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An Improved Method to Determine the Age-of-Air from Tracer-Gas Measurements

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SUMMARY

In order to determine the local mean age or the room average age of air in a ventilated space from tracer gas measurements it is necessary to calculate moments about the origin of time-dependant tracer-gas concentration curves $c_p(t)$ from different points p in the room, i.e. integrals of the form $I^{(m)} = \int_0^\infty t^m\, c_p(t)\, dt$. Because of the limited measuring time, the exponential part of the concentration curves $c_p(t)$ has to be extrapolated to obtain the residual part of the above mentioned integrals. Literature concerning this topic lacks of detailed information on what procedure should be used.

This paper discusses an improved method to obtain the residual part of the integrals $I^{(m)}$ and is based on the so called "Nordtest Method", but in contrast to it, quantifies and minimizes the error that arises from the extrapolation of the concentration curves. The principle of the method is as follows: The evaluation of the concentration curve $c_p(t)$ at a certain point p in the room is carried out after measuring every new concentration value. In this way time-dependant curves of the respective determined values of $I^{(m)}$ can be achieved. By means of these curves a definite criteria can be deduced to stop the measurement.

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INTRODUCTION

Tracer-gas measurements are a relatively new technique which enables us to make quantitative statements concerning the quality of the local and the overall ventilation of a room or a limited space (see Grieves, 1989). These statements can either be made for certain points p in the room by determining the local-mean-age $\bar{\tau}_p$ of the air, or for the room as a whole by determining the room-average-age $< \bar{\tau} >$. The age-of-air is a statistical measure of how long it takes, in average, for air molecules to get from the supply duct to certain points p in the room.
The age-of-air can be obtained by measuring the time-dependant change of the concentration of a tracer-gas that has been or is being artificially distributed in the room. Such a time-dependant change of the concentration can be achieved by a step-down or a step-up test.

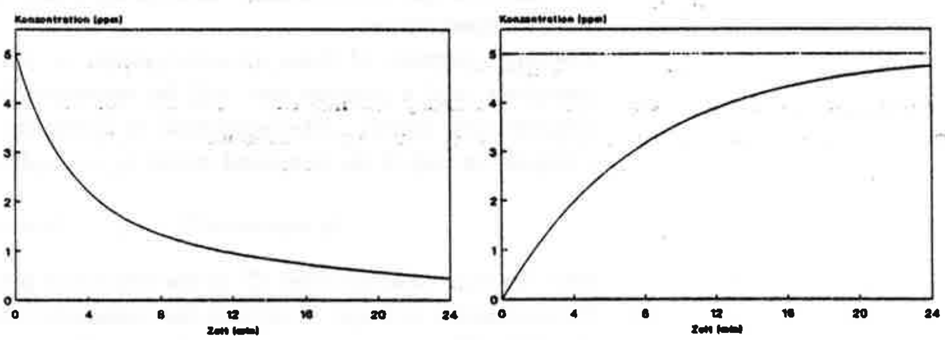

fig. 1 Time-dependant courses of the concentration of a tracer-gas at a certain point p when performing a step-down (left) or a step-up test (right)

During a step-down test you are observing the decay of the concentration of a tracer-gas that was evenly distributed in the entire room at the beginning of the measurement. During a step-up test you are observing the increase of the concentration of a tracer-gas that is continuesly fed into the supply duct of the room. If the entire air supplied to the room is maintained by means of a mechanical ventilation system, both tests should yield the same results.

