6184

MATHEMATICAL MODELLING OF HEAT AND AIR PROCESSES IN MECHANICALLY VENTILATED SPACES

G.M.Pozin
Research Institute of Occupational Safety,
St.Petersburg, Russia

INTRODUCTION

In recent years due to increased sanitary-hygienic and technological requirements for industrial environment and to the construction of large integrated production departments ventilation systems have considerably grown in size and become more sophisticated. Greater importance is being attached to mechanically operated ventilation systems. Under such circumstances the problems of rational air supply and its effective utilization in ventilated spaces are becoming more and more vital.

The characteristics of heat, moisture and contaminants distribution within ventilated and air heated spaces are determined mainly by the generated air streams, which in their turn depend upon the accepted method of air-exchange set-up.

MATHEMATICAL APPROACH TO DESIGNING HEAT AND AIR REGIME IN A SPACE

Jet streams in a ventilated space affect significantly the circulation pattern of the air currents and contribute to heat and mass transfer in the space thus playing an important part not only in determining the velocities and temperatures in the occupation zone but also in solving various problems related to heat and air regime in the space /1/.

In order to calculate this regime a procedure for constructing approximate mathematical models has been developed. Such models are systems of heat and mass balance equations applicable to specific volumes and surfaces which take into account a great number of factors forming the environment in a ventilated space, such as the distribution of excess heat and heat losses through different zones in the space; the characteristics of inflow and convective jets; the operation of the local exhaust ventilation systems, air showers and heating systems; infiltration and inward leakage from the ajacent rooms /2,3/.

At the present stage of scientific development approximate models allow to perform integral evaluation of above factors affecting the heat and air regime in a space, which is still impossible to do to-day with the help of more sophisticated and precise models based on the solution of Navier-Stokes equations. In this sense approximate models posses certain advantages over precise models.

Approximate mathematical models have been developed for the main methods of air supply into production areas equipped with mechanical ventilation systems: directly into the occupation zone; concentrated air supply into the upper zone; air supply with vertical jets; inclined jets; jets clinging to the ceiling /2,3/.

AN EXEMPLE OF AN APPROXIMATE MATHEMATICAL MODEL

A design diagram of thermal and air processes which occur in an enclosed space in the case of air supply with vertical jets is shown as an exemple in Fig.1. The corresponding mathematical model (the principal system of equations) includes heat and air balances for five specific volumes (I - occupation zone; II - inflow jet; III - circulation zone; IV - convective jet above the heat source; V - convective flow at the wall) and two constraint equations. One of them defines the temperature in the occupation zone as a weighted temperature over the inflow jet area $\left(\bar{F}=\frac{F_i}{F_{space}}\right)$ and that of the back flow; the other one - the temperature of the air leaving the upper zone, depending upon relation between the convective flow rate and the flow rate of the air extracted from the space.

