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THE AIR FLOW PATTERN CREATED BY A VENTILATOR UNIT REINFORCED BY A JET FLOW

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SUMMARY

In this paper a computational mathematical model of the air flow pattern created by a two-dimensional Aaberg exhaust hood ventilator unit, as described by [1], is presented. The model is obtained in terms of the stream function for a turbulent injection of fluid. Streamlines and lines of constant speed deduced from the model are examined for various heights of the ventilator unit above the floor surface of the workplace and predictions are given as to the region of the workplace and over what area of floor space the contaminated air can be successfully exhausted. The effect of the injection of fluid on the air speed along the floor surface and the ventilator's centre-line is examined for various heights of the ventilator unit above the floor surface.

NOMENCLATURE

a	radius of exhaust flange
A	effective suction area
d	diameter of the exhaust flange
G	dimensionless operating parameter, $G = \frac{\sqrt{3}}{2m}\left(\frac{Ka}{\sigma}\right)^{1/2}$
h	height of ventilator unit above floor surface
H	dimensionless height of ventilator unit above floor surface, H=h/a
K	flux of kinematic momentum of jet
m	volumetric flow rate of exhaust
q	resultant air speed
Q	dimensionless resultant air speed
s	semi-width of the exhaust inlet
S	dimensionless semi-width of the exhaust inlet
u, v	components of velocity
U, V	dimensionless components of velocity
U_i	dimensionless upstream air speed due to exhaust flow
v_i	fluid speed at face of exhaust inlet
(x, y)	cartesian coordinates with origin at the centre of the exhaust face
(X, Y)	dimensionless coordinates, X=x/a, Y=y/a
ν	kinematic viscosity of fluid
σ	experimentally determined constant, σ=7.67
ψ	stream function
ψ^*	dimensionless stream function, $\psi^*=\psi/m$
ψ_i	stream function across face of exhaust inlet
ψ_j	stream function along turbulent jet boundary

INTRODUCTION

Many industrial processes are exothermic and produce buoyant plumes. Such processes, for example reheat furnaces, can be equipped with a ventilator that functions as a hood to receive the hot plume of contaminant. In these and other processes, where it may be beneficial to draw the contaminant vertically upwards, it has been demonstrated experimentally [1] that a jet reinforced exhaust system, such as one employing the Aaberg principle, may, under the correct operating conditions prove advantageous over traditional hoods to draw the contaminant more effectively into the exhaust opening.

The Aaberg Principle

The discovery in 1965 of a new exhaust system, by C.P. Aaberg, a Danish manufacturer, has made it possible to create a controlled air movement over greater distances than possible with conventional exhaust hoods by combining exhaustion and injection in a correctly balanced ratio. The Aaberg exhaust hood is fitted with a flange through which air can be ejected radially from a narrow slit (see Fig. 1). Air is entrained by the jet, reducing the suction area of the exhaust and thereby concentrating the suction in a region along the longitudinal axis of the hood. This results in quite new velocity profiles in front of the exhaust opening and a directional exhaust capable of creating a flow towards the opening at distances of up to 10 times the exhaust diameter [1]. Although replacement air should still be supplied the Aaberg exhaust works with significantly smaller quantities of air than traditional exhausts. This, together with a higher concentration of pollutant in the exhaust air make the Aaberg process for limiting pollutant emission less expensive and more effective than traditional methods.

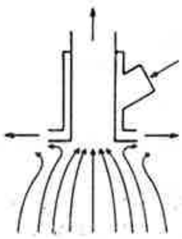

Fig. 1. The Aaberg Principle.

Fig. 2. A Ventilator Unit placed about 3.5d above the floor on which smoke powder is burnt.

The Ventilator Unit

The Aaberg ventilator unit (see Fig. 2), consists of an Aaberg exhaust hood suspended from the ceiling of the workplace vertically above the source of contaminant. Originally the Aaberg exhaust hood ventilator unit was three-dimensional axisymmetric in design but in this paper, for simplicity, we concentrate on a two-dimensional hood but with same principles of operation as for the three-dimensional hood.