In order to determine the age-of-air it is necessary to calculate moments about the origin of the measured time-dependant tracer-gas concentration curves $c_p(t)$, i.e. integrals of the form (Sandberg and Sjöberg, 1983)

$$I^{(m)} = \int_0^\infty t^m \, c_p(t) \, dt \qquad with: \; m \geq 0$$

Since the measuring time-interval $0 \leq t \leq t_{meas}$ is limited, the concentration curves $c_p(t)$ have to be extrapolated to infinity to obtain the residual parts of the above mentioned integrals $I^{(m)}$. Thus two integrals have to be calculated.

$$I^{(m)} = \int_0^{t_{meas}} t^m \, c_p'(t) \, dt + \int_{t_{meas}}^\infty t^m \, c_p''(t) \, dt$$

$$\Rightarrow \quad I^{(m)} = I^{(m)'} + I^{(m)''}$$

The value of $I^{(m)'}$ can be determined with any numerical integration method (Trapezoidal or Simpson's Rule) and shouldn't be any problem as long as the data sampling rate is high enough. But literature concerning the determination of the value of $I^{(m)''}$ lacks of detailed information.

PRELIMINARY REMARKS

In order to simplify the evaluation of the different concentration curves it is useful to transform the concentration readings $c_p(t)$ and the measurement time t into dimensionless values.

The first purpose of these transformations is that no distinction between a step-down and a step-up test will be necessary during the evaluation of the concentration curves. The evaluation of a step-up test corresponds to that of a step-down test, if the measured values $c_{p,\,step-up}(t)$ are transformed as follows †

$$c_{p,\,step-down}(t) = c_p^\infty - c_{p,\,step-up}(t)$$

with the final concentration c_p^∞ at the respective point p (see fig. 1).

Furthermore it is easier to perform the evaluation with dimensionless concentrations $c_p^*(t)$. The concentration values of a step-down test $c_{p,\,step-down}(t)$ are standardized according to the initial concentration $c_{p,\,step-down}(t=0) = c_p(0)$ at the respective point p, whereas the concentration readings of a transformed step-up test

† This only holds, if the entire air supplied to the room is maintained by means of mechanical ventilation, i.e. no infiltration may occur.

are standardized with regard to the final concentration $c_{p,\,step-up}(t \rightarrow \infty) = c_p^{\infty}$.

$$c_p^*(t)\,[-] = \frac{c_{p,\,step-down}(t)}{c_p(0)} \tag{1a}$$

$$c_p^*(t)\,[-] = 1 - \frac{c_{p,\,step-up}(t)}{c_p^{\infty}} \tag{1b}$$

Furthermore it is more convenient to state the measurement time t not in real-time, but as a multiple of the nominal time-constant τ_n. The nominal time-constant τ_n is the reciprocal value of the air-change-rate $n = Q_s/V_R\,[s^{-1}]$ with the supply flow rate Q_s and the net volume of the room V_R.

$$t^*\,[-] = \frac{t}{\tau_n} \tag{2}$$

This standardization allows and simplifies a) the comparison of different local-mean-ages $\bar{\tau}_p$ with regard to the room-mean-age $< \bar{\tau} >$ and b) the comparison of both the local-mean-ages $\bar{\tau}_p$ and the room-mean-age $< \bar{\tau} >$ at different air-change-rates n.

Making use of the transformations according to *eq.* (1*a*) , (1*b*) and (2) the moments about the origin $\mu_p^{(m)}$ can be calculated as (Sandberg and Sjöberg, 1983; Sutcliffe, 1990)

$$\mu_p^{(m)} = \tau_n^{m+1} \int_0^{\infty} (t^*)^m\, c_p^*(t^*)\, dt^* \qquad with:\ m \geq 0 \tag{3}$$

Thus, the local-mean-age $\bar{\tau}_p$ can be determined as

$$\bar{\tau}_p\,[s] = \mu_p^{(0)} = \tau_n \int_0^{\infty} c_p^*(t^*)\, dt^* \tag{4}$$

The local-mean-age in the exhaust air duct $\bar{\tau}_e$ is always equal to the nominal time-constant τ_n (Sandberg and Sjöberg, 1983; see *eq.* (4))†

$$\tau_n = \bar{\tau}_e = \mu_e^{(0)} \tag{5}$$

The room-mean-age $< \bar{\tau} >$ can be determined by means of an evaluation of the concentration curve in the exhaust air duct $c_e^*(t^*)$. Making use of *eq.* (4) and (5) with $\mu_e^{(0)}/\tau_n = \int_0^{\infty} c_e^*(t^*)\, dt^* = 1$ yields

$$< \bar{\tau} >\,[s] = \frac{\mu_e^{(1)}}{\mu_e^{(0)}} = \tau_n \int_0^{\infty} t^*\, c_e^*(t^*)\, dt^* \tag{6}$$

† This only holds in general, if the entire air supplied to the room is maintained by mechanical ventilation.

