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NATURAL CONVECTION IN UNCLOSED AIR CAVITY TAKING ACOUNT WALLS' RADIATION

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SUMMARY

The particularity of heat transfer in unclosed cavity which is ventilated by natural convection is researched.

Temperature of one vertical boundary is known, but temperature of second vertical boundary is defined from heat balance conditions. Temperature of horizontal boundaries is given function.

Connected system of Boussinesq equation for air and linear algebraic equations for walls' heat radiation is the mathematical model of process. We cast the equations in the vorticity - stream function form. This equations are solved numerically, using finite difference technique.

Detailed results of the stream function, isotherms and velocities as well as vertically averaged and local values of Nusselt numbers are given. The natural convection air flow through region as a function of Grashof number is determined.

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INTRODUCTION

It is often necessary to find natural convection in unclosed region for investigation some problems of air - and temperature distribution in rooms. This problem has some peculiarity [1]. On the one hand the flow of convection aire is a priori unknown and must be found in numerical process. On the other hand it is necessery to take into account walls' radiated heat. In addition the unknown temperature of some boundary must be defined with the help of heat balance.

As example this problem in this paper the movement and heat transfer in convected canal of panel heating system are investigated. The results, discussed in the following, can be used in analisys of natural convection in unclosed air cavity.

DESCRIPTION OF PROBLEM

Fig. 1 shows general scheme of panel heating system with convected canal. Hot panel 1 with temperature t_p transfers heat immediately to room from inner surface as well as to convected canal 2 from another surface. Definition of this addition heat is general aim in the physical problem. The enceinte wall 3 separates the room and convected canal from surroundings with outdoor temperature t_n .

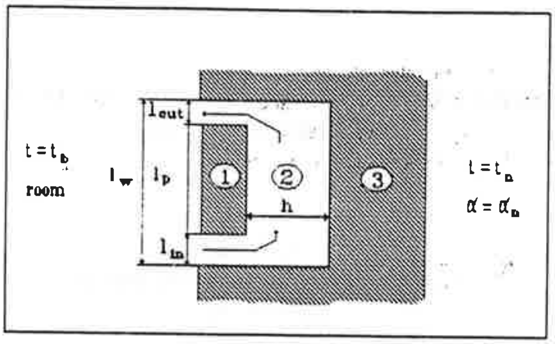

Fig. 1. General scheme of panel heating system(1 Hot panel 2 Convected canel 3 Enceinte wall).

Let t_b is the room air temperature, h - the width of canal, l_p and l_w - lenght of panel and vertical boundary of wall, l_{in} and l_{out} - dimensions of inlet and outlet of canal.

To formulate matematical model we'll require, that following assumption are realized.
1. Air movement and heat transfer in canal by Boussinesq approximation are described.
2. Air in canal is transparent for radiated heat of walls. The rest of the asumption are described in reference [2] .

Estimations show, laminar natural convection is realized for using parameter h=0.05m, l_p = 0.5 m , t_b = 20 ^oC , t_n = - 20 ^oC , t_p = 30 - 70 ^oC . Furthermore a little part of the heat transfer is dissipated through the wall. It allows to consieder this heat transfer only approximately. This procedure is described below.

MATHEMATICAL MODEL

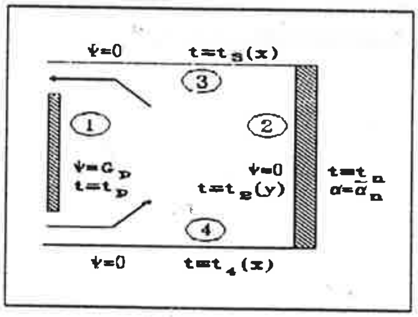

Fig. 2. Calculation scheme.

Fig. 2 shows the calculation scheme of problem. The air cavity borders are vertical wall surface 1 , 2 and horizontal wall surface 3 , 4 , inlet and outlet of canal. In this scheme enceinte wall 3 in Fig. 1 is replaced by partition 2 with great heat transfer resistance. However this average resistance is expressed by the formula

$$\tilde{\alpha}_n = \alpha_n + \frac{1}{R_n} \tag{1}$$

where α is surface coefficient, R_n is average thermal resistance of enceinte wall. Temperature $t_2(y)$ along partition 2 is result of its heat interaction with other surfaces and air cavity by convected and radiated heat.

