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NEW MODEL CONCEPT TO CONTROL THE ENERGY AND MASS TRANSFER IN A THREE-DIMENSIONAL IMPERFECTLY MIXED VENTILATED SPACE

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Keywords : Control of micro-environment around a living organism, ventilated spaces, modelling, energy and mass transfer in imperfectly mixed fluids, greenhouses, livestock buildings

1 INTRODUCTION

The desired micro-environmental conditions in a process or in a ventilated space can only be achieved by good control of the process control inputs (Berckmans D., 1991). The tools in modern control theory to develop a process controller are based on the assumption that a dynamic mathematical model of the process to be controlled is available (Elgerd O.I., 1967).

From previous work it can be stated that the useful existing models have at least one of the following shortcomings when used for control purposes (Berckmans D., 1991):

1. they are based on the overall assumption of a perfectly mixed airspace in which no air flow pattern is considered; they assume that the ventilation rate is constant or is a linear function of inside temperature, which is not possible when the ventilation rate is considered to be a control input and which is almost never the case in modern controllers, they do not consider the aspect of high dynamic behaviour of the process for control purposes, they are not usable as a basis for adaptive control strategies since they are not compact enough to be implemented in a controller, they are not achievable for control purposes since they need too much calculation power and time.

2 OBJECTIVE

The objective of this paper is to present the theoretical considerations behind a model concept that should permit a deeper understanding of the process of micro-environment in a non-perfectly mixed three-dimensional fluid or an imperfectly ventilated air space (Figure 1). More specifically it is the objective to present a mathematical model concept :

1. in which a quantitative approach is made for the process of micro-environmental conditions in a non-perfectly mixed air space,

2. in which the existence of three dimensional air flow patterns in a non-isothermal ventilated space is considered,

3. in which the local micro-environmental variables to be controlled are : inside temperature, inside humidity and gas concentration,

4. that predicts the total (including transient) high frequency response of the micro-environmental variables defined above to non-linear variations of the process inputs : ventilation rate and heat supply,

5. in which every parameter has a physical meaning.

6. that is compact enough to be implemented in a controller.

3. THE METHOD

3.1. Hypothesis

Taking into account the objective of the desired model and knowing the shortcomings of existing models (Berckmans D., 1991), the following assumptions were based on familiarity with real systems (Fishman G.S., 1967):

1. In a non-perfectly mixed fluid process there exist relationships between the global process inputs and the local dynamic concentration distributions in the space.

2. The physical process behind these relationships forms the crucial part of the dynamic response of micro-environment in a non-perfectly mixed fluid. The stated physical process in mainly based on a mechanism of mass transport.

3. The new model concept offers a method for modelling and understanding these basic physical processes including the response of the fluid flow pattern.

3.2 The model concept

According to what has been shown in literature (Timmons M.B., 1980; Randall J.M., 1981; Randall J.M., 1979), the total volume of the ventilated space (subsequently called "the building volume") is considered to be a non-perfectly mixed air space. Consequently it is assumed that there is a three dimensional air flow pattern in the building volume as well as (related) gradients in the local micro-environmental parameters (temperature, humidity, gas concentration). Although the building volume is a non-perfectly mixed air volume and is considered as such in the model, it is always possible to define a control volume as being the maximum three-dimensional volume in which, by definition, there is perfectly mixed air. This means that within this control volume there are no gradients of temperature, humidity, gas concentration, air velocity etc. Consequently from the theoretical viewpoint this control volume is supposed to be infinitely small. It will be shown by experimental results that in reality this is not the case.

It is clear that according to this definition the perfectly mixed control volume can be positioned anywhere in the global non-perfectly mixed building volume. Since it is the objective to achieve a model for control purposes we consider the control volume being the maximum three-dimensional volume which is perfectly mixed and which is positioned around the temperature sensor of the control system.

If fresh air enters the building volume much of this air passes over, under, or alongside the control volume. Only a part $\dot{V}_{cv}$ of the global ventilation rate $\dot{V}$ enters the control volume (Figure 2). The rest of this global ventilation rate $\dot{V}$ leaves the building without ever passing through the considered control volume at the specified position. Similar behaviour is assumed for the internal moisture production by the occupants. As explained below the hygroscopic behaviour of the surrounding construction is neglected and the variable W is considered to be the total inside moisture production.