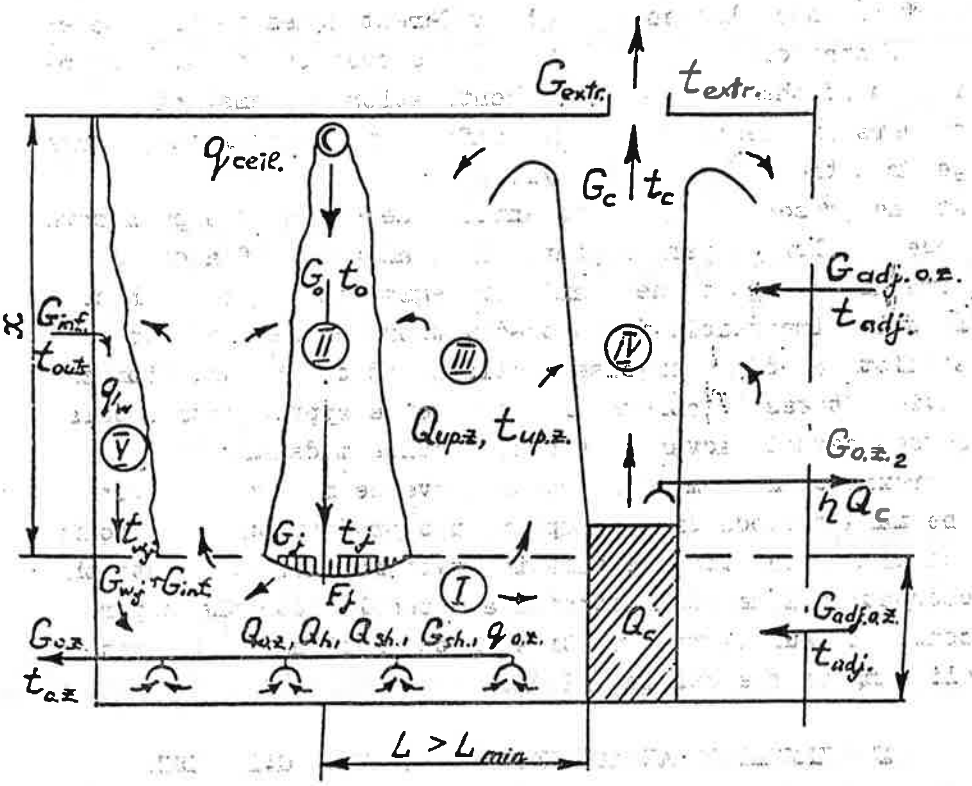

Fig. 1. Design diagram of thermal and air processes in an enclosed space in case of air supply with vertical jets

Seven equations of the principal system are linear in relation to the differences between the temperatures in specific volumes and the inflow air temperature which are assumed to be unknown values: $\Delta t_{o.z.}$ - for the occupation zone; Δt_{extr} - for the extracted air; $\Delta t_{up.z}$ - for the circulation zone; Δt_j and $\Delta t_{w.j}$ - for the inflow jet and the flow at the wall at their entering the occupation zone; Δt_b - for the back flow; Δt_c - for a convective jet under the

ceiling.
After transforming the system of equations and eliminating the unknown values Δt_c and Δt_{wj} its augmented matrix becomes:

$$
\begin{array}{cccccc}
\Delta t_{oz} & \Delta t_{extr} & \Delta t_{up.z.} & \Delta t_j & \Delta t_\ell & \\
\hline
\end{array}
$$

$$
\left\| \begin{array}{cccccc}
cG_{oz_1} + q_{oz} & 0 & -cG_{ij} + q_w & -cG_j & \Delta G_\ell & Q_{oz}^{tot} + (cC_{int} + q_w)\Delta t_{outs} \\
0 & 0 & c(G_j - G_o) & -G_j & 0 & 0 \\
cG_{oz_1} + q_{o.z.} & cG_{extr} & cG_{oz_q} + q_w + q_{ceil} & 0 & 0 & Q_{oz}^{tot} + Q_{up.z}^{tot} \\
1 & 0 & 0 & -\bar{F} & -(1-\bar{F}) & 0 \\
0 & a & -a & 0 & 0 & (1-\eta)Q_c
\end{array} \right\| \quad (1)
$$

In matrix (1) $a = max\{cG_{extr}, cG_c\}$;

$$Q_{oz}^{tot} = cG_{sh}\Delta t_g + cG_{adj\,o.z.}\Delta t_{adj} + Q_{oz} + Q_{sh} + Q_h + q_{oz}\Delta t_{outs};$$

$$Q_{up.z} = cG_{inf}\Delta t_{outs} + cG_{adj\,o.z.}\Delta t_{adj} + (1-\eta)Q_o + Q_{up.z} + (q_w + q_{ceil})\Delta t_{outs}$$

C - specific heat of the air.
The model describes heat and air processes in a space in the case of vertical air supply considering the relationship between the parameters taken into account in accordance with the diagram in Fig.1 which affect the formation of heat and air regime in the space, namely: $Q_{o.z.}, Q_{up.z.}, Q_c$ - heat input from the technological equipment; - specific heat losses; G_{inf} - the amount of infiltration air; $G_{adj.o.z.}$ and $G_{adj.up.z}$ - inward air leakage from the ajacent rooms; G_w and G_ℓ - air flow rates in the downward flow and in the back flow; Q_{sh} - heat consumption for air showers; Q_h - heat consumption for the heating system; G_{oz_1}, G_{oz_2} - air flows from local exhausts; $G_j\bar{F}$ -