Hence, both values $\bar{\tau}_p$ and $< \bar{\tau} >$ will be calculated as a multiple of the nominal time-constant τ_n!

Inserting *eq.* (4) and (6) into the definition of the air-exchange-efficiency η_a gives

$$\eta_a\,[-] = \frac{\tau_n}{2 < \bar{\tau} >} = \frac{1}{2 \int_0^\infty t^* \, c_e^*(t^*) \, dt^*} \tag{7}$$

with $\eta_a = 0.5$ for a perfectly mixed room and $\eta_a < 0.5$ for a room with short-circuiting, i.e. supply air bypassing the working area.

Thus, the determination of the different local-mean-ages $\bar{\tau}_p$ just requires the calculation of the 0^{th} moment at the respective points p and the determination of the room-mean-age $< \bar{\tau} >$ and the air-exchange-efficiency η_a just requires the calculation of the 1^{st} moment in the exhaust air duct.

PHYSICAL BASIS OF THE EXTRAPOLATION METHOD

After a certain time the concentration decay $c_p^*(t^*)$ at all points p in the room may be described by one single exponential term,

$$c_p^*(t^*) = a \, e^{\lambda \, t^*} \tag{8}$$

because a logarithmic plot of the curve $c_p^*(t^*)$ will turn into a straight line with the slope λ (with the value of λ being negative and dimensionless). The value of λ can be determined using a linear regression of the concentration values that belong to the exponential decay phase. The usual procedure is to extrapolate this fitted exponential function beyond the point being last measured $c_p^*(t^*_{meas})$ to infinity. Inserting *eq.* (8) thus yields the residual parts $\mu_p^{(n)''}$ (Nordtest NT VVS 047, 1985)

$$\mu_p^{(0)''} = \tau_n \int_{t^*_{meas}}^\infty c_p^*(t^*) \, dt^* = -\frac{c_p^*(t^*_{meas})}{\lambda} \tau_n \tag{9}$$

$$\mu_e^{(1)''} = \tau_n^2 \int_{t^*_{meas}}^\infty t^* \, c_e^*(t^*) \, dt^* = -\frac{c_e^*(t^*_{meas})}{\lambda} \left(t^*_{meas} - \frac{1}{\lambda} \right) \tau_n^2 \tag{10}$$

with $t^*_{meas} = t_{meas}/\tau_n$. In general and especially when evaluating scattered concentration readings it is more sensible not to use the last measured concentration $c_p^*(t^*_{meas})$, but the value that can easily be calculated by means of the linear regression according to *eq.* (8). In this way the measurement error of the last measured value can be minimized.

This is basically the procedure of extrapolation being described by Nordtest NT VVS 047 (1985) and Nordtest NT VVS 019 (1988), which are the only available extrapolation methods described in literature (besides another one described by

Raatschen and Walker, 1991). But the procedure of evaluation explained by the Nordtest reports just provides a rough guide-line and it can only be marked as being vague and uncertain.
According to Nordtest NT VVS 047 (1985) the moment to stop the measurement as well as the selection of the concentration values used for the linear regression are just determined by eye using a logarithmic plot of the measured concentrations $c_p(t)$. Furthermore it is impossible to carry out a following check whether these assumptions have been justified or not, because just one linear regression with arbitrary selected concentration values is undertaken.
This procedure is acceptable for field measurements where you just have a pocket calculator accessible to get a general idea of the values of $\bar{\tau}_p$ and $< \bar{\tau} >$, but by using a PC this procedure can be optimized.

