

# Numerical Simulation of Air Flows in Gymnasia

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**Abstract**

The designers of ventilation systems need to predict the air flow patterns in order to optimize design and to ensure a healthy interior. Numerical simulation is a powerful tool to obtain the air flow patterns. In the present study, a computer program which solves the three-dimensional conservation equations of mass, momentum, energy, and contaminant concentration is used. The program is based on the k-ε turbulence model with wall function expressions for solid boundaries. Flow fields are computed for two gymnasia, of 24×12×9 m<sup>3</sup> and 44×23×10 m<sup>3</sup>, with variations in the ventilation rate, the arrangement of inlet and outlet, heating system, and the number of occupants. The simulation gives the field results for air velocity, temperature, contaminant concentration, percentage of dissatisfied people due to draught, and predicted percentage of dissatisfied due to thermal comfort.  
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## Introduction

Accurate prediction of air velocity, temperature, contaminant concentration distributions and thermal condition in a room is indispensable for designing high-quality air conditioning and ventilation systems from the viewpoint of comfort, health and energy saving. As computer resources have increased, more numerical simulations of air flows have appeared [1], but there are still very few studies of large enclosures, such as theatres, gymnasia and auditoria because such simulation requires much computer storage and CPU time, and is expensive. However, researchers are beginning to identify and understand the problems associated with the numerical simulation of buoyant flow in large spaces, and this paper considers large-scale airflow simulation based on the computational results of 18 cases for two gymnasia.

## Numerical Approach

### Physical Equations

The air flow in a gymnasium is a three-dimensional turbulent flow. In the field of numerical simulation of air flow in buildings, the k-ε turbulence model is most commonly used [1-6], and several studies have indicated the appropriateness of this model [7-10].

In the present study, the PHOENICS computer program, as documented by Ludwig et al. [11], has been employed to solve the system of Navier-Stokes equations with the standard k-ε turbulence model which is presented in Appendix A.

A designer is not only interested in the distributions of the basic variables such as velocity, temperature and contaminant concentration, but also needs information on derived quantities to assess occupant comfort. For this purpose, we programmed the evaluation of several comfort parameters directly in the output routine of PHOENICS: the PMV (predicted mean vote) and PPD (pre-

dicted percentage of dissatisfied) for the thermal comfort [12], and the PD (percentage dissatisfied people) due to draught sensation [13]. The mathematical models of these comfort parameters are listed in Appendix B.

### Cases Considered

The configurations of the two gymnasia are illustrated in figures 1 and 2. Eighteen cases were computed for the two gymnasia with variation in ventilation rate, the arrangement of inlet and outlet ports (including a well-mixed system, a displacement ventilation system, and a ceiling supply system), heating system (including floor heating and radiator heating), and the number of occupants. All of these cases refer to winter conditions. The principal information for the 5 cases mentioned in this paper is given in figure 3.

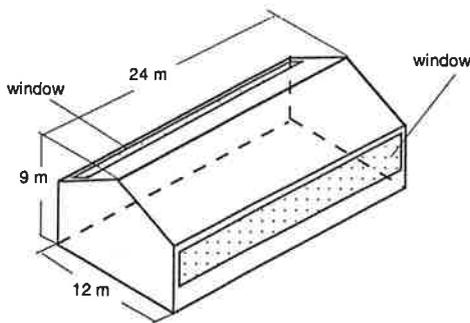


Fig. 1. The configuration of gymnasium 1 (Gym 1).

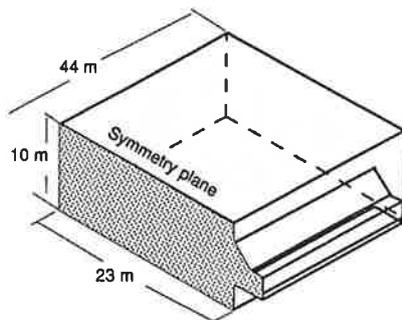


Fig. 2. The configuration of gymnasium 2 (Gym 2).