characteristics of the jet flows formed by the air distribution equipment.
The unknown values in the system of equations (1) are found from Cramer's formulae:

$$\Delta t_{o.z} = \frac{d_1}{d}; \quad \Delta t_{extr} = \frac{d_2}{d}; \quad \Delta t_{up.z} = \frac{d_3}{d};$$
$$\Delta t_j = \frac{d_4}{d}; \quad \Delta t_j = \frac{d_5}{d}, \tag{2}$$

where d – determinant of the system; $d_1 \ldots \ldots d_5$ – determinants obtained by substituting in d a column of absolute terms for the first . . . fifth column, respectively.

Using these mathematical models one can solve various problems related to the design of heat and air regimes in ventilated spaces.

DETERMINATION OF AIR-EXCHANGE RATE

Determination of scientifically grounded amount of inflow air G_o has become one of their major applications:

$$G_o = G_{o.z} + \frac{Q - G_{o.z}\,\Delta t_{o.z}}{cK_L\,\Delta t_{o.z}} \tag{3}$$

where the air exchange coefficient

$$K_L = \frac{\Delta t_{extr}}{\Delta t_{o.z}} \tag{4}$$

It follows from (2) and (4) that

$$K_L = \frac{d_2}{d_1} \tag{5}$$

which is a function of all parameters included into the mathematical models.

PRINCIPLES OF SIMULTANEOUS DESIGNING AIR-EXCHANGE AND AIR DISTRIBUTION

Matrix (1) includes the parameters of jet flows $\bar{G}_j$ and $\bar{F}$ which depend upon the type of air distribution equipment characterized by the velocity coefficient m and temperature coefficient n ; its design area F_0 ; the distance between x and the occupation zone; the data characterizing the degree of confinement and non-isothermality of a jet.

Thus, in order to calculate air exchange rates needs to know a number of parameters related to air distribution, such as the type, the dimensions and the number of inflow air supply devices and their location, which, in their turn, are impossible to find without knowing G_0 .

Close relationship between air exchange and air distribution is observed when solving all problems concerning environmental parameters: uniform distribution of velocities and temperatures across the space area; circulation patterns of air currents; the extreme parameters for a jet entering the occupation zone, etc.

It can be concluded from the above that considering the present level of developments in ventilation science and research the calculations of the amount of inflow air and of its distribution through an enclosed space should be interrelated, that is, the principle of simultaneous designing air exchange and air distribution schemes should be applied /4/.

Practical realization of this principle is shown in the block diagram in Fig 2.

The relations found between the solutions of air exchange problems and those of air distribution which characterize this principle are indicated in the diagram with horizontal lines.

The realization of the proposed principle allows to make an optimal choice of air exchange schemes and methods of supplied air distribution in a ventilated space.

The mathematical model which allows simultaneous determination of air exchange rates and air distribution design includes:

- approximate models of thermal and air processes in ventilated spaces;
- laws of jet flows behaviour;

- a number of restrictions (limits for v_x, Δt_x, v_0 values, etc.);
- relationships describing specific character of the ventilation process (ensuring the designed circulation schemes, etc.).