PRINCIPLE OF THE OPTIMIZED EXTRAPOLATION METHOD

The principle of the evolved method is as follows (Jung, 1991):

- The evaluation of the concentration curve $c_p^*(t^*)$ is carried out after measuring every new concentration value, i.e. from every new measured concentration value a linear regression of the last n_{fit} concentration readings is carried out.
- Keeping n_{fit} constant, a new or current value of λ, $\mu_p^{(0)} = \mu_p^{(0)\prime} + \mu_p^{(0)\prime\prime}$, $\mu_e^{(1)} = \mu_e^{(1)\prime} + \mu_e^{(1)\prime\prime}$ can be assigned to each time t.
- By plotting these values of λ, $\mu_p^{(0)}$ and $\mu_e^{(1)}$ versus time one will get steadily increasing curves approaching constant final values.
- The number of concentration readings n_{fit} determines both the uncertainty and how fast these curves approach their respective final values.

SENSITIVITY TESTS WITH ARTIFICIAL CONCENTRATION HISTORIES

This procedure has been converted into a computer code. With this code several artificially generated concentration curves have been examined. Using simplified assumptions for specific air flow patterns in the room (2-zone, 3-zone model) and varying air flows between the zones (see fig. 2), the concentration curves have been generated using the computer code CONTAM86 (Axley, 1987).

The setup in fig. 2 is a good example to test and to verify different extrapolation methods, because the exponential decay of the different concentration curves $c_p^*(t^*)$ in the three zones doesn't begin until 5 nominal time-constants (see fig. 3). The setup is also a typical example of extreme short-circuiting, because the air-exchange-efficiency is well below 50 % ($\eta_a = 0.2$).

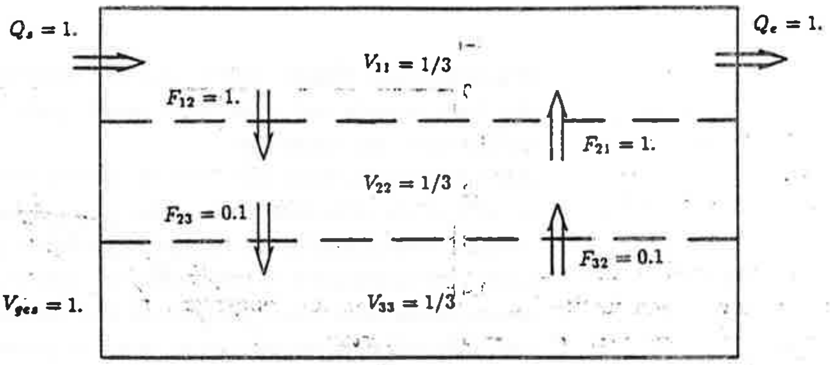

fig. 2 Setup of a simplified model of a short-circuit stream in order to generate artificial concentration curves of a step-down and step-up test