### Boundary Conditions and Heat Source

Boundary conditions are specified as follows. At the air inlet, a fixed mass flow rate is specified and also the values of velocity, enthalpy, the kinetic energy of turbulence, the energy dissipation rate, and contaminant. The inlet kinetic energy of turbulence,  $k_{in}$ , is fixed at:

$$k_{in} = (0.1V_{in})^2/2$$

and inlet energy dissipation rate,  $\epsilon_{in}$ , at:

$$\epsilon_{in} = 0.09k_{in}^{3/2}/0.5478L_{in}$$

$$L_{in} = 2ab/(a+b)$$

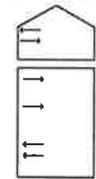
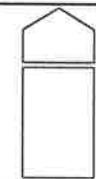
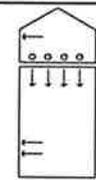
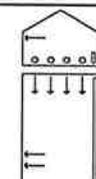
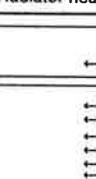
Case	Arrangement of ventilation and heating systems	Input data
1	 Floor heating	Ventilation rate: 0.36 ach Inlets: 2x0.15(m)x0.40(m) Inlet air velocity: 2.3 m/s Outlets: 2x0.40(m)x0.40(m) Inlet air temperature: 16.0°C Window temperature: 12.4°C Wall temperature: 15.2°C Roof temperature: 15.3°C Floor temperature: 17.7°C Number of occupant: 23
2	 Floor heating	Ventilation rate: 0.00 ach (natural convection) Window temperature: 12.4°C Wall temperature: 15.2°C Roof temperature: 15.3°C Floor temperature: 17.7°C Number of occupant: 23
3	 Floor heating	Floor temperature: 17.7°C  Ventilation rate: 0.70 ach Inlets: 4x1.5(m)x0.40(m) Inlet air velocity: 0.23 m/s Outlets: 2x0.40(m)x0.40(m) Inlet air temperature: 16.0°C Window temperature: 12.4°C Wall temperature: 15.2°C Roof temperature: 15.3°C Number of occupant: 23
4	 Radiator heating	Floor temperature: 15.8°C
5	 Radiator heating	Ventilation rate: 0.14 ach Inlets: 7x0.071(m)x0.071(m) Inlet air velocity: 5.5m/s Outlets: 0.071(m)x23.0(m) Inlet air temperature: 16.0°C Window temperature: 12.4°C Wall temperature: 15.2°C Roof temperature: 15.3°C Floor temperature: 15.7°C Number of occupant: 40

Fig. 3. The principal information for the 5 cases.

where  $a$  and  $b$  are the height and width of the inlet, respectively.

At the outlet a fixed pressure is imposed. The logarithmic law of the wall is applied near walls. At the walls, temperature and the convective heat transfer coefficient are fixed.

The heat and contaminant ( $\text{CO}_2$  or tobacco smoke) released by people are considered as uniform sources located in the layer between 0.3 and 1.6 m above the floor. The source strengths representative of a person are listed in table 1.

#### Mesh System and Computer Time

In order to represent supply location and near wall region by fine grid, a non-uniform grid system is used. Figure 4 shows the grid system definition of Gym 1 in which a stair-shaped, heavy line represents the slope of the roof.

Because the flow is strongly buoyant, two thousand or more sweeps (or iterations) are required to reach a converged solution for all cases.

The mesh system and CPU time for each case are shown in table 2.

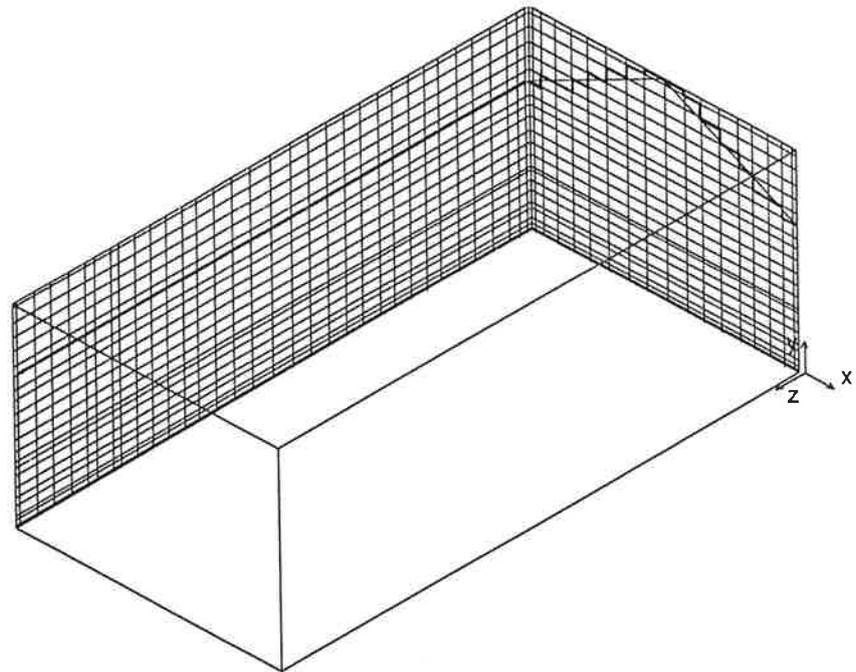


Fig. 4. Grid system definition of Gym 1.