THE MATHEMATICAL MODEL

The primary aim of this paper is to obtain a two-dimensional model of the air flow pattern created by the Aaberg exhaust hood ventilator unit. The ventilator unit is modelled as a two-dimensional flanged opening suspended a height h above the floor surface of the work place. The cartesian coordinate system adopted is with the origin at the face of the exhaust inlet, the x-axis along the flange of the hood and the y-axis along it's centre-line, see Fig.3. The fluid velocities in the x and y-directions are denoted by u and v, respectively. The centre-line of the hood and the solid floor surface of the workplace are streamlines of the flow through which the fluid may not cross and along these lines we may take the stream function $\psi=0$. Owing to the symmetry of the problem about the centre-line the flow need only be determined in the region $x\geq0$, $0\leq y\leq h$.

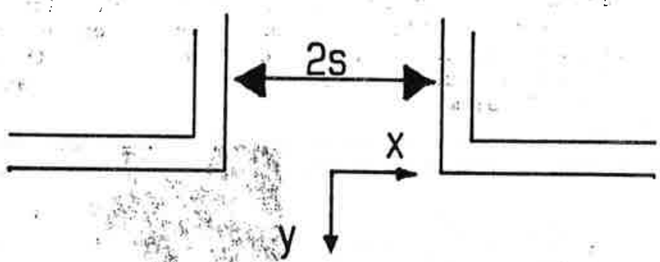

Fig.3. The geometry and coordinate system used.

In order to model the fluid flow it is necessary to make a number of simplifying assumptions, namely the fluid is assumed to be inviscid and incompressible and the flow to be steady. The problem is split into three parts:

(i) To model the suction opening and exhaust flow.
The suction opening is modelled as a finite-sized slot of width 2s, at the centre of a plane, the plane representing the flange of the exhaust. The flow induced by the flux of fluid, m, into the opening, modelling the exhaust flow, is assumed to be an inviscid potential flow.

(ii) To model the injection.
The injection of fluid is modelled as a two-dimensional turbulent jet issuing from the ends of the exhaust flange. The stream function for the turbulent jet is determined analytically using boundary-layer theory, see [2]. The solution is then used to find the value of the stream function at the edge of the jet boundary-layer.

(iii) To model the flow induced by the injection.
Owing to the friction developed at its boundary, the emerging jet carries with it some of the surrounding fluid which was originally at rest. The stream function for the jet-induced flow is found by assuming that the flow induced by a slender (i.e. high Reynolds number) jet is an inviscid potential flow [3].

The stream function modelling the total flow created by the Aaberg exhaust hood ventilator unit is then given by the combination of the exhaust flow and the jet induced flow.

Modelling the Suction Inlet and Exhaust Flow

In cartesian coordinates the continuity equation may be written

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

and we therefore introduce the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} . \qquad (2)$$

It is assumed that the flow is inviscid and irrotational and hence the equation governing the fluid motion is Laplace's equation,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 . \qquad (3)$$

At the face of the exhaust inlet the fluid velocity is assumed to be uniformly distributed with magnitude v_i, hence

$$v_i = -\frac{m}{2\,s} \qquad (4)$$

Denoting the stream function across the face of the exhaust inlet as ψ_i we obtain the following boundary condition

$$\psi_i = \frac{mx}{2s} \qquad 0 \le x \le s \, , \; y=0 \qquad (5)$$

Non-dimensionalising all lengths with respect to the radius of the exhaust flange, a, and the stream function with respect to m, we introduce the dimensionless quantities:

$$X = \frac{x}{a} \, , \quad Y = \frac{y}{a} \, , \quad H = \frac{h}{a} \, , \quad S = \frac{s}{a} \, , \quad \psi^{*} = \frac{\psi}{m} \qquad (6)$$

Hence, the dimensionless form of the boundary condition across the face of the exhaust inlet may be written

$$\psi_i^{*} = \frac{\psi}{m} = \frac{X}{2\,S} \qquad 0 \le X \le S \, , \; Y=0 \qquad (7)$$

The boundary condition for the flow far upstream created by the flow into the exhaust inlet is found by assuming that for $X \gg 1$, the flow is purely horizontal and has constant velocity, say $-U_i$, across the depth of the workplace. Under these assumptions the dimensionless form of the stream function far upstream is given by:

$$\psi_i^*(X,Y) = \frac{1}{2}\left(1 - \frac{Y}{H}\right) \qquad X \gg 1\ ,\ 0 \le Y \le H \qquad (8)$$

The remaining boundary condition along the ceiling of the workplace is taken to be

$$\psi^* = 0 \quad \text{for} \quad X > S\ ,\ Y=0 \qquad (9)$$

The solution of Laplace's equation subject to boundary conditions (7) (8), (9) and $\psi^*=0$ at the flow boundaries along the floor surface and ventilator centre-line will give a description of the flow into the exhaust.