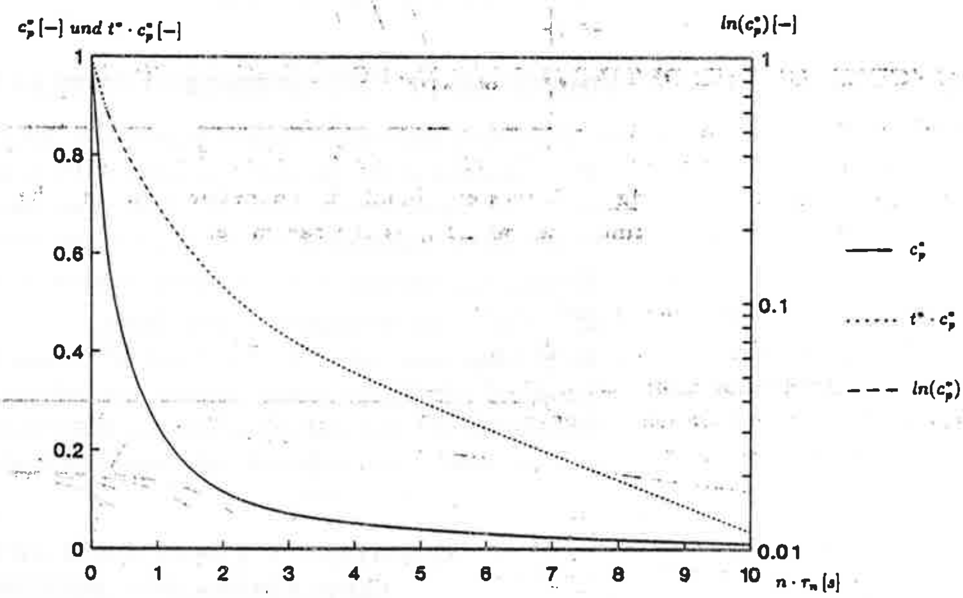

fig. 3 Curves of $c_e^*(t)$, $t^*\, c_e^*(t)$ and $ln(c_e^*(t))$ in the exhaust air duct according to the setup in fig. 2

The concentration history $c_e^*(t^*)$ in the exhaust air duct has been evaluated with a data sampling rate of 20 concentration readings per nominal time-constant τ_n. With the number of $n_{fit} = 20, 40, 60, 80$, the linear regression has been performed with the last measured values out of a time-interval of $\Delta t_{fit} = 1, 2, 3, 4\, \tau_n$.
Looking at the figures 4, 5 and 6 one can see the basic idea of the proposed extrapolation method: The measurement should not be stopped until the curves of λ, $\mu_p^{(0)}$ und $\mu_e^{(1)}$ have approached constant final values, i.e. concerning the moment to stop the measurement one may not orientate at the concentration curves $c_p^*(t^*)$ or to any empirical values (an often proposed rule-of-thumb value was $2\,\tau_n$).

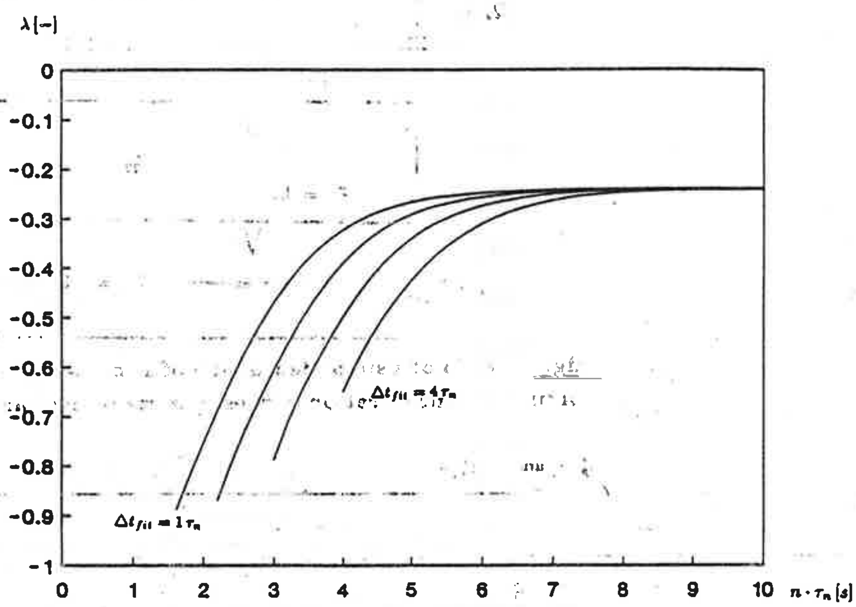

fig. 4 **Curves of λ in all three zones according to the setup in fig. 2 using the time-interval Δt_{fit} as the parameter**

