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SIMULATION ON TEMPERATURE AND FLOW FIELD IN THE ATRIUM (PART1. COMPUTATION OF SOLAR RADIATION, RADIATIVE HEAT TRANSFER, AIR FLOW, AND TEMPERATUR)

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SUMMARY

In the effective control of indoor climate in the atrium, it is very important to predict the temperature and flow fields at a design step. One of the methods for the prediction is numerical simulation. We have developed the numerical simulation system, which consists of three-subsystems as followings;

(1) the simulation system of the direct solar radiation, the diffused solar radiation and the diffused reflection of the solar radiation,

(2) the simulation system of the radiative heat transfer,

(3) the simulation system of the temperature and flow distribution which is based on the k - ε model of turbulence solved with GSMAC3D-FEM[1,2].

In this paper (Part 1), we will describe the numerical methods and the techniques for computation of above three sub-systems. In the second paper (Part 2), the accuracy and reliability of those systems are confirmed by comparison between the numerical results and measurements, and applications for the actual design of atrium will appear[3,4].

1 INTRODUCTION

In recent years, the large space with the glass roof in the building, so called atrium, has been popular. But it is difficult to predict the thermal environment in the atrium at the design step since the thermal environment gets much effect from the solar radiation. Moreover, because the scale of the atrium is large, the three dimensional distribution of temperature and air velocity must be considered for the control of the environment. Therefore the three dimensional calculation is required for the analysis of the detailed distribution.

In this paper (part 1), the numerical methods for the calculation of solar radiation, radiative heat transfer, and flow and temperature field are going to be mentioned. The calculation of solar radiation gives the solar radiation absorbed with considering amount of the direct solar radiation, the diffused solar radiation, and the diffused reflection of the solar radiation as the boundary conditions for the analysis of flow and temperature field. By the calculation of the radiative heat transfer based on the results from the solar radiation absorbed, amount of radiative heat transfer among boundaries can be obtained. In the analysis of the flow and temperature distribution using the k-ε model of turbulence as the basic equations, the temperature and flow velocity in the atrium is solved by the GSMAC3D finite element method.

An advantage of this system is in the calculation based on the finite element method. Therefore the solar radiation, the radiative heat transfer, and the temperature and flow field calculation can be applied for the atrium having geometrically complicated indoor spaces.

2 OUTLINE OF THE THERMAL ENVIRONMENT ANALYSIS SYSTEM

This simulation system is based on the finite element method(FEM). The FEM has a big advantage in case of computation of geometrically complicated domain. Therefore, this system can be applied for the calculation about solar radiation, radiative heat transfer, flow and temperature field in a complex-shaped domain very easily. The calculation procedure is expressed in Fig.1. However, this system is restricted to the stationary analysis.

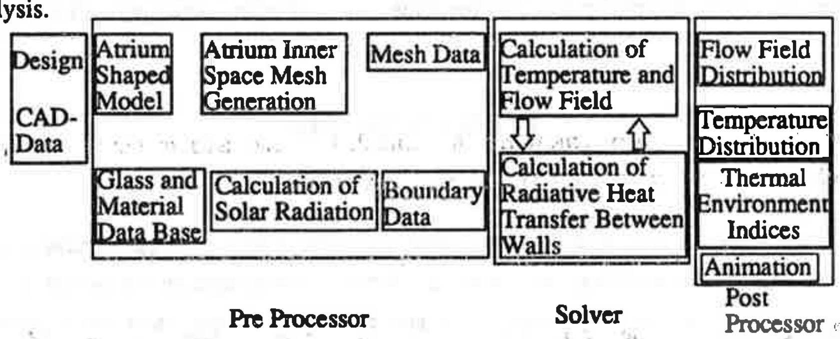


Fig.1 Numerical analysis on thermal environment.

3 COMPUTATION OF THE SOLAR RADIATION ABSORBED [5]

The direct solar radiation, the diffused solar radiation, and the diffused reflection on boundary elements are required in the calculation of the absorbed heat quantity. In this section, the calculation techniques of these three quantities are mentioned. Here, the boundary elemet is the boundary of the finite element mesh. In case of the hexahedral element mesh, the boundary elemet is quadrate (Fig.2).