Modelling the Injection of Fluid

The value of the stream function at the edge of the turbulent jet boundary-layer, determined from [2], is of the form:

$$\psi_j = \frac{\sqrt{3}}{2}\left(\frac{K}{\sigma}\right)^{\frac{1}{2}}(x-a)^{1/2} \qquad x>a \qquad (10)$$

In dimensionless form the boundary condition may be written

$$\psi_j^* = G(X-1)^{1/2} \qquad X>1 \qquad (11)$$

where, G denotes the dimensionless operating parameter which is defined as

$$G = \frac{\sqrt{3}}{2m}\left(\frac{Ka}{\sigma}\right)^{1/2} \qquad (12)$$

Boundary condition (11) is very important as it determines the flow induced by the turbulent jet. The dimensionless operating parameter G, which may be thought of as a parameter which models the ratios of the injection to exhaustion strengths, may be varied and its effect on the flow pattern investigated.

Modelling the Jet-Induced Flow

The equation of motion modelling the inviscid flow induced by the jet is the Laplace equation. In this simple model the slender jet is assumed to have zero thickness so that the boundary condition along the edge of the turbulent jet boundary-layer, given by equation

(11), is imposed along the X-axis.

The boundary condition modelling the flow far upstream created by the jet-induced flow is found by using asymptotic methods and to a second-order approximation is given by:

$$\psi^{*} = G(X-1)^{1/2}\left(1 - \frac{Y}{H}\right) - \frac{G}{4}(X-1)^{-3/2}\left(\frac{HY}{3} - \frac{Y^2}{2} + \frac{Y^3}{6H}\right)$$

$$- \frac{15}{16}\,G(X-1)^{-7/2}\left(\frac{H^3Y}{45} - \frac{HY^3}{18} + \frac{Y^4}{24} - \frac{Y^5}{120H}\right) \qquad (13)$$

for $0 \leq Y \leq H$ and $X \gg 1$.

A Two-Dimensional Model of the Aaberg Ventilator Unit

In practice this type of hood arrangement would be used for the capture of buoyant plumes of contaminant. However, in this paper the problem of modelling the fundamental air flow pattern is addressed and therefore the effects of buoyancy are neglected.

Owing to the complexity of the problem an analytical solution is not possible and finite-difference techniques are employed to solve the problem numerically. Once the approximations to the stream function at the grid nodes of the finite-difference mesh have been determined subject to the appropriate boundary conditions, the dimensionless velocity components U and V can be approximated and from them the resultant fluid speeds created by the ventilator predicted.

RESULTS AND DISCUSSION

The operating conditions at which [1] ran the ventilator unit have not been made available and so the operating conditions at which [4] ran a local Aaberg exhaust hood are used. The operating conditions lead to the following approximate values of the physical quantities a, m and K:

$$a \simeq 0.15\text{m} \qquad (14a)$$

$$m \simeq 0.05\text{m}^2\text{s}^{-1} \qquad (14b)$$

$$K \simeq 0.50\text{kgm}^3\text{s}^{-2} \qquad (14c)$$

and since the fluid used was air at about 20°C we have the kinematic viscosity of ν=1.7x10^{-5} Pa s. These quantities result in the operating condition which has $G \simeq 2$. It should be noted that the operating condition G=0 is equivalent to a traditional ventilator, i.e. it models a ventilator operating under suction alone.

Sets of streamlines describing the air flow pattern created by an Aaberg exhaust hood ventilator unit, deduced from the computational model, are shown in Figs.5a, 5b and 5c for a

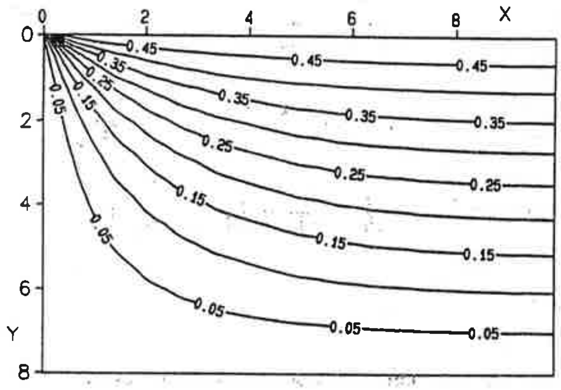

Fig. 5a. Streamlines modelling the Aaberg flow, G=0, H=8.