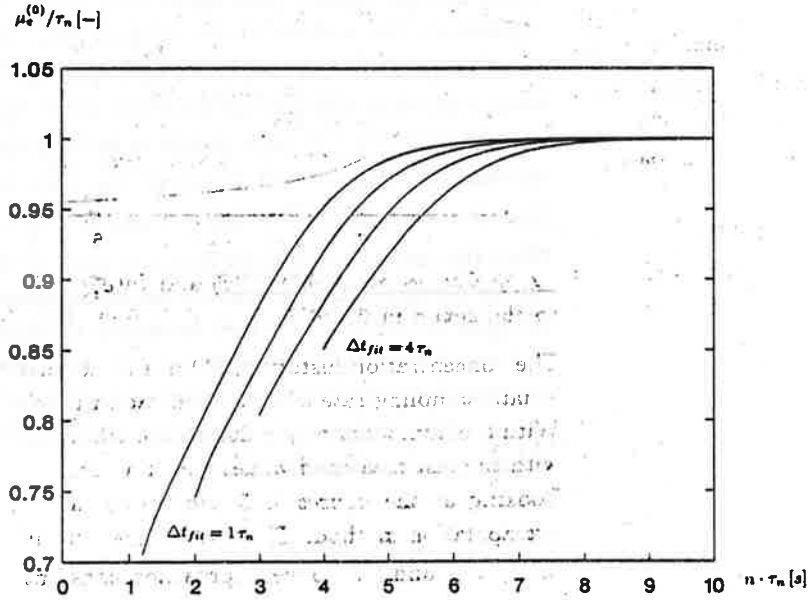

fig. 5 **Curves of $\mu_e^{(0)}/\tau_n$ in the exhaust air duct according to the setup in fig. 2 using the time-interval Δt_{fit} as the parameter**

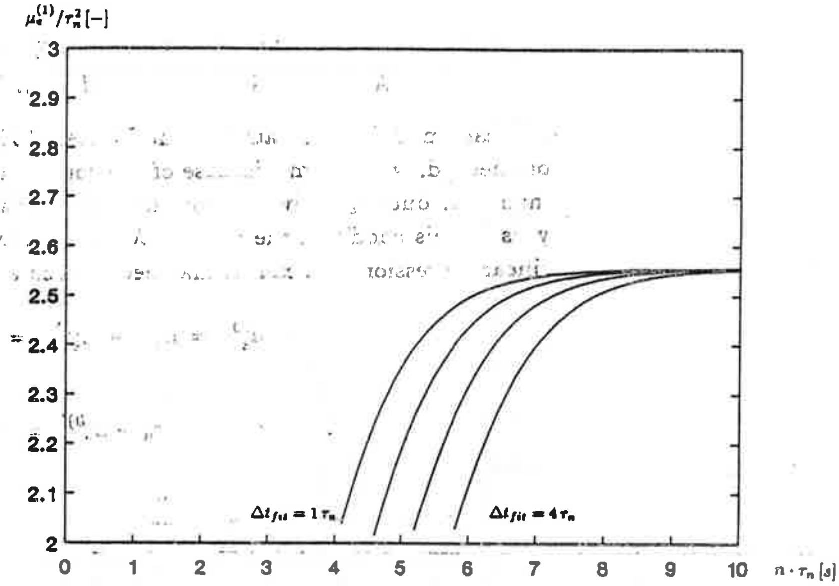

fig. 6 Curves of $\mu_e^{(1)}/\tau_n^2$ in the exhaust air duct according to the setup in fig. 2 using the time-interval Δt_{fit} as the parameter

The time-interval Δt_{fit} is important when evaluating scattered concentration curves: If Δt_{fit} is too small, the respective values of λ, $\mu_p^{(0)}$ and $\mu_e^{(1)}$ might clearly deviate around their mean final values. By increasing the range of Δt_{fit}, respectively the number of the readings n_{fit} used for the linear regression, the uncertainty of the residual parts $\mu_p^{(0)''}$ and $\mu_e^{(1)''}$ can be minimized, because the linear regression will become more and more reliable. However, the measurement time-interval $0 \leq t \leq t_{meas}$ needs to be increased accordingly. But regardless of the value of Δt_{fit} one will always get the same final values of λ, $\mu_e^{(0)}$, $\mu_p^{(0)}$ and $\mu_e^{(1)}$ (here $\lambda = -0.2400$, $\mu_e^{(0)} = 1.000\,\tau_n$ and $\mu_e^{(1)} = 2.556\,\tau_n^2$; see figures 4, 5 and 6).
With this procedure definite limits of error of the residual parts $\mu_p^{(0)''}$ and $\mu_e^{(1)''}$ can be determined. These errors can be minimized to any desirable value (even if the data sampling rate is low) by simply extending Δt_{fit}.
But one has to take into account that the extrapolation cannot compensate for any mistakes arising from an incorrect numerical integration, because the determination of the numerically integrated values $\mu_p^{(0)'}$ and $\mu_e^{(1)'}$ and the extrapolation procedure are independant.