Only the part W_{cv} of the total moisture production W in the building volume enters the control volume. The same occurs with the internal heat production Q where it has to be noted that there are three sources of heat production : the occupants (animals in case of a livestock building), the heating system to supply additional heating (or cooling) and the building construction which is a source of (positive or negative) heat flow. Only the part Q_{cv} of the total heat production enters the defined control volume. The remaindes of the global heat production Q leaves the building volume without passing through the control volume. This means that Q_{cv} is considered to be the sum of the total heat flow arising from the three different sources and entering the control volume.

The moisture flow W_{cv} as well as the heat flow Q_{cv} entering the control volume can each be the sum of different flows (moisture from the manure-system or from the hygroscopic behaviour of the surrounding construction, heat from an additional heating system or from the long term behaviour of the building construction), but the concept is still valid. Indeed the temperature sensor in the control volume does not permit the identification of the heat source. Furthermore, this information is not of great importance for the resulting micro-environmental conditions (as defined above) within the control volume since they are measured continuously.

In this model concept there are two types of input variables : "global inputs" and "local inputs" (Figures 2 and 3). Initially the global inputs are ventilation rate V through the building volume and the heat supply Q_{hs} from the heating system to the building volume. There may be an air inlet as a third control input to control the airflow pattern in the building volume. The inside micro-environmental variables have to be controlled by these global control inputs. In addition to these, the local inputs to the control volume are the part V_{cv} of the global ventilation rate that enters the control volume, the part Q_{cv} of the total internal heat production Q that enters the control volume and the part W_{cv} of the total moisture production that enters the control volume.

3.2.1 Equation for inside humidity in the control volume.

Applying the law of mass conservation on the control volume defined above results in the **water mass equation** :

$$\boxed{\frac{dX_i}{dt} = -\alpha \,.\, V_{cv} \,.\, X_i + \alpha \,.\, V_{cv} \,.\, X_o + \frac{c_c}{\gamma_i \,.\, \varepsilon_i} \qquad (5)}$$

With :

c_c : $\frac{H_L \,.\, N_1}{Vol_{cv}}$: the volumetric concentration of moisture production in the control volume by the occupants ($J/s.m^3$)

γ_o : density of the fresh air entering the control volume (kg/m^3)

γ_i : density of the inside air in the control volume (kg/m^3)

t : time (s)

$V_{cv} = \dfrac{\dot{V}_{cv}}{Vol_{cv}}$: volumetric concentration of fresh air flow in the control volume (m^3 fresh air/s.m^3)

X_o : humidity ratio of the fresh air (kg water/kg dry air)

X_i : humidity ratio of the inside air in the control volume (kg water/kg dry air)

$\alpha = \dfrac{\gamma_o}{\gamma_i}$

ε_i : heat of vapouration of water at inside temperature (J/kg water)

3.3 Equation for temperature

When considering an infinitely small control volume surrounding a sensor it is possible from a theoretical standpoint to define theoretical inlet sections and outlet sections. It is considered that the mean time value of the velocity vectors of the air particles is perpendicular to those sections. By definition it is stated that within the control volume the physical properties of the fluid are equal over the whole volume and consequently over all the total inlet section as well as over the total outlet section (figure 3). Applying the general law of total energy conservation (internal, kinetic and potential energy) results in a temperature equation to be written as :

$$\boxed{\frac{dT_i}{dt} = -\beta \cdot V_{cv} \cdot T_i + \beta \cdot V_{cv} \cdot T_o + \frac{w_c}{\gamma_i \cdot c_{li}} \qquad (2)}$$

In which

c_{li}: specific heat of inside air (J/kg, K)

T_i: inside temperature in control volume (C)

T_o: outside temperature in control volume (C)

t: time (s)

V_{cv}: volumetric concentration of fresh air flow in control volume (m^3/s m^3)

w_c: volumetric concentration of total sensible heat flow in the control volume (J.s^{-1}.m^{-3})

γ_i: density of inside air (kg/m^3)

β: physical constant

3.4 Equation for gas concentration

A gas concentration equation, similar to the moisture balance equation (2), for the control volume can be written as :