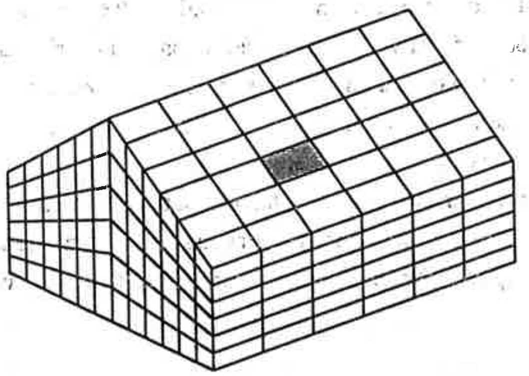


Fig.2 Boundary element .

3.1 Pre-computation of the Solar Radiation

We input two datatum; position of atrium (latitude, longitude), date of computation (month, day, hour, miniute), then compute the direct solar radiation tonormal plane, namely I_{dn} , the diffused solar radiation to horizontal plane, namely I_{sky} by equation (1);

$$I_{dn} = I_0 \, P^{1/\sin h},$$
$$I_{sky} = \sin h \, (I_0 - I_{dn})(0.66-0.32\sin h)\{0.5+(0.4-0.3P)\sin h\}, \quad (1)$$
$$I_0 = 1178 \, \{ 1+ 0.033 \cos (2\pi n/365) \},$$

where P is permeability of atmosphere, h is altitude, I_0 is solar radiation out of atmosphere, n is number of days counted from January 1st.

3.2 The Shape Model and the Properties on the Wall

The boundaries of the atrium are represented as surface models, and the buildings around the atrium are represented as solid models (Fig.3).

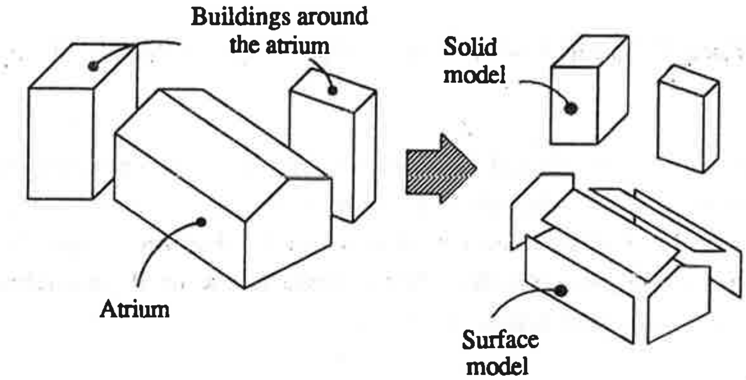


Fig.3 Modeling of atrium and buildings.

Since the surface models are expressed by arbitary polygons, the complicated shape of atrium can be modeled accurately. On the other hand, the boundaries of the atrium are also represented as the boundary elements, namely the boundaries of the finite elemet mesh. These boundary elements are seperated into the several groups belonging to the surface models. We input the properties on the walls to the surface models consisting of the atrium boundaries. The number of surface models is no more than several hundreds. But the number of boundary elements is no more than ten thousands. Then our system automatically put the properties in the surface models to the boundary elements. These properties are absorbance, transmissivity of solar radiation, and over all heat transfer coefficients.

3.3 Computaion of the Direct Solar Radiation

The direct solar radiation reaching the boundary elements depends on the interruption by other walls. Jugement of interrupting the solar radiation is carried out by intersecting between straight-lines directed to the sun from boundary and other walls (Fig.4).

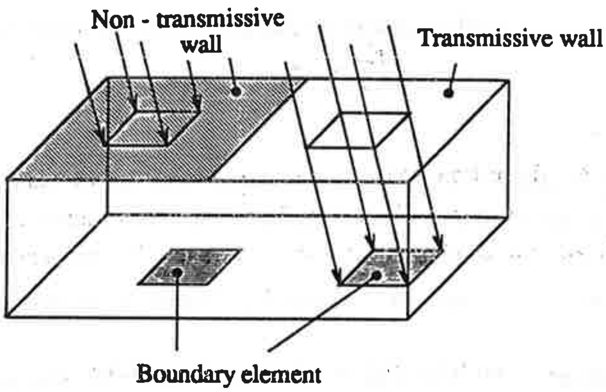


Fig.4 The direct solar radiation reaching the boundary elements.