In this assumption for laminar incompessible viscous flow equations in non-dimensional form become [2]

$$\frac{\partial(u\omega)}{\partial x} + \frac{\partial(v\omega)}{\partial y} = \Delta\omega + Gr\frac{\partial\theta}{\partial x} \tag{2}$$

$$\frac{\partial(u\theta)}{\partial x} + \frac{\partial(v\theta)}{\partial y} = \frac{1}{Pr}\Delta\theta \tag{3}$$

$$\Delta\psi + \omega = 0 \tag{4}$$

$$\frac{d^2\theta_2}{dy^2} - M_n\theta_2 - M_2\left[\frac{\partial\theta}{\partial x}\right]_2 - Mq_{r2} = 0 \tag{5}$$

$$q_{rm} - \sum_{j=1}^{N}(1 - \varepsilon)q_{rj}\varphi_{mj} = \frac{\varepsilon C_0 h}{\lambda}\sum_{j=1}^{N} b_{mj}\varphi_{mj}(\theta_m - \theta_j) \tag{6}$$

$$(j = 1, 2, \ldots, N)$$

Where scales of length, velocity, temperature and flux heat density are h, v/h, $t_b - t_n$, $\lambda(t_b - t_n)/h$. Other designations: u, v, w, ψ, θ are components of velocity, vorticity, stream function and temperature

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x}, \quad w = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \quad \theta = \frac{t - t_n}{t_b - t_n}$$

$$Gr = \frac{\beta g h^3(t_b - t_n)}{\nu^2}, \quad Pr = \frac{C_p\nu\rho}{\lambda}$$

are Grashof and Prandtl numbers; β, ν, ζ, λ, C_p - coefficients of cubical expansion , kinematic viscosity, density, conduction and specific heat of air; M_n, M_2, M are values such as Bio number; $q_{r2}(y)$ is flux density of radiated heat of surface 2, q_{rm} (m = 1, 2, . . . N) is local flux density of radiated heat, N - quantity of isothermal segments on surfacis 1, 2, 3, 4, which are used for calculation of radiated heat. The rest of designations and details of calculation are described in [2]. The equation's system (6) and equation (5) describe flux density of radiated heat and temperature distribution along surface 2 accordingly.

Boundary conditions for inlet of air cavity are

$$t = t_b \; , \; \frac{\partial\psi}{\partial x} = \frac{\partial\omega}{\partial x} = 0 \tag{7}$$

and for outlet of air cavity

$$\frac{\partial\psi}{\partial x} = \frac{\partial w}{\partial x} = \frac{\partial t}{\partial x} = 0 \quad \left(\text{or} \; \frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 t}{\partial x^2} \right) \tag{8}$$

Boundary condition for t and ψ are shown in Fig. 2. Value of stream function G_p on panel surface for natural convection must be determined in calculation process. Some methods for definition G_p are described in [3]. Side by side this methods in this paper another method is used. It is based on the correlation

$$h(\rho_b - \rho_c)g = \rho_c V^2/2 \, (\Sigma C_m + C_f) \tag{9}$$

where ρ_b, ρ_c are air density in room and average air density in canal, V is average velocity, C_f - viscous friction coefficient on the wall, ΣC_m is the sum of local air resistance, g - acceleration due to gravity. The values C_f and r_c are calculated numerically from equations (2) - (4). $P = \Sigma C_m$ is external parameter. It characterizes resistance toward convection stream outside canal. Equation (9) may by expressed as $F(G_p) = 0$. G_p is defined from this equation by iterations.

RESULTS

Numerical results were obtained for the next parameters Pr = 0.7, $\varepsilon = 0.9$, $t_b = 20^oC$, $t_n = -20^oC$. Two cases P = 0.66 and P = 2.16 were investigated. The former takes into account only turnings in cavity. The letter in addition takes into account local resistance of air inlet and outlet outside the cavity. The results of computing of the mean velocity V_c and the dimensionless air flow G_p for natural convection in the cavity are shown on the Fig. 3. They are expressed as a functions of Grashof number $Gr_{pb} = \beta g h^3 (t_p - t_b)/\nu^2$.