It can also be written that (Berckmans, 1986):

$$\boxed{\frac{dg_i}{dt} = \delta \cdot v_{cv} \cdot g_o - \delta \cdot v_{cv} \cdot g_i + \frac{g_c}{\gamma_i} \qquad (3)}$$

with

$$\delta = \frac{j_o}{j_i}$$

$v_{cv} = \frac{\dot{V}_{cv}}{Vol_{cv}}$: volumetric concentration of ventilation rate in the control volume ($m^3/s.m^3$)

$g_{cv} = \frac{G}{Vol_{cv}}$: volumetric concentration of gas production in the control volume ($l/s.m^3$)

With :

g_i : acceptable inside gas concentration in the control volume (l/kg)

g_o : gas concentration in the entering fresh air (l/kg)

γ_i : density of inside air (kg/m^3)

γ_o : density of fresh air (kg/m^3)

G : gas production in the control volume (l/s)

$\dot{V}_{cv}$: volumetric ventilation rate through the control volume (m^3/s)

Vol_{cv} : volumetric content of the control volume (m^3)

3.5 Physical meaning of this model concept

In the three equations (1), (2) and (3), every parameter has a physical meaning. According to the basic assumptions of this model it is considered that there is a non-perfectly mixed air-space in the building volume. But, at a certain position in space, there is a perfectly mixed space : namely in the control volume (figure 2). Although the air in the global building volume is non-perfectly mixed, a control volume can always be defined as the maximum volume enveloping the control system's sensor in which, by definition, there is a perfectly mixed air space. From the theoretical viewpoint, this control volume is supposed to be infinitely small. But, from measurements, it can be shown that this volume can have significant dimensions (up to 60 m^3) for certain values of the process control inputs (Berckmans D., 1986).

If the amount of fresh air $\dot{V}$ enters the global building volume at every time step, much of this air passes over, under or alongside the control volume. Only the part $\dot{V}_{cv}$ of the global ventilation rate $\dot{V}$ enters in one way or another the three-dimensional control volume. The passage of this ventilation rate $\dot{V}_{cv}$ through the control volume generates the volumetric concentration of fresh air v_{cv} in the control volume. Similar behaviour is assumed for the internal heat, gas and moisture production in the building volume. In this way there is generated the v_{cv}, w_{cv}, c_{cv} and g_{cv} which consequently have a physical meaning in the equations (1), (2) and (3). In these equations all the Greek symbols α, β and δ are well-known physical constants such as density, specific heat, heat of vaporisation etc.

In this model concept it is assumed that the resulting values of the micro-environmental parameters T_i, X_i and g_i are mainly influenced by the energy and mass transfer realized by the part $\dot{V}_{cv}$ (of the global fresh air flow rate $\dot{V}$) that enters the control volume (figure 2). As long as this fresh air does not reach the control volume, it can be distinguished from the space air because of the non-perfect mixing. Other physical transport mechanisms (mass transport, conduction, radiation, etc.) are ignored because the hypothesis is that it is mainly the transport of fresh air that determines the considered micro-environmental variables. At the same time it is assumed that it is not possible to describe these mechanisms with physical laws in a useful way for control purposes.

By comparing the moisture equation (2) with the gas equation (3) it can be seen that, since both are mass equations, are very similar. It can also be stated that every other mass variable (dust, micro-organisms, chemical solution in a liquid fluid mixture, etc.) can be modelled in a similar way. To solve the model there is a requirement for two equations as will come clear. Because of the similarity between all mass equations, the two equations that are considered below are the temperature equation (2) and the moisture equation (3). Indeed every system consists of energy and mass variables.

The equations deduced in the previous sections and figure 2 can be represented schematically as in figure 3. It is clear that the part of the model describing the control volume is based on physical laws. It has three unknown parameters whose physical meaning has been explained. As concentration variables there value is directly related to the so called local input variables as explained above. The value of these three unknown variables V_{cv} w_{cv} and c_{cv} has to be determined before they can be used to model the process outputs T_i and X_i (figures 2 and 3). Since neither the size nor the shape of the control volume is known, these three unknown parameters can never be physically measured in a practical way. Even if the size V_{con} and shape of the control volume was known, it would still be physically impossible to measure the sum of the three dimensional fresh air flows that enter the three-dimensional control volume in one or another direction. Similarly the total sum of the heat flow coming from the heating element and from the animals and the heat exchange with the building envelope never can be measured. Identical reasoning applies to the moisture flow that enters the control volume.