The direct solar radiation reaching the boundary element is represented by equation (2);

$$I_d = I_{dn} \cdot \prod_{i=1}^{m} t_i \cdot \cos\ \theta\ , \qquad (2)$$

where m is total number of interrupting walls, t_i is transmissivity depended on the angle of incidence for each interrupting wall, θ is the angle of incidence for the boundary element.

3.4 Computation of the Diffused Solar Radiation

Amount of the diffused solar radiation reaching the boundary element is computed by the diffused solar radiation to horizontal plane. Amount of the diffused solar radiation reaching the boundary element is decided by solid angle to the transmissive wall (Fig.5).

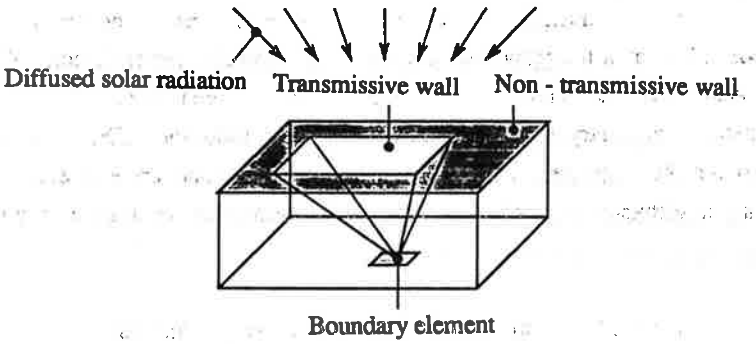


Fig.5 The diffused solar radiation reaching the boundary elements.

Amount of the diffused solar radiation reaching the boundary element, namely I_s is represented by equation (3);

$$I_s = \sum_{j=1}^{N} F_{ij} \cdot t_j^* \cdot 0.91 \cdot I_{sky}\ , \qquad (3)$$

where F_{ij} is view factor from element i to element j, t_j^* is transmissivity of the solar radiation in case of the vertical incidence of wall belonging to boundary element j, 0.91 is the characteristic angle of incidence in transmissivity, and N is the number of boundary elements.

3.5 Computation of the Diffused Reflection

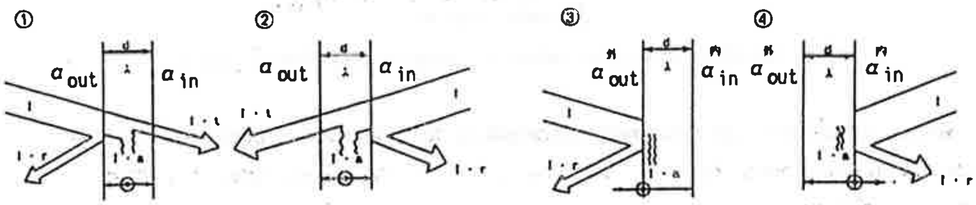
Amount of the diffused reflection between boundary elements is computed by the radiosity method[6] considering the diffused reflection on each wall. Modeling of the diffused reflection on each wall is represented by equation (4);

$$J_{s_i} = I_{d_i} + I_{s_i} + \sum_{j=1}^{N} F_{ij}\, G_{s_j},$$
$$G_{s_i} = J_{s_i} \cdot r_i \,. \tag{4}$$

In equation (4), unknown variables, J_{si} and G_{si}, are amount of the energy reaching and reflecting on the i-th boundary element, respectively. I_{di}, I_{si}, and r_i are given variables which mean amount of the direct solar radiation, diffused solar radiation reaching the boundary element, and reflectivity of the solar radiation on the boundary element, respectively. Here, all the variables except r_i are given for an hour and unit area.

3.6 Computation of the Solar Radiation Absorbed

Absorbed heat quantity of solar radiation on boundary element is computed basing on the amount of solar radiation reaching the boundary element. Computation of the absorbed heat quantity is separated four cases by the transmissive performance of walls and the direction of the solar radiation (Fig.6).



(a) In the transmissive wall (b) In the non-transmissive wall

Fig.6 Computation of the solar radiation absorbed.

In case of solar radiation reaching the transmissive wall from inner side of boundary element, the absorbed heat quantity is computed by multiplying amount of solar radiation by absorbtivity on wall. In case of solar radiation reaching the non-transmissive wall from outer side of the boundary element, the absorbed heat quantity is equivalently converted to one of inner walls.