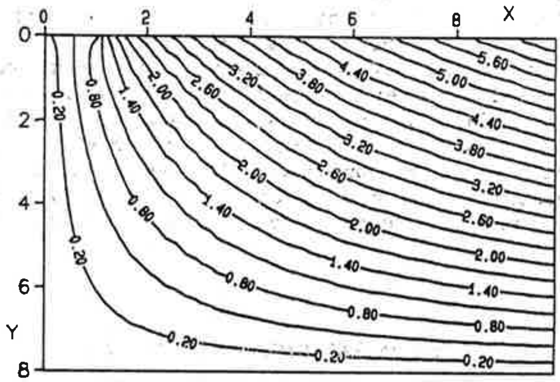

Fig. 5b. Streamlines modelling the Aaberg flow, G=2, H=8.

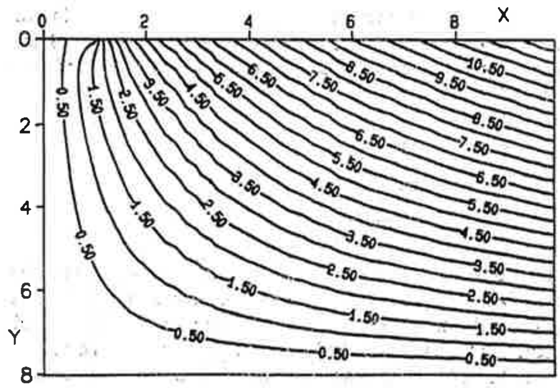

Fig. 5c. Streamlines modelling the Aaberg flow, G=4, H=8.

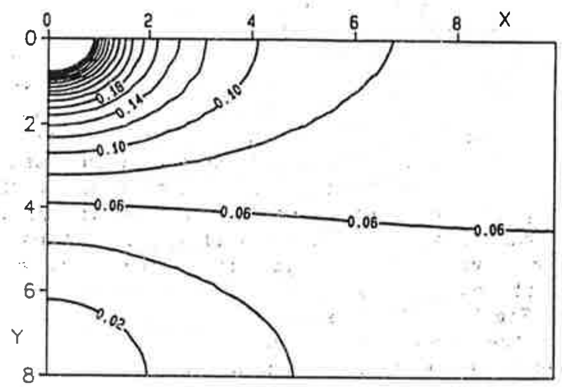

Fig. 6a. Lines of constant speed in the workplace, G=0, H=8.

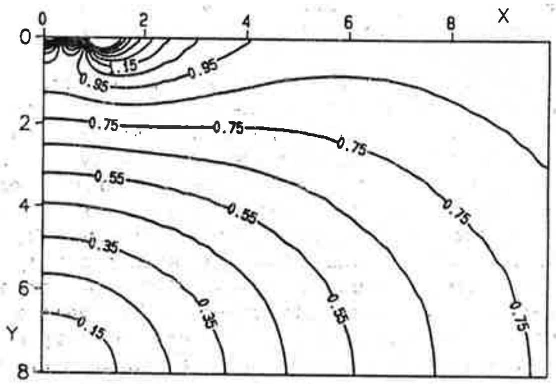

Fig. 6b. Lines of constant speed in the workplace, G=2, H=8.

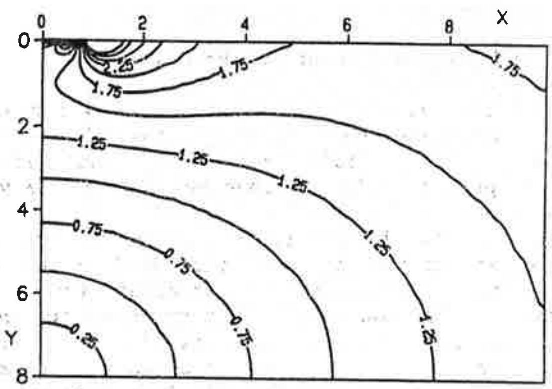

Fig. 6c. Lines of constant speed in the workplace, G=4, H=8.