The process outputs, inside temperature T_i and inside moisture X_i, however can be measured as well as the total process inputs, ventilation rate $\dot{V}$ through the building volume and heat supply Q by the heating element (figure 5).

The problem can now be split up into two steps.

1. determination of the values of the variables v_{cv}, w_{cv} and c_{cv} starting from measurement results of T_i, X_i, T_o and X_o. This is a problem of parameter estimation of the model given by equations (1) and (2) (see 3.6).

2. determination of the relationship between the total process inputs $\dot{V}$, Q, $H_s . N_1$ and $H_1 . N_1$ and the three variables c_{cv}, V_{cv} and w_{cv}. This is a problem of identification of the physical system that relates the total process inputs and the local concentrations V_{cv}, w_{cv} and c_{cv}. It is in other words the modelling of the energy and mass transport by the fluid in the three-dimensional space to the control volume.

It can be noted here that the control volume, a more general part of the system, is described with a model based on physical laws. In contrast to this first part, the second part of the system is very particular for the considered building or space. Furthermore the relationship between V_{cv}, w_{cv} and c_{cv} on the one side and $\dot{V}$ and Q on the other, is constantly changing in time and space (varying air flow pattern, moving occupants, etc.). Therefore this part of the system is modelled by on-line parameter estimation. This is from the reasoning that this part of the system is too complex to be modelled in a useful way by a model based on physical laws. Nevertheless the total model gives an improved physical understanding because every parameter in the model proposed here has a physical meaning.

3.6 Parameter estimation V_{cv}, w_{cv} and c_{cv}

It can be shown mathematically (Berckmans D., 1986) that starting from the continuous equations (1) and (2) :

$$\frac{dX_i}{dt} = -\alpha \,.\, V_{cv} \,.\, X_i + \alpha \,.\, V_{cv} \,.\, X_o + \gamma \,.\, c_c \qquad (1)$$

$$\frac{dT_i}{dt} = -\beta \,.\, V_{cv} \,.\, T_i + V_{cv} \,.\, T_o + \delta \,.\, w_c \qquad (2)$$

it is possible to obtain for one measurement, discrete equations: when using the transformations :

$$K_1 = \frac{\gamma \,.\, c_{cv}}{\alpha \,.\, v_{cv}}$$

$$K_2 = \frac{\delta \,.\, w_{cv}}{\beta \,.\, v_{cv}} \qquad (4)$$

$$A_1 = -\alpha \,.\, v_{cv}$$

$$B_1 = \alpha \,.\, v_{cv} \,.\, x_o + \gamma \,.\, c_{cv}$$

$$A_2 = -\beta \,.\, v_{cv} \qquad (5)$$

$$B_2 = \beta \,.\, v_{cv} \,.\, T_o + \delta \,.\, w_{cv}$$

it is possible to obtain the set of discrete equations :

$$(1 \;\; X_{o(k)} \;\; X_{i(k)} \;\; X_{i(k+1)}) \,.\, \begin{bmatrix} K_1 \,.\, (1 - e^{A_1 . \Delta t}) \\ 1 - e^{A_1 . \Delta t} \\ e^{A_1 . \Delta t} \\ -1 \end{bmatrix} = [0]$$

and (6)

$$(1 \;\; T_{o(k)} \;\; T_{i(k)} \;\; T_{i(k+1)}) \,.\, \begin{bmatrix} K_2 \,.\, (1 - e^{A_2 . \Delta t}) \\ 1 - e^{A_2 . \Delta t} \\ e^{A_2 . \Delta t} \\ -1 \end{bmatrix} = [0]$$

Once the values of A_1, K_1, A_2 and K_2 are determined, as shows figure 4, v_{cv}, w_{cv} and c_{cv} can be found by using sets (4) and (5).

As represented in figure 4 and proved by mathematical analysis (Berckmans 1980), the application of the Singular Value Decomposition (S.V.D.) is the key to solving three problems. Initially S.V.D. is used to estimate the noise on the measured data. Secondly, S.V.D. permits determination of the unknown variables K_1, A_1, K_2 and A_2 by using the transformations with equations (5), (4) and (6) the variables Vol_{cv}, w_c and c_c. Finally S.V.D. allows us to define a criterion which determines the number of observations m that should be handled in the matrices to achieve a mathematically reliable solution (See figure 4).