FURTHER APPLICATIONS OF THE INTRODUCED METHOD: 1. MATCHING THE NOMINAL TIME-CONSTANT

As already mentioned Raatschen and Walker (1991) presented another extrapolation method, which is making use of the condition that the local-mean-age in the exhaust air duct $\bar{\tau}_e$ is always equal to the nominal time-constant τ_n (see *eq.* (5)). By using this condition the value of λ doesn't have to be calculated by means of a linear regression, but can be matched in such a way that *eq.* (5) is fulfilled.

$$\mu_e^{(0)} = \mu_e^{(0)'} + \mu_e^{(0)''} := \tau_n$$

$$\Rightarrow \quad \mu_e^{(0)''} = \tau_n - \mu_e^{(0)'} = -\frac{c_e^*(t_{meas}^*)}{\lambda}\tau_n$$

$$\Rightarrow \quad \lambda = -\frac{c_e^*(t_{meas}^*)}{1 - \frac{\mu_e^{(0)'}}{\tau_n}} \tag{11}$$

Inserting the value of λ in *eq.* (9) and (10) again yields the same final values of $\mu_p^{(0)}$ and $\mu_e^{(1)}$, achieving similiar courses to the ones shown in fig. 4, 5 and 6. This method is consistent in itself, but one has to know or presume the value of τ_n in advance.

The curves of λ are very sensitive towards small inaccuracies of the presumed value of τ_n (even if they are in the region of $\pm 2\,\%$). Small uncertainties in τ_n lead to the fact that the curves of λ will no longer approach a constant final value (Jung. 1991). Hence, this extreme sensitivity allows in turn an easy way of evaluating the value of τ_n. In this way the results of a separate air flow measurement can either be corrected or confirmed.

2. MATCHING THE FINAL CONCENTRATION OF A STEP-UP TEST

The procedure described above to obtain the nominal time-constant τ_n can also be used in principle to get the final concentration c_p^{∞} of a step-up test, because the curves of λ are also sensitive towards corresponding inaccurate assumptions of the final-value c_p^{∞} (see figures 7 and 8).

The simulation of a step-up test with the setup in fig. 2 followed by an evaluation with the "true" assumption $c_p^{\infty} = 1\,c_s$ yields the same curves and final values of λ, $\bar{\tau}_p$ and $< \bar{\tau} >$ like the corresponding step-down test (c_s marks the concentration in the supply duct). But if the evaluation of the step-up test is carried out with a final concentration c_p^{∞} being too small or too high, the curves of λ will no longer approach a constant final value.