ventilator suspended at the height of H=8 and for the parameters G=0, 2 and 4, respectively. Fig.5a which shows sets of streamlines modelling the flow created by a ventilator operating under suction alone (i.e. for the operating parameter G=0) illustrates that under these conditions the ventilator is non-selective, drawing air from all directions into the exhaust inlet. Fig.5b shows sets of streamlines modelling the flow created by a ventilator operating under combined exhaustion and injection, for the parameter G=2. In Fig.5b it can be seen that the contaminated air is now selected from an area immediately above the floor surface and drawn by the exhaust into the inlet. Fig.5c shows the effect of further increasing G from G=2 to G=4. The figure shows that the ventilator operating at G=4 now selects fluid from a very narrow layer immediately above the floor surface and draws it towards the inlet.

By comparing Fig.5a with Figs.5b and 5c it can be clearly seen that the effect of the injection of fluid on the air flow is to force the dividing streamline, $\psi = 1/2$, which divides the flow travelling towards the inlet from that travelling towards the ejector flow, towards the floor surface and thereby concentrating the suction along the floor area of the workplace. With combined exhaustion and injection the height of the suction area decreases, implying an increased fluid velocity along the floor surface towards the centre-line of the ventilator.

Figs.6 show lines of constant air speed in the workplace for a ventilator suspended at the height of H=8 above the floor surface, operating with zero injection (G=0) and with combined injection and exhaustion for the parameters G=2 and G=4, respectively. The effect of the injection of fluid on the lines of speed is quite complicated. By comparing Figs.6b and 6c with Fig.6a it can be seen that for the whole area of the workplace the air speeds developed by combining injection and exhaustion (G=2 and G=4) are significantly increased over those developed under exhaustion alone (G=0) and increasing the parameter G further increases the air speeds. The regions where the increased air speeds developed by the Aaberg principle are most significant for the effective control of the contaminant are along the floor surface and along the ventilator centre-line. These air speeds are now investigated for a ventilator operating under exhaustion alone and under combined injection and exhaustion.

Air Speeds along the Floor Surface of the Workplace

The air speeds along the floor surface of the workplace are of particular interest as once they are known the effective capture range of the hood at the floor level, for neutrally buoyant contaminants, may be predicted for various heights of the ventilator above the floor surface.

Fig.7a illustrates how the resultant air speed along the floor surface of the workplace varies, as a function of X, as the height of the ventilator above the floor surface is increased; for a ventilator operating under exhaustion alone (G=0) suspended at the heights of H=1,2,4 and 8 above the floor surface. The figure clearly illustrates how the air speed along the floor surface, created by the exhaustion, falls dramatically as the height of the ventilator above the floor surface is increased.

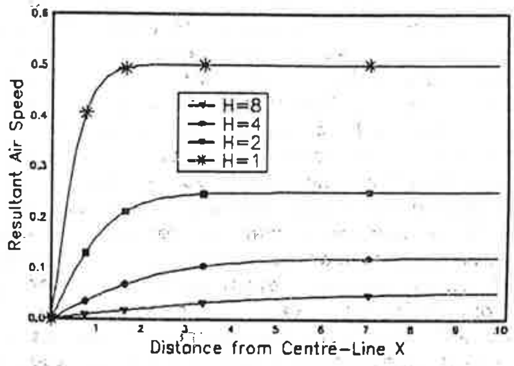

Fig. 7a. Variation in the air speed along the floor surface as a function of the ventilator height for G=0.

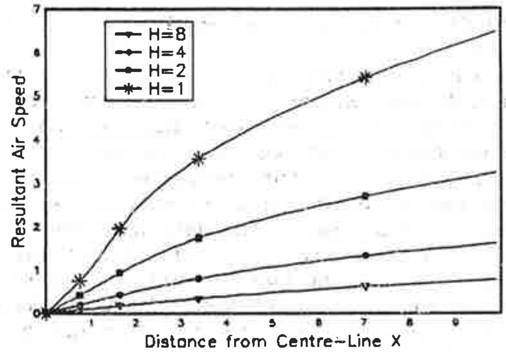

Fig. 7b. Variation in the air speed along the floor surface as a function of the ventilator height for G=2.