4 DISCUSSION

In the ventilated space considered as an example, the dynamic response of three physical micro-environmental parameters T_i, X_i and g_i has been modelled. The main objective was to create a model that is useful for control purposes and which should be compact. As opposed to most models resulting from on-line modelling techniques, this model is not a black box model. Every parameter in the model has a physical meaning which improves the physical understanding of the system and supports it applicability.

The reasoning behind this model concept is that every imperfectly mixed fluid-space has a more "general part" which can be considered as a perfectly mixed fluid-space and which is, by definition, the control volume. This more general part can thus be modelled by equations based on physical laws (figure 2 and 3). This part describes the dynamic behavior of the micro-environmental variables of interest.

The second part of the system is very specific for the specific fluid and in this particular bio-space. It is the relationship between the systems, control inputs and the parameters to be used in the first model of the (more general) control volume. This relationship consists of a complex physical process and is changing constantly as a function of time and of space. This physical process is too complex to be modelled in an accurate or useful way by a model based on physical laws. The specific model part is modelled with an on-line parameter estimation technique as a mathematical identification procedure.

The combination of the model based on physical equations (equation (1) and (2)) with the mathematical identification procedure results in a model concept that models the dynamic response of the physical micro-environment to variations of the process control inputs. More specifically to predict (in an on-line procedure) the high frequence dynamic response of inside temperature and humidity (and gas concentration) to variations of the control inputs ventilation rate $\dot{V}$ and heat supply Q. This is achieved by using continuous measurements of T_i, X_i and the total control inputs ventilation rate $\dot{V}$, heat supply Q, outside temperature T_o and outside humidity X_o. The other process inputs H_a . N and H_l . N are assumed not to change so fast. This means that the variables V_{cv}, w_{cv} and c_{cv} can be calculated continuously. Since they have a physical meaning it means that, as well as a certain factor that equals the size of the control volume, there is a known value for the total heat flow entering the three-dimensional control volume, the part of the fresh air $\dot{V}_{cv}$ entering the control volume, and the total moisture flow W_{cv} entering the control volume. Consequently at every instant it is possible to calculate the relationship between the local inputs ($\dot{V}_{cv}$, W_{cv} and Q_{cv}) or the model input parameters V_{cv}, w_{cv} and c_{cv} and the total system control inputs $\dot{V}$ and Q. In addition it can be shown that these relationships between local and total inputs allow modelling of the transient behaviour of air flow pattern (Berckmans et al., 1988, Illinois).

5 CONCLUSIONS

From this paper it can be concluded that:

1. A model concept (model and modelling technique) has been presented to model the dynamic response of physical micro-environmental variables (Namely : air temperature, air humidity and gasconcentration), to variations of the control inputs ventilation rate and heat supply.

2. The model concept is based on a mathematical model based on physical laws in combination with a mathematical identification procedure. The general part of the air space is described by equations based on physical laws : an energy-equation (temperature) and a mass equation (moisture). The more specific physical process of energy and mass transport related to the individual fluid space and its state is modelled by an on-line mathematical procedure.

3. Although the model concept is partly based on an on-line modelling technique, every parameter has a physical meaning. Consequently a deeper physical understanding of the process is generated and wider application of the concept is possible.

4. The basic assumptions for this model are : the fluid is a non-perfectly mixed air space, the control inputs ventilation rate and heat supply can vary in a non-linear way as most controllers do today.

5. The Singular Valve Decomposition is used to calculate the noise on the measurements, and to solve the unknown variables which correspond to three-dimensional energy and mass flows.

6. A criterion has been presented to calculate the number of measured samples that have to be used in the data-matrices (one for mass-variable and for the energy-variable) for mathematically reliable solution to be achieved.