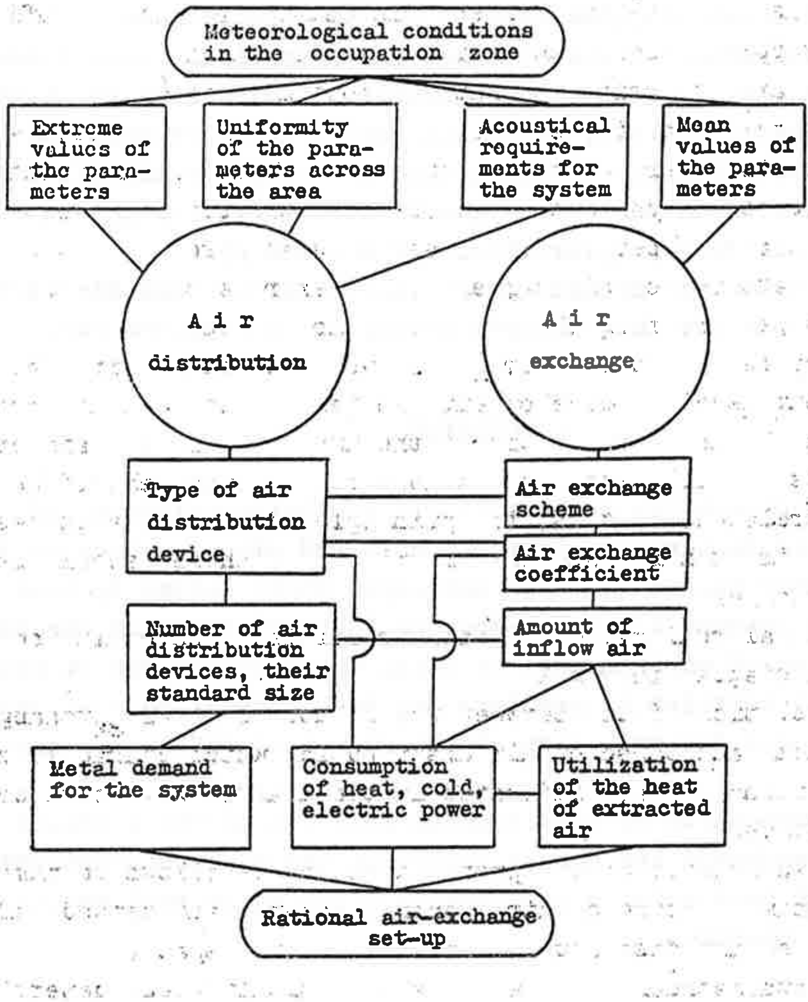

Fig. 2. Block diagram of a rational air-exchange set up

The mathematical problem consists in finding a solution for a system which includes linear and nonlinear algebraic equations and a number of inequalities. The method of successive approximations is used in the solution with obtaining one or several results which would satisfy the formulated problem. There are cases however when there can be no solution for a system.
Upon optimizing the results of the calculations on the basis of some indices (thermal and electrical power consumption, metal demand, possibilities of utilizing the heat of the extracted air, structural and other features) the most rational variant of organizing air exchange and air distribution in an enclosed space is arrived at, using the system analysis proposed by A.A.Rymkevich /5/.
The described methodological approach has been realized in full measure in a program system for a computer /6/.

CONCLUSION

A procedure for constructing approximate mathematical models designed to describe heat and air processes in ventilated spaces has been developed which allows to take into account a great number of factors affecting the formation of environment. By using these models one of principal ventilation problems has been solved, that of science-based determination of the amount of inflow air required.
The necessity of simultaneous solution of the problems of air-exchange and air distribution has been substantiated, which need using a computer in order to be realized in full measure.

LITERATURE

/1/ Grimitlin M.I. Air distribution in enclosed spaces// M.: Stroiizdat, 1982. - 164 s.

/2/ Pozin G.M. Principles of developing an approximate mathematical model for thermal and air processes in ventilated spaces // Izvestia vuzov. Stroitelstvo i arhitektura. - 1980. - No. 11. - S. 122-127.

/3/ Pozin G.M. Mathematical modelling of thermal and air processes in enclosed spaces // Industrialnye sitemy ventiliatsii i konditsionirovania vozduha: Sb. nauch. tr. / Vsesoyuzn. nauch. issled. institut gidromehanizatsii, san.-tehn. i spets. stroit. rabot (VNIIGS) L., 1990. - S. 17-23.

/4/ Pozin G.M. Principle of simultaneous designing air exchange and air distribution schemes // Nauchnye problemy ohrany truda na sovremennom etape: Sbornik nauchnyh rabot institutov ohrany truda VTsSPS. - M.: Profizdat, 1984. - S. 43-46.

/5/ Rymkevich A.A. System analysis of optimizing general-exchange ventilation and air-conditioning. - M.: Stroiizdat, 1990. - 299 p.

/6/ Pozin G.M. Computer-aided design of air-exchange and air distribution schemes // Mezhotraslevye voprosy bezopasnosti truda: Sbornik nauchnyh rabot institutov ohrany truda VTsSPS. - M.: Profizdat, 1984. - S. 13-18.