Table 1. The heat and contaminant source per person [14]

	Athlete	Audience	
		non-smoker	smoker
Convective heat, W	30	22.5	22.5
Contaminant, l/h	30	15.0	75.0

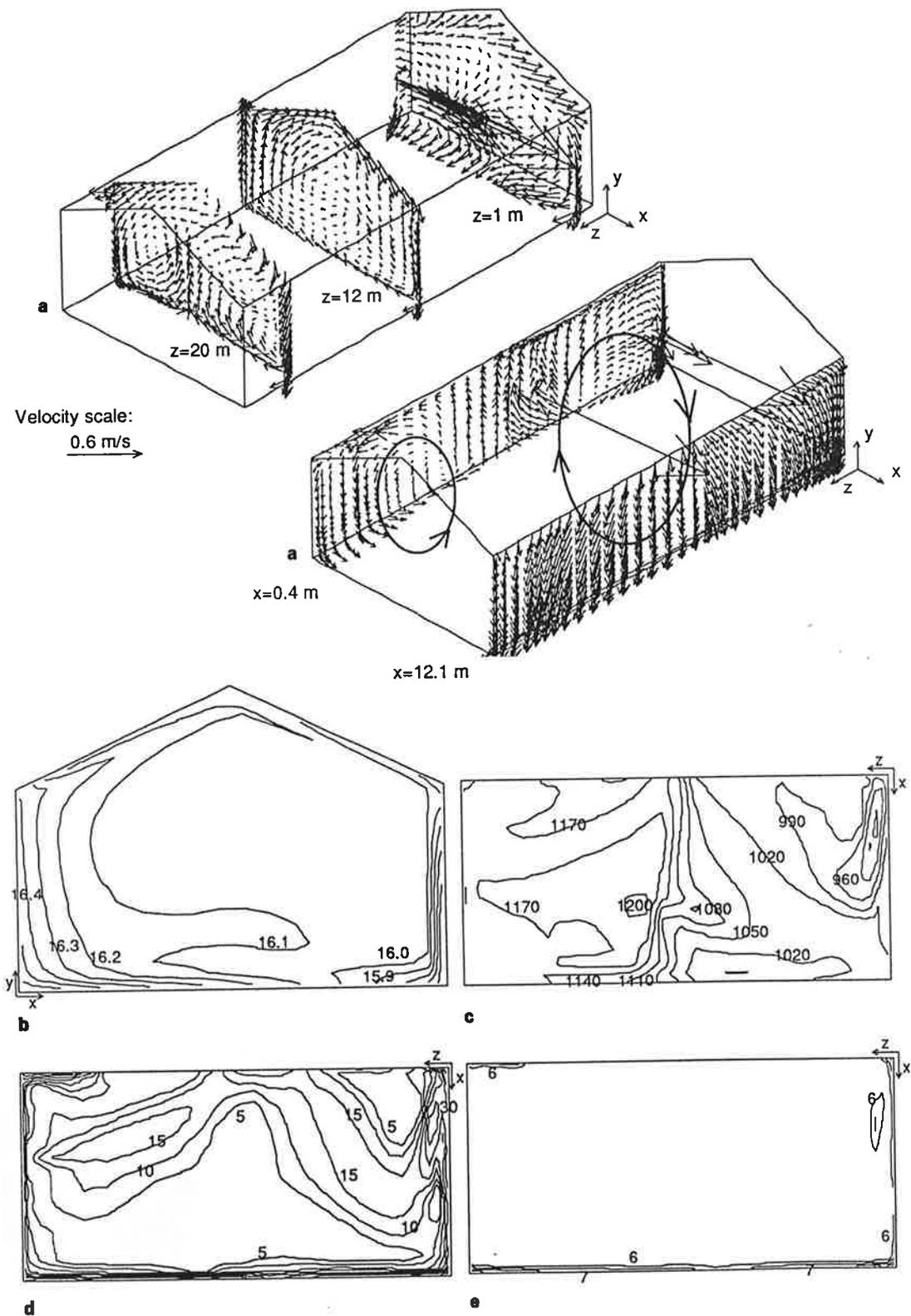
Table 2. Mesh system and CPU time

Case	Mesh system	CPU time/case	Sweep
1 <sup>1</sup> (Gym 1)	18×16×34	60 h on CONVEX <sup>2</sup>	3,600
2, 3, 4 (Gym 1)	18×22×34	33 h on CONVEX	2,000
5 (Gym 2)	29×17×32	5 h on CRAY <sup>3</sup>	6,000

<sup>1</sup> In this case body-fitting coordinates are used.