4 COMPUTATION OF THE RADIATIVE HEAT TRANSFER

4.1 Computation of the View Factor

The view factors depend upon the relative locations among all boundary elements. Number of pair boundary element is very large. Therefore, computation efficiency is required to be higher for saving the computation time. Each boundary element belongs to one of the surface model mentioned in previous section. In stead of computing the view factors between the boundary elements, the view factors between surface models are calculated first. And if the view factors between the surface models are not equal to zero, the calculation of view factors belonging to the surface models can be done (Fig.7). By this method, the computation time can be reduced. View factor F_{12} from boundary element A_1 to A_2 is defined by equation (5);

$$F_{12} = \frac{1}{\pi A_1} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{r^2} dA_1 dA_2 \ . \qquad (5)$$

Since equation (5) can not be integrated analytically, it is needed to be numerically. Therefore, we approximate equation (5) to equation (6);

$$F_{12} \sim \frac{1}{\pi A_1} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \frac{(R_{ij}\cdot N_1)\cdot(-R_{ij}\cdot N_2)}{|R_{ij}|^4} dA_i^1 dA_j^2 \ . \qquad (6)$$

where $R_{ij} = P_j^2 - P_i^1$, P_i^1, P_j^2 are centroid of divided small element dA_i^1, dA_j^2, N_1, N_2 are unit normal vector of boundary element A_1, A_2, n_1, n_2 are partition number of boundary element A_1, A_2, respectively (Fig.8).

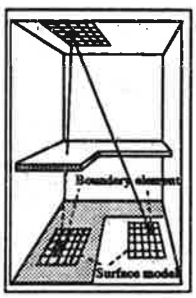


Fig.7 Geometrical position of surface model and boundary element.

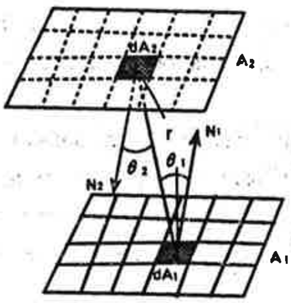


Fig.8a View facter F_{12}.

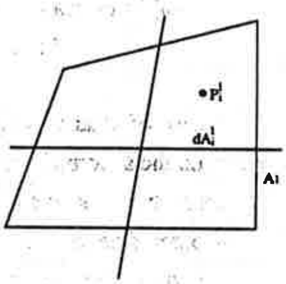


Fig.8b Partition of element A_1 (n_1=4).

Partition number n_1, n_2 of boundary element are defined by its size,shape, position (Table1).

Table 1 Equation for calculating number of partitions n_1, n_2 .

$$\text{Aspect partition ratio of an element} = \begin{cases} k_a : k_a A & (A < 4), \\ k_b : k_b A & (A \geq 4) \end{cases}$$

$$k_a = \begin{cases} 2 & (p \geq 0), \\ 4 & (p < 0) \end{cases}$$

$$k_b = k_a \ / \ 2,$$

$$p = r - k_p \max (A_1^{1/2} A_2^{1/2}),$$

A : Aspect ratio of element,

r : distance between the center of elements,

A_1,A_2 : each element area,

k_p : constant

When r is to be zero, $1/r^2$ is to be singularity. To overcom this problem, the partition number, n_1, n_2 are increased automatically.

4.2 Computation of the Radiative Heat Transfer

Model of radiative heat transfer on boundary element is;

$$\begin{aligned} J_i &= \sum_{j=1}^{N} F_{ij} G_j , \\ G_i &= \sigma \varepsilon T_i^4 + (1-\varepsilon) J_i , \\ \alpha_{ci}(T_{in} - T_i) + J_i - G_i + I_i &= K' (T_i - T_{out}) , \\ \int_s \alpha_{ci} (T_i - T_{in}) \, ds &= Q . \end{aligned} \qquad (7)$$

Here temperature, irradiation and radiosity in each boundary element are constant. Convective heat transfer is simply modeled in terms of $\alpha_{ci}(T_{in}-T_i)$. Unknown variables are J_i, G_i, T_i, T_{in}, and given variables are I_i, σ, ε, α_{ci}, K', T_{out}, S, Q, where J_i is irradiation of boundary element, G_i is radiosity, T_i is surface temperature, T_{in} is room air temperature, I_i is solar radiation absorbed, σ is Stefan-Boltzmann's constant, ε is emissivity, α_{ci} is the coefficient of convective heat transfer, K' is over-all heat transfer coefficient without contribution from inner total heat transfer, T_{out} is outer side reference temperature, S is boundary of domain, Q is heat source or sink caused by air conditioner or ventilation.