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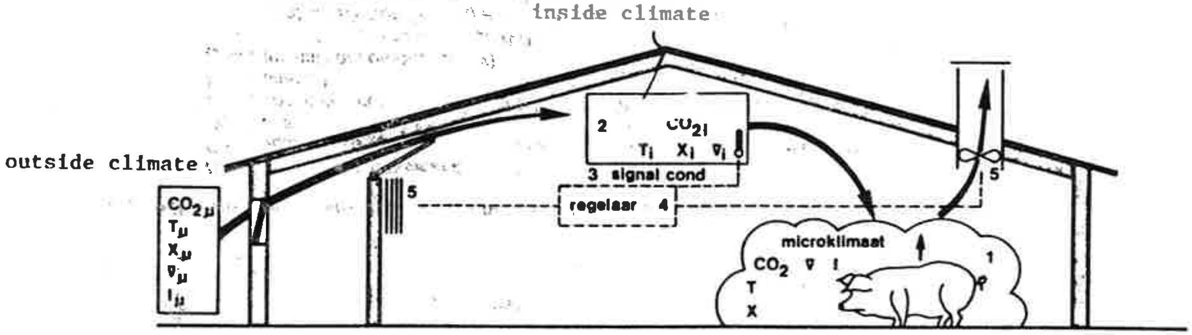

1 micro climate

2 sensor

3 signal conditioning

4 controller

5 actuators: ventilation and heating system

Figure 1. The considered process with its control inputs and its outputs to be controlled

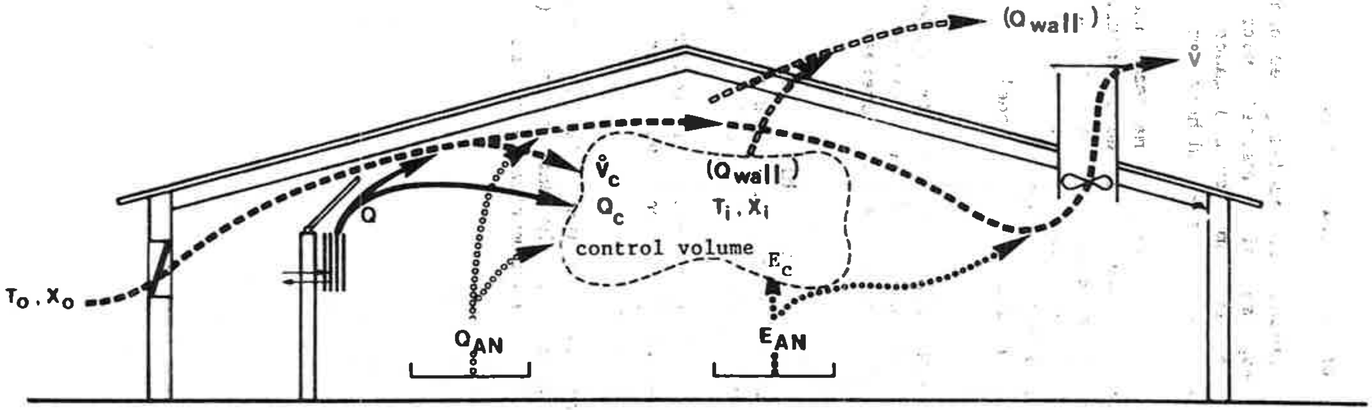

Figure 2. Representation of the energy and mass flows in the building volume and to the control volume as modelled in this model-concept

