenuation due to cavity leakage", *Journal of the Acoustical Society of America*, 84(6).

Sherman, M.H., Wilson, D.J. and Kiel, D.E. (1986) "Variability in residential air leakage". In: Trechsel/Lagus (eds), *Measured Air Leakage of Buildings*, Philadelphia, PA, American Society for Testing and Materials, pp. 348 (STP 904).

Sherman, M.H. and Grimsrud, D.T. (1980) "Infiltration-pressurization correlation: simplified physical modeling". *ASHRAE Transactions*. **86**,Pt II, 778-807 (Lawrence Berkeley Laboratory Report LBL-10163).

Sherman, M.H. (1980) *Air Infiltration in Buildings*, Ph.D Thesis, University of California (Lawrence Berkeley Laboratory Report LBL-10712).

Walker, I.S. and Wilson, D.J. (1990) *AIM-2: The Alberta Air Infiltration Model*, University of Alberta (Department of Mechanical Engineering Report 71).

Wilson, D.J. and Pittman, W. (1983) "Correlating measured infiltration for wind from a single direction", *ASHRAE Transactions*, **89**.