Strictly speaking, the radiation field is to be coupled with the convection field. However, in this analysis, we adopted the next simple procedure. First we solve the radiation field, and next we solve the convection field.

Equation (7) is solved by iterative method explicitly to make the computation faster. This method is expressed by equation (8);

$$
\begin{aligned}
G_i^{n+1} &= \sigma\, \varepsilon\, T_i^{n4} + (1-\varepsilon)\, J_i^n\,,\\
J_i^{n+1} &= \sum_{j=1}^{N} F_{ij}\, G_j^{n+1},\\
R_i^{n+1} &= J_i^{n+1} - G_i^{n+1}\,,\\
R_i^{n+1} &= \beta\, R_i^{n+1} + (1-\beta)\, R_i^n\,,\\
\int_s \alpha_{ci}\, (T_i^n - T_{in}^{n+1})\, ds &= Q\,,\\
\alpha_{ci}\, (T_i^{n+1} - T_{in}^{n+1}) + R_i^{n+1} + I_i &= K'\, (T_i^{n+1} - T_{out})\,,\\
T_i^{n+1} &= \beta\, T_i^{n+1} + (1-\beta)\, T_i^n\,,
\end{aligned}
\tag{8}
$$

where n is number of iteration, β is coefficient of relaxation. The succesive over relaxation , SOR, is adopted in equation (8). SOR is known to give a quick convergence in iterative methods.

Initial condition is given by equation (9);

$$
J_i^0 = 0,\;\; R_i^0 = 0,\; T_i^0 = T_w, \tag{9}
$$

where T_w is surface temperature after absorbing solar radiation.

Jugement of convergence in the iterative computation is carried out by the absolute error. The absolute error is defined by equation (10);

$$
E = \left| U^{n+1} - U^n \right|\,, \left| U \right| = \{ \sum_{i=1}^{N} U_i^2 / N \}^{1/2}, \tag{10}
$$

where U is variable to be solved. Here E_T, E_Q is the absolute error of temperature and amount of radiative heat transfer (J_i-G_i), respectively defined by above equation (10). We choose following values for E_T, E_Q;

$$
E_T < 0.01 \quad \text{and} \quad E_Q < 0.1\,. \tag{11}
$$

5 COMPUTATION OF TEMPERATURE AND FLOW DISTRIBUTION

In computation of flow and temperature field, non-isothermal k - ε model of turbulence is used as basic equations shown in Table 2.

Table 2 Basic equations (k-ε model of turbulence).

$$\frac{\partial U_i}{\partial x_i}=0$$

$$\frac{\partial U_i}{\partial t}=-\frac{\partial}{\partial x_i}(P+\frac{2}{3}k+\frac{1}{2}U_kU_k)+\varepsilon_{ijk}U_j\omega_k+\frac{\partial}{\partial x_j}\{(\frac{1}{Re}+\nu_t)(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i})\}+A_rT\delta_{i3}$$

$$\frac{\partial k}{\partial t}+U_j\frac{\partial k}{\partial x_j}=\frac{\partial}{\partial x_j}(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j})+(P_k+G_k)-\varepsilon$$

$$\frac{\partial \varepsilon}{\partial t}+U_j\frac{\partial \varepsilon}{\partial x_j}=\frac{\partial}{\partial x_j}(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j})+\frac{\varepsilon}{k}(C_1P_k+C_3G_k)-C_2\frac{\varepsilon^2}{k}$$

$$\frac{\partial T}{\partial t}+U_j\frac{\partial T}{\partial x_j}=\frac{\partial}{\partial x_j}\{(\frac{1}{Pe}+\frac{\nu_t}{\sigma_T})\frac{\partial T}{\partial x_j}\}$$

$$P_k=\nu_t(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i})\frac{\partial U_i}{\partial x_j}, \quad G_k=-A_r\frac{\nu_t}{\sigma_T}\frac{\partial T}{\partial x_i}\delta_{i3}$$

$C_D=0.09$, $C_1=1.44$, $C_2=1.92$, $\sigma_k=1.0$, $\sigma_\varepsilon=1.3$, $\sigma_T=1.0$

$C_3=0$ for $G_k\le 0$

$C_3=C_1$ for $G_k>0$

(Notation)