<sup>2</sup> CONVEX C120, about 4 MFLOPS with actual PHOENICS code.

<sup>3</sup> CRAY X-MP, about 50 MFLOPS with actual PHOENICS code.



**Fig. 5.** Field distributions in Case 1. **a** Velocity distribution in vertical sections. **b** Temperature in  $z = 12\text{ m}$ ,  $^{\circ}\text{C}$ . **c** Contaminant concentration in  $y = 1.4\text{ m}$ , ppm. **d** Percentage dissatisfied people due to draught in  $y = 1.4\text{ m}$ . **e** Predicted percentage of dissatisfied due to thermal comfort in  $y = 1.4\text{ m}$ ; metabolism = 2.5 met, clothing = 0.71 clo.

## Results

This section presents the computed field results for air velocity, temperature, contaminant concentration, percentage dissatisfied people due to draught, and predicted percentage of dissatisfied due to thermal comfort. Of the 18 cases computed, only five have been selected for the following discussion because these demonstrate some of the typical flow phenomena.

Figure 5 shows the results of Case 1 (the existing situation in Gym 1), where a floor heating system is used, and 2 inlets and 2 outlets are located on the long wall (fig. 3). The velocity distribution indicates that the cold window causes air to move downwards and the warm floor raises it. Two eddies are seen in the velocity vector fields. The eddy near the inlets is much larger than the one near the outlets. The maximum air velocity outside the supply air jets is about 0.25 m/s and occurs in the area near the window and the floor.

The temperature contours in the middle x-y plane imply that the temperature variations in this section are very small (less than 1 °C).

The profiles of contaminant concentration and percentage dissatisfied people due to draught at the cross-section located 1.4 m above the floor show that the region near inlets is clear but has a higher draught risk.

The predicted contours of percentage of dissatisfied due to thermal comfort indicate that the thermal conditions are excellent for dancing (metabolism = 2.5 met, clothing insulation = 0.71 clo) or other physical activities.

The results of Case 5 (the existing situation in Gym 2) are shown in figure 6. The figure only gives the results of the half symmetric hall. In this case, 14 inlets with diameter 0.08 m are located on the front wall, and two outlets at the ceiling. The radiators located on the rear wall are used for heating (fig. 3).

The air flow pattern may be deduced from the velocity distribution: there is a large eddy in the half symmetric hall which is driven by buoyancy and induced by inlets. Except for the area near the inlets, the maximum velocity is about 0.65 m/s which occurs near the radiators.

As in the smaller gymnasium (Case 1), in Case 5 the profiles of temperature, the contaminant concentration, the percentage dissatisfied people due to draught, and the predicted percentage of dissatisfied due to thermal comfort imply almost uniform mixing. Temperature differences in the occupied zone are less than 1.5 °C. The region near inlets is clear but has a higher draught. And the thermal conditions are excellent for dancing or other physical activities.

From the results of numerical simulation of the 18 cases, we conclude:

- (1) the air flow pattern depends mainly on the buoyancy induced by the cold window and the heating system;
- (2) the temperature distribution is very uniform;
- (3) the arrangement of inlet and outlet influences the contaminant concentration distribution, but the average concentration in the occupied zone is almost independent of it;
- (4) from the viewpoint of comfort, all cases are acceptable except for a small region near the inlets.

The analysis of the above conclusions follows in the next section.