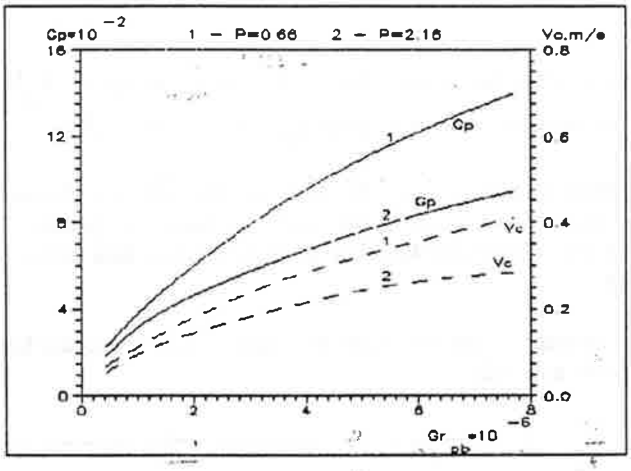

Fig. 3. Natural convection of air in a cavity as a function Grashof number Gr_{pb}.

It is obvious from physical consideration that natural convection for P = 0.66 must be the largest for Gr = const. The rest of natural convection conditions lie below curve P = 0.66.

Fig. 4 shows velocity distribution in different canal sections for various Grashof numbers Gr_{pb}. The curves correspond to different values of $Z = Gr_y/Re_y^2$, where $Gr_y = \beta g y^3(t_p-t_b)/v^2$ and $Re_y = uy/v$ are local Grashof and Reynolds numbers.

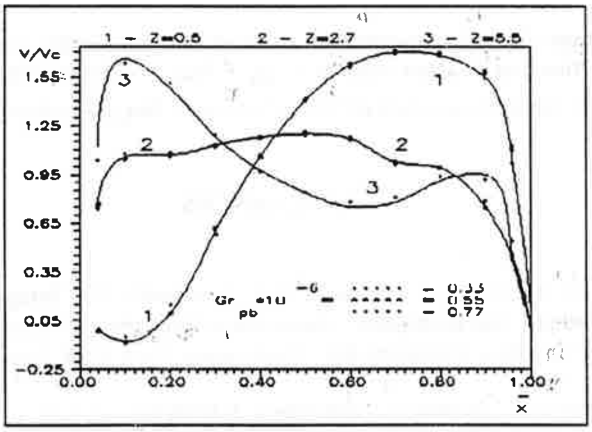

Fig. 4. Velocity distribution as function of dimensionless value $Z=Gr_y/Re_y^2$ for various Grashof numbers Gr_{pb}.

The velocity profiles for various Gr_{pb} are similar, if values Z are identical. This similarity executes only for great values Gr_{pb} ($Gr_{pb} > 0.3*10^6$).

In initial sections of canal Archimedes force is little. There is the turning character of flow in this sections is pressed to the wall. Increasing the parameter Z , the role of Archimedes force is increased too. The air is accelerated near hoting walls and reduce speed near axis.

Fig. 5 shows average Nusselt numbers for radiated and convected heat on the vertical surfaces of panel and wall.

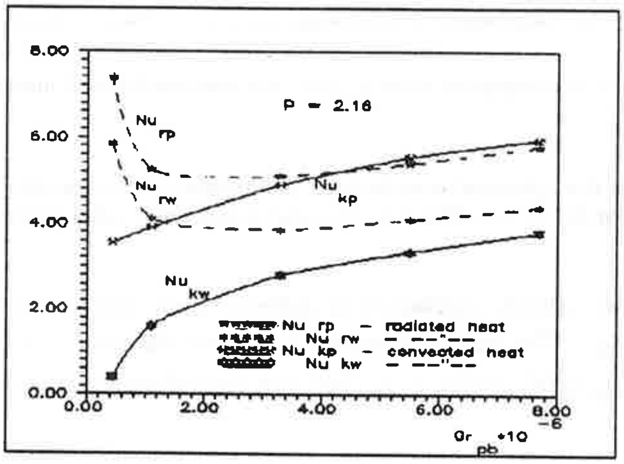

Fig. 5. Average Nusselt numbers for radiated and convected heat on the vertical surfaces as a function Grashof number Gr_{pb}. (Nu_{rp} - radiated heat in panel, Nu_{rw} - radiated heat in wall, Nu_{kp} - convected heat in panel, Nu_{kw} - radiated heat in wall).

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