U_i : Mean air velocity, P : Mean pressure, k : Turbulence kinetic energy, ε : Turbulence dissipation rate, T : Mean temperature, ν_t : Kinetic eddy viscosity, P_k : Production term of k, G_k : Production term of k caused by buoyancy effect, ε_{ijk} : Permutation tensor, ω_k : Vorticity vector, Re : Reynolds number (= $\frac{U_0 \cdot L_0}{\nu}$), Ar : Archimedes number (= $\frac{g\beta\Delta T_0 L_0}{\nu}$), Pe : Pechlet number(Re•Pr), Pr : Prandtle number, U_0 : Reference air velocity, L_0 : Reference length, ν : Kinetic viscosity, g : Gravitational acceleration, ΔT_0 : Reference temperature difference, β : Thermal expansion rate of air.

Momentum equation is described in rotational form. These equations in Table 2 are discretized by Galerkin weight residual formulation with hexahedral elements and solved by the GSMAC3D code. Boundary conditions are shown in Table 3. Boundary conditions on walls are approximated by the generalized log-law and the wall function method. The details of discretization and numerical computation are reported in reference [1,2].

Table 3 Boundary conditions .

Supply outlet Boundary	$U_i = U_i^{in}, T = T^{in}, k = k^{in}, l = l^{in}, \varepsilon = \varepsilon^{in} = C_D \frac{(k^{in})^{3/2}}{l^{in}}$
Exhaust inlet Boundary	$U_i = U_i^{out}$ T, k, and ε are under free slip condition.
Openning boundary on a wall	$P = P_0$ $T = T_0$, $k = k_0$ and $\varepsilon = \varepsilon_0$ for inflow case. T, k, and ε are under free slip condition for outflow case. Judgement of inflow or outflow depends upon the sign of inner product of outward normal vector and air velocity vector.
Wall boundary	(Velocity field) First u^{*2} is calculated from eq (i) , then the shear stress on the wall boundary is obtained. The boundary condition about velocity field is given by substituting the shear stress for the boundary integration term of the momentum equation. $\frac{U_{1t}.(C_D^{1/2}k_1)^{1/2}}{u^{*2}} = \frac{1}{\kappa}\ln\{\frac{E \cdot h_1 \cdot (C_D^{1/2}k_1)^{1/2}}{\nu}\}$ (i) $U_{1n} = 0$ $\frac{\partial k}{\partial n} = 0,\ \varepsilon_1 = \frac{C_D^{3/4} \cdot k_1^{3/2}}{\kappa \cdot h_1}$, $E = 9.0$, $\kappa = 0.42$ (Temperature field) Thermal equilibrium equation eq.(ii) is assumed: $q_2 = q_1 + Q$ $q_2 = k'(T_w - T_{ref})$ (ii) $q_1 = \alpha_{ic}(T_i - T_w)$ Calculating q_1 from k' ,T_{ref}, Q and T_i in previous step, the boundary condition is given by substituting q_1 for the boundary integration term of the convective diffusion equation for temperature. The linkage with the calculation of solar radiation, radiative heat transfer and air convection is performed through the heat quantity, Q. If q_1 is fixed, it is to be substituted directly for the boundary integration term. In case of the adiabatic wall, q_2 is to be zero.

(Notation)

U_i^{in}, T^{in}, l^{in}, ε^{in} : Boundary values at supply outlet ,U_i^{out}: Boundary values at exhaust inlet, l : Length scale of turbulence, P_0 : Pressure at openning boundary on the wall, T_0 , k_0 , ε_0 : Boundary values at openning boundary on the wall, u^* : Friction velocity ($=\sqrt{\frac{\tau_w}{\rho}}$), τ_w : shear stress on the wall boundary, ρ : Density of air, U_{1t} , U_{1n} : Tangential and normal velocity on the wall boundary, k_1 , ε_1: k and ε values on wall boundary, h_1 : Length from the wall surface to the boundary of the adjacent ellement, κ : von Karman's constant.

6 CONCLUSION

In this paper, we describe the numerical methods and the techniques of computation for (1)solar radiation,(2)radiative heat transfer, (3)temperature and flow distribution in a very complicated shape atrium for the control of the thermal environment. The accuracy and reliability of the simulation are confirmed by comparison between the numerical results and measurements, and applications for the actual design of atrium will appear in the part 2 of this report.

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