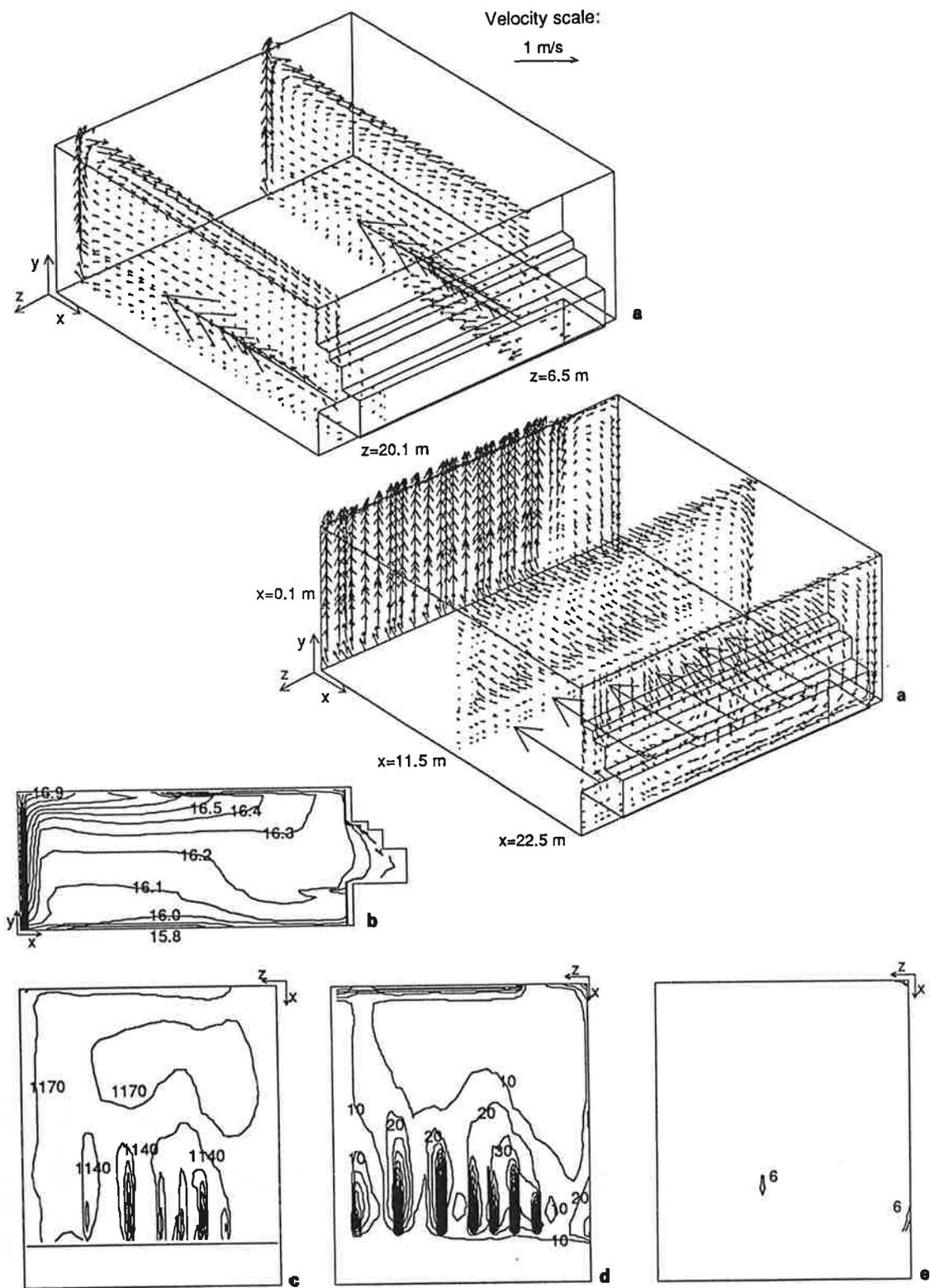
## Discussion

### *Some Features of Air Flows in Gymnasia*

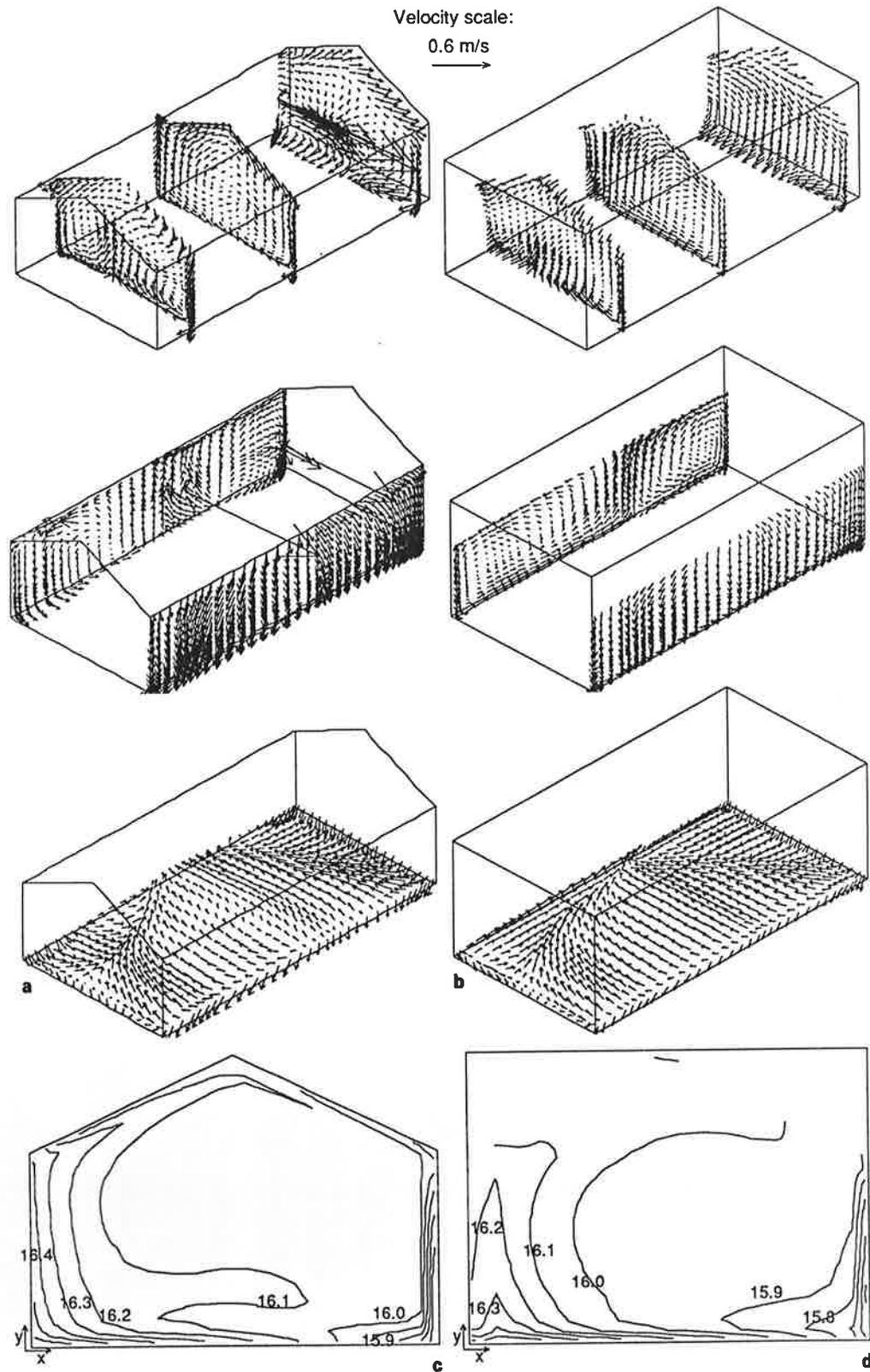
*Flow Pattern Depends on Buoyancy.* A gymnasium is a large enclosure, and the air flow in a gymnasium is a high Rayleigh number flow. For example, for Case 1,  $Ra = 3.2 \times 10^{11}$ . The height of the gymnasium is used as the characteristic length, and the characteristic temperature difference is taken between the surface temperatures of floor and window. For Case 5,  $Ra = 6.8 \times 10^{11}$ . The characteristic temperature difference here is the difference between the highest air temperature and the window surface temperature. Correspondingly, the Grashof numbers are  $4.5 \times 10^{11}$  and  $9.6 \times 10^{11}$  for Case 1 and Case 5, respectively.

In winter, the ventilation rate is very low in a gymnasium, so the inlet Reynolds number is also low. For Case 1,  $Re = 5.6 \times 10^4$  (the characteristic length is here taken  $L_{in}$ , as defined above), and for Case 5,  $Re = 1.7 \times 10^5$ .