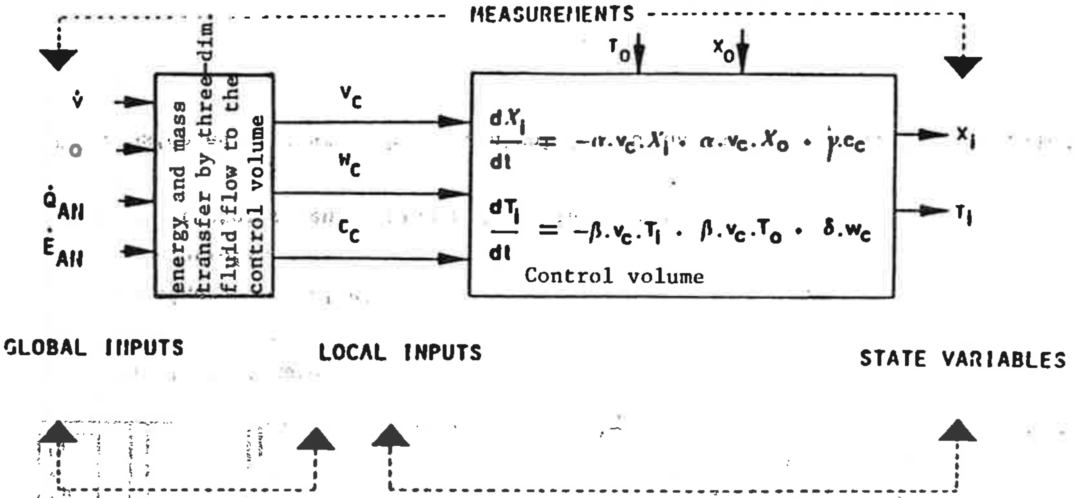

Figure 3 : *Schematic representation of the two modelling parts.*

Greek symbols : physical constants

C_C : *volumetric concentration of moisture flow in the control volume* ($Js^{-1}m^{-3}$).
V_C : *volumetric concentration of fresh air flow rate in control volume* ($m^3s^{-1}m^{-3}$).
W_C : *volumetric concentration of total sensible heat flow in control volume* ($Js^{-1}m^{-3}$)
t : *time (s).*
$\dot{Q}_{an}$: *animal heat production (W).*
$\dot{E}_{an}$: *animal moisture production (W).*
$\dot{Q}$: *heat supply from heating element to the building volume* ($Js^{-1}m^{-3}$).
$\dot{V}$: *ventilation rate through the building* (m^3s^{-1}).
T_i : *inside temperature in control volume (C).*
T_o : *outside temperature in control volume (C).*
X_i : *humidity ratio of inside air in control volume (kg water* kg^{-1} *dry air).*
X_o : *humidity ratio of outside air in control volume (kg water* kg^{-1} *dry air).*

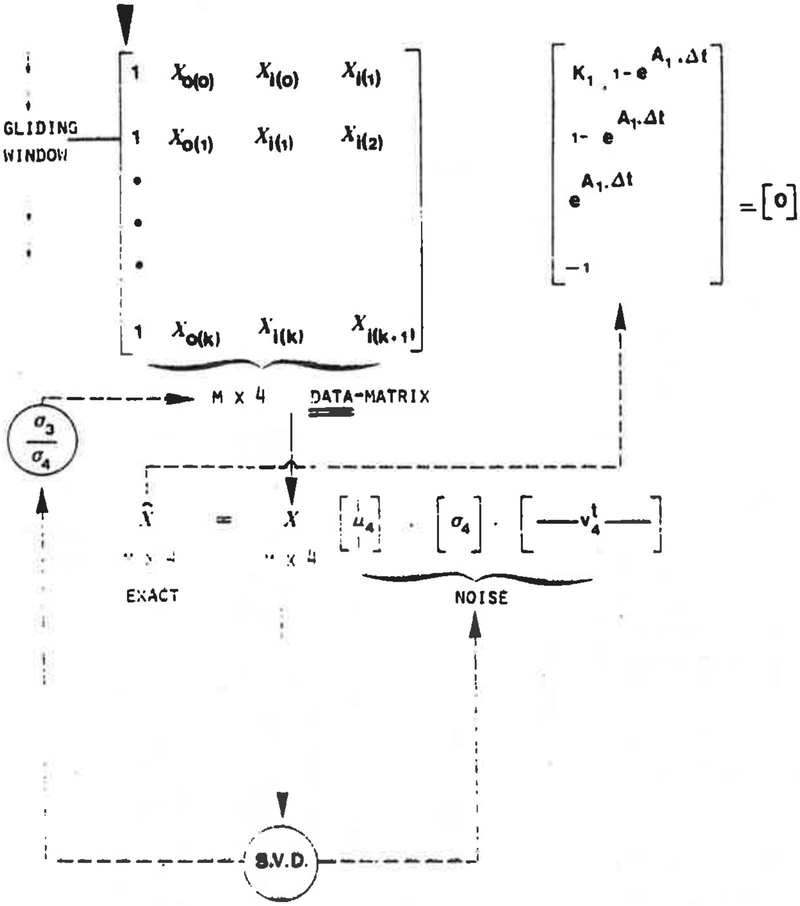

Figure 4. S.V.D. persuits: to estimate the noise on the data (1), to solve the unknown parameters (2) and to determine the number of rows needed in the datamatrix to obtain a reliable mathematical solution (3)