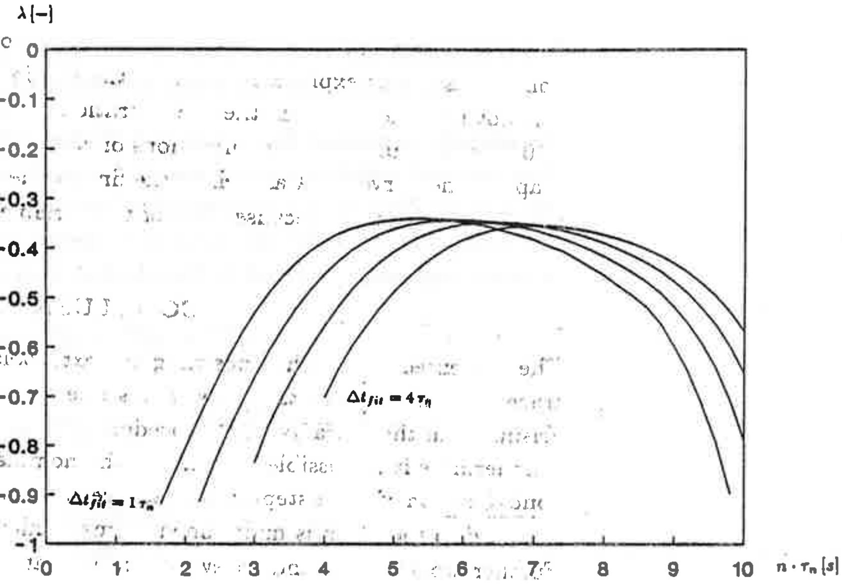

fig. 7 **Curves of λ according to the setup in fig. 2 using the time-interval Δt_{fit} as the parameter (here for a step-up test assuming a final concentration $c_p^\infty = 0.99\, c_s$)**

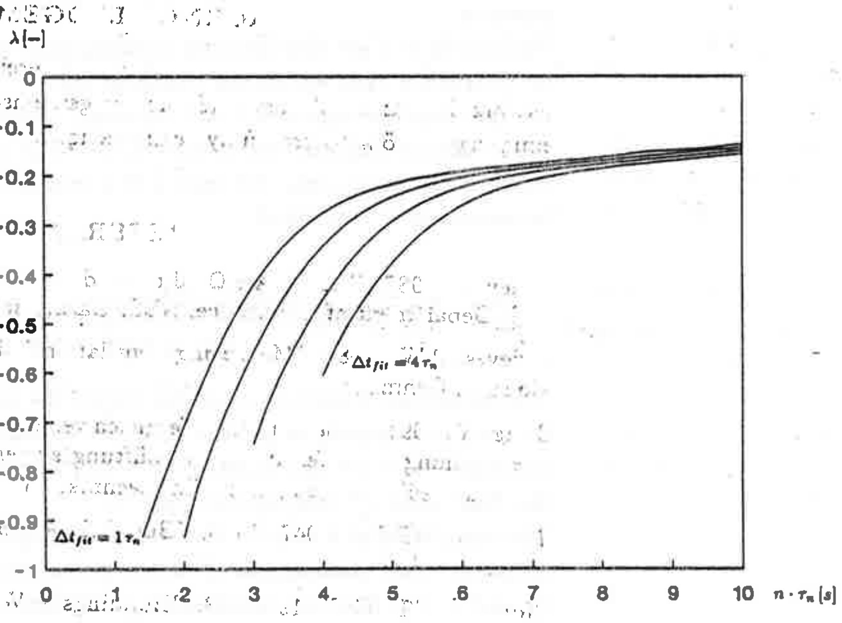

fig. 8 **Curves of λ according to the setup in fig. 2 using the time-interval Δt_{fit} as the parameter (here for a step-up test assuming a final concentration $c_p^\infty = 1.01\, c_s$)**

This characteristic should also make it quite simple to match the final concentration c_p^{∞}, as it was explained above for the nominal time-constant τ_n.
It is not possible to match the concentration at the beginning of a step-down test $c_p(0)$, because inaccurate assumptions of $c_p(0)$ don't have any affects towards the shape of the curves of λ as well as the final value of λ. One will still get the same constant value for λ, because both of the variables $c_p(0)$ and λ are independant.

CONCLUSION

The presented approach concerning the extrapolation of concentration curves of tracer-gas measurements makes it possible to quantify and minimize the errors arising from the extrapolation procedure.
Furthermore it is possible to evaluate the nominal time-constant τ_n and the final concentration c_p^{∞} of a step-up test at a certain point p provided that the entire air supplied to the room is maintained by mechanical ventilation.
Further work will intend to evolve an evaluation procedure in such a way that all variables of a tracer-gas measurement ($c_p(0)$, c_p^{∞}, τ_n, $\mu_p^{(0)}$, $\mu_e^{(0)}$ and $\mu_e^{(1)}$) are consistent to one another.

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