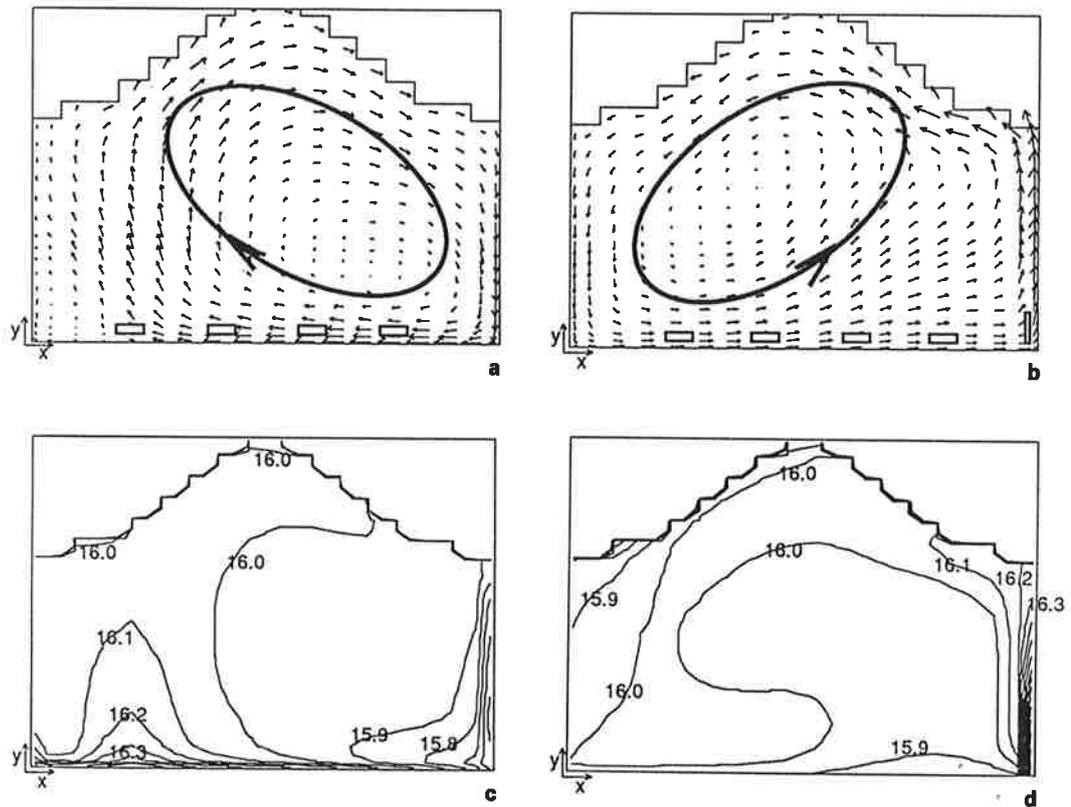
It can then be found that the proportion of  $Gr$  to  $Re^2$ , a relative measure of buoyant to inertial forces in the flow [16, 17] is relatively large; for Case 1,  $Gr/Re^2 = 144$ , and for Case 5,  $Gr/Re^2 = 34$ . The value is so large that the air flow in Case 1 can be considered to be governed by natural convection! There is a strong buoyancy effect in the flow. The comparison of the flow patterns between Case 1 and Case 2 (natural convection) (fig. 7) shows very little difference between them. In contrast, figure 8 gives completely different flow patterns between Cases 3 and 4. The two cases are identical except for the heating systems. So we can say that the flow pattern mainly depends on buoyancy and on the heating system.



**Fig. 6.** Field distributions in Case 5. **a** Velocity distribution. **b** Temperature in  $z = 10$  m,  $^{\circ}\text{C}$ . **c** Contaminant concentration in  $y = 2$  m, ppm. **d** Percentage dissatisfied people due to draught in  $y = 1.4$ . **e** Predicted percentage of dissatisfied due to thermal comfort in  $y = 1.4$  m; metabolism = 2.5 met, clothing = 0.71 clo.



**Fig. 7.** Comparison between mixed convection (Case 1; **a, c**) and natural convection (Case 2; **b, d**) for velocity distribution (**a, b**), and temperature in  $z = 12$  m, °C (**c, d**).



**Fig. 8.** Comparison between floor heating (Case 3; **a, c**) and radiator heating (Case 4; **b, d**) for velocity distribution (**a, b**) and temperature, °C (**c, d**) in  $z = 12$  m.

*Air Temperature Distribution Is Uniform.* In the two gymnasia considered, the temperature differences between walls and windows are less than 6 °C. In addition, the supply air temperature in most cases is the same as the average temperature of the room air. Therefore the air temperature distribution is almost uniform.

*The Averaged Contaminant Concentration in the Occupied Zone Is Independent of the Arrangement of Inlet and Outlet.* In gymnasia, usually, the contaminant source is located in the occupied zone (gases released by the occupants) and is modelled as a uniform source. Even if the so-called displacement ventilation system with fresh outside air is employed, at low ventilation rates more than 90% of the air passing through the occupied zone is recirculated within the room, rather than fresh air. Thus, the ventilation system does not perform as a displacement ventilation system should. The inlets can only influence very small regions. Therefore, the averaged contaminant

concentration in the occupied zone is almost independent of the arrangement of inlet and outlet as in a well-mixed ventilation system.

#### *Convergence Behaviour of Simulation of Air Flows in Large Enclosures*

Divergence is a common problem in numerical simulation. Under-relaxation with which the changes in dependent variables are slowed down in the iteration procedure is the most important technique to avoid divergence. But it is very difficult to use appropriately because convergence is very sensitive to the under-relaxation factors, or the size of the time step. (The calculation finds the steady state solution in progressing in different steps along a hypothetical time; of course these time steps have no real significance.) Sometimes, changing the time step by 10% will lead to different results.