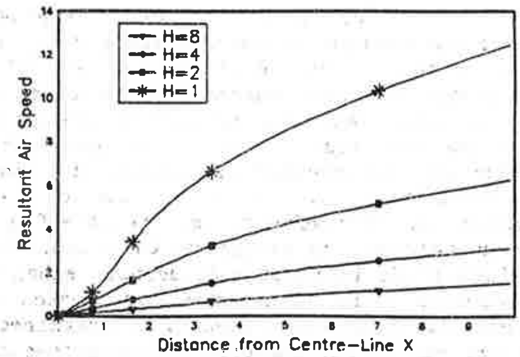

Fig. 7c. Variation in the air speed along the floor surface as a function of ventilator height for G=4.

Figs.7b and 7c illustrate how the resultant air speed along the floor surface varies, as a function of X, as the height of the ventilator above the floor surface is increased; for a ventilator suspended at the heights of H=1,2,4 and 8 operating with combined exhaustion and injection for the parameters G=2 and G=4, respectively. By comparing Fig.7a with Figs.7b and 7c the dramatic effect of the injection of fluid on the air speed along the floor surface is clearly visible. The injection effect significantly increases the air speed along the floor surface for each height of ventilator considered. Thus, contaminated air from either side of the ventilator's centre-line can be drawn, from a region immediately above the floor surface, towards the centre-line at an increased speed with combined injection and exhaustion than can be achieved with exhaustion alone. The increased air speeds developed when operating at G=2 and G=4 also serve to increase the lateral range of the ventilator, allowing contaminated air to be sampled from greater distances either side of the centre-line. The effect of increasing G is to increase the air speeds along the floor surface and the lateral range of the ventilator.

Air Speeds along the Centre-Line of the Ventilator

To examine how the turbulent injection of fluid effects the air speed along the ventilator centre-line, air speeds were calculated for a ventilator operating at G=0, G=2 and G=4 and the results obtained from the operating situations compared.

The variation in the centre-line speed, as a function of Y, for a ventilator suspended at the heights of H=1,2,4 and 8 above the floor surface and operating under exhaustion alone are shown in Fig.8a. Fig.8a illustrates that at each height of ventilator considered the air speed along the centre-line decays very rapidly as one moves away from the inlet; from a speed of Q=2 at the face of the inlet to Q=0 at the floor surface. The dramatic effect on the air speed along the ventilators centre-line obtained by combining injection and exhaustion is clearly illustrated in Figs.8b and 8c; which show the air speed along the centre-line for the ventilator suspended at the heights of H=1,2,4 and 8 for G=2 and G=4, respectively. Figs.8b and 8c show that, initially, as for a ventilator operating under exhaustion alone, the air speeds fall very sharply as one moves away from the inlet along the centre-line. However, at only a very small distance from the inlet (of the order of 0.25a) the injection of fluid begins to influence the flow, reducing the sharpness in the decay of the centre-line air speed.

By comparing Fig.8a with Figs.8b and 8c it can clearly be seen that with a ventilator at the height of H=1 above the floor surface the injection effect only slightly enhances the centre-line air speed; at this height the suction effect dominates the flow. However, as the ventilator is raised further above the floor surface the effect of the injection of fluid is to considerably enhance the flow into the exhaust opening, with increased air speeds along the centre-line predicted at each height of ventilator examined.

Thus, the model predicts that the effect of the injection on the air flow along the centre-line is only 'felt' after some minimum distance from the inlet has been exceeded. Once this distance has been exceeded the injection effect considerably increases the air

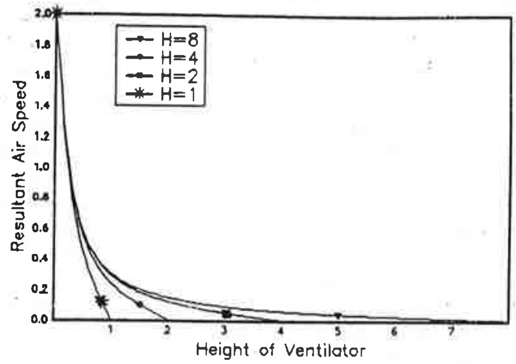

Fig. 8a. Variation in the air speed along the centre-line as a function of the ventilator height for G=0.

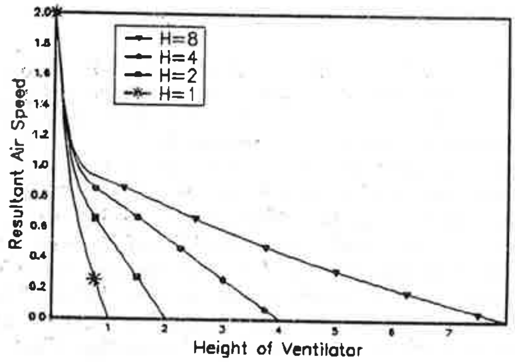

Fig. 8b. Variation in the air speed along the centre-line as a function of the ventilator height for G=2.

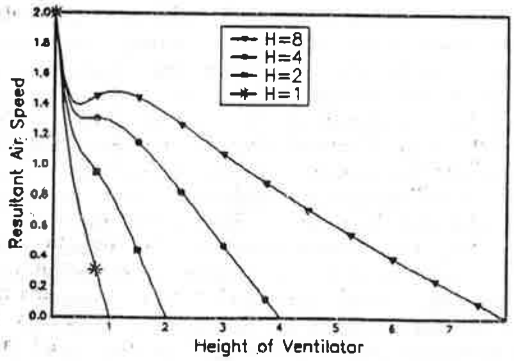

Fig. 8c. Variation in the air speed along the centre-line as a function of the ventilator height for G=4.

speeds along the centre-line. The role of the suction is then only to draw the contaminated air that distance along the centre-line, towards the inlet, over which the injection of fluid has little effect. The model also predicts that a ventilator operating under a correctly balanced ratio of injection and exhaustion will draw contaminated air, from a region immediately above the floor surface, towards the centre-line and then upwards into the exhaust inlet at greatly enhanced speeds compared to those of a conventional ventilator operating under suction alone.

The Effective Suction Area

Owing to the nature of the contaminant and to the background air movements the air speed along the floor surface of the workplace and upwards into the exhaust inlet must exceed a minimum speed, known as the capture speed, in order for the contaminated air to be sampled. Under normal practical conditions (for a neutrally-buoyant contaminant) the capture speed is of the order of $0.25ms^{-1}$, see [1,2]. A capture speed of $0.25ms^{-1}$ corresponds to a non-dimensional speed of $Q_c \approx 0.75$ for the operating conditions given in expressions (14). Thus, we can define the effective suction area A, from which the neutrally-buoyant contaminated air will be drawn into the inlet and successfully removed from the workplace, to be the area bounded by the line of constant speed $Q_c=0.75$ and the dividing streamline $\psi^*=1/2$. We can see from Figs.5 and Figs.6 how the shape of this area changes as G changes. On increasing the value of G from G=2 and G=4 we see that the height of the suction area decreases and its length increases. Hence increasing the value of G means that the lateral range of the ventilator across the floor area of the workplace increases. Detailed examination of Fig.5c and Fig.6c show that for X>4 the contaminant is drawn towards the ventilator centre-line in a fluid layer immediately above the floor surface and then enters a region where the air speed developed by the hood is less than the capture speed. In this region the contaminated air is free to randomly wonder and here diffusion effects will dominate the fluid motion. Contaminated air drawn upwards towards the inlet which has been successfully contained in the effective suction area then enters a zone where the air speed is greater than the capture speed and may then be sampled.

In this model the air flow pattern modelling the case of a neutrally-bouyant contaminant has been considered, however, including the effects of a non-neutrally-bouyant contaminant is possible but leads to complicated two-phase problems. It should be noted that, although in this model the effects of diffusion have been neglected, in some regions of the flow diffusion is the dominant effect. For example, when the fluid enters a region where the air speed developed by the hood is less than the capture speed the contaminant will be free to wonder randomly and here the diffusion effects will dominate the fluid motion. The effective suction area outlined above models that which would be obtained in ideal conditions, i.e. of neutrally-buoyant contaminant and negligible background air disturbances.

CONCLUSIONS

A computational mathematical model for the fluid mechanics of a two-dimensional Aaberg exhaust hood ventilator has been developed and the parameter G which characterises the flow identified. This simple model allows us to predict from what area of the floor surface of the workplace the neutrally buoyant contaminated air can be successfully removed as a function of this parameter. The model also predicts that in all aspects a much improved air flow pattern is developed when the traditional exhaust is reinforced with jet flow.

ACKNOWLEDGEMENTS

The authors would like to thank the Health and Safety Executive, Sheffield, England, for their guidance and for the financial support of part of this project.

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