It was also observed that the higher the Rayleigh number, the more difficult it was to get converged results. In addition, the complexity of flow and the proportion of the largest to the smallest grid distance also influence convergence.

Our experience shows that it is efficient to choose large time steps at the beginning and then to decrease their size as the number of sweeps increases.

## Conclusion

Numerical simulation has been used to calculate the air flows in 2 gymnasia for 18 cases. The numerical method is based on the  $\kappa$ - $\epsilon$  turbulence model. The air flows in this study have a high Rayleigh number, and the flow is mainly driven by buoyancy. The averaged contaminant concentration in the occupied zone is almost independent of the arrangement of inlet and outlet.

The numerical simulation of air flow in large enclosures is feasible, but needs much computing time and storage. Under-relaxation is the most important technique to avoid divergence. The convergence behaviour is sensitive to the size of time step. Multiple solutions were often observed in the numerical simulation of these cases.

## Acknowledgments

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## Appendix A: The Standard k- $\epsilon$ Turbulence Model

The governing differential equations of the standard k- $\epsilon$  turbulence model is expressed in the following form:

$$\frac{\partial}{\partial t}(\rho\phi) + \text{div}(\rho\vec{V}\phi) - \Gamma_{\phi}\text{grad}\phi = S_{\phi}$$

where  $\phi$  stands for velocity components (u, v, w), enthalpy (h), the kinetic energy of turbulence (k), the energy dissipation rate ( $\epsilon$ ), or contaminant concentration (C).  $\rho$ ,  $\vec{V}$ ,  $\Gamma_{\phi}$ , and  $S_{\phi}$  are density, velocity vector, diffusivity, and source for  $\phi$ , respectively.

## Appendix B: The Mathematical Model of PMV, PPD, and PD

(1) Fanger defined a subjective temperature sensation parameter, PMV, by the following formula [15]:

$$\begin{aligned} \text{PMV} = & (0.303e^{-0.036M} + 0.028)\{M - W - 3.05 \times 10^{-3}[5733 - \\ & 6.99(M - W) - P_a] - 0.42(M - W - 58.15) - 1.7 \times \\ & 10^{-5}M(5867 - P_a) - 0.0014M(34 - T) - 3.96 \times \\ & 10^{-8}f_{cl}[(T_{cl} + 273)^4 - (T_r + 273)^4] - f_{cl}h_c(T_{cl} - T)\} \end{aligned}$$

where M is metabolism [W/m<sup>2</sup>], W external work [W/m<sup>2</sup>], P<sub>a</sub> partial water vapour pressure [Pa], T air temperature [°C], T<sub>r</sub> mean radiant temperature [°C], and f<sub>cl</sub>, T<sub>cl</sub> and h<sub>c</sub> are determined by the following equations:

$$\begin{aligned} f_{cl} = 1.05 + 0.645I_{cl} & \quad \text{for } I_{cl} \geq 0.078, \\ f_{cl} = 1.00 + 1.290I_{cl} & \quad \text{for } I_{cl} < 0.078. \end{aligned}$$

$$T_{cl} = 35.7 - 0.028(M - W) - I_{cl}\{3.96 \times 10^{-8}f_{cl}[(T_{cl} + 273)^4 - (T_r + 273)^4] + f_{cl}h_c(T_{cl} - T)\}$$

$$\begin{aligned} h_c = 2.38(T_{cl} - T)^{0.25} & \quad \text{for } 2.38(T_{cl} - T)^{0.25} \geq 12.1\sqrt{V}, \\ h_c = 12.1\sqrt{V} & \quad \text{for } 2.38(T_{cl} - T)^{0.25} < 12.1\sqrt{V}. \end{aligned}$$

where I<sub>cl</sub> and V are clothing insulation [m<sup>2</sup>·°C/W] and air mean velocity [m/s], respectively.

(2) PPD can be calculated from:

$$\text{PPD} = 100 - 95e^{-(0.03353\text{PMV}^4 - 0.2179\text{PMV}^2)}$$

(3) The mathematical model of draught risk, PD, is expressed by [13]:

$$\text{PD} = (34 - T)(V - 0.05)^{0.62}(3.14 + 0.37VI) \quad [\%]$$

for V < 0.05 m/s insert V = 0.05 m/s, for PD > 100% use PD = 100%, where I is the turbulence intensity [%] which is defined as the velocity fluctuation over the mean velocity, i.e.,

$$I = 100(2k)^{0.5}/V \quad [\%]$$

Note that these equations depend on the system of units and are not dimensionally consistent.

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