

# IMPROVED WALL TREATMENTS IN TWO EQUATION FINITE ELEMENT MODELLING OF TURBULENT FLOW OVER CURVED BOUNDARIES

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## 1. INTRODUCTION

All high Reynolds number formulations of turbulence modelling require some form of special treatment in the region close to a solid wall. This treatment is normally based on the logarithmic law-of-the-wall, which in many finite element formulations is applied as a slip-velocity condition at a small distance from the wall with, say, zero gradient for  $k$  and a Dirichlet condition on  $\epsilon$ . As in finite volume procedures, various techniques have developed for applying these boundary conditions.

Betts and Haroutunian [1] found that, over a range of standard turbulent flows past smooth surfaces at laboratory scale, the computations converged more stably when the velocity wall-law was applied as a shear stress condition than as an iterated imposed slip velocity. However, when they extended their work to three-dimensional atmospheric flows over a rough ground of varying elevation [2], they encountered severe computational stability problems. Subsequent analysis has suggested that the cause of the instability was an interaction between the traction boundary condition on the horizontal velocity components and the pressure checkerboarding, which is endemic in the low-order linear velocity-constant pressure formulation. This interaction became unbounded when the mesh on the ground of fluctuating elevation became too coarse.

## 2. IDENTIFICATION OF THE CAUSE OF THE INSTABILITY

The finite element code FEMSET, developed at UMIST, uses an explicit time marching procedure such that velocity components parallel (or near-parallel) to the ground wall are imposed as traction conditions, while the normal velocity is set to zero. In the turbulent code, the laminar shear term  $\mu \partial u / \partial n$  is replaced by minus the wall shear stress, obtained from turbulent kinetic energy at the previous time step. The traction condition boundary integral then takes the form

$$-\int (wP \, dy + w \, \tau_w \, dx),$$

where  $w$  is the Galerkin weighting function and  $\tau_w$  the wall shear stress. With constant pressure elements, the pressure  $P$  automatically appears in the boundary integral, owing to the need to integrate pressure gradient terms by parts. This must also be obtained from the value at the earlier time.

For flow over horizontal ground, the pressure term is irrelevant since  $dy$  is zero. However as indicated on Fig.1, when ground level varies, wall pressure force can have a component in the  $x$  direction. Moreover with a coarse mesh and significant changes in gradient between adjacent elements, contributions from adjacent element boundaries can be significantly different. This provides a mechanism by which the checkerboard mode (described in the previous summary) can interact with the velocity boundary condition. When the elevation changes between adjacent nodes become sufficiently large, the CB mode appears to become unbounded so that the velocity solution is unstable.

## 3. OVERCOMING THE INSTABILITY

Tests have been undertaken on two-dimensional flows, which were found to be significantly more stable than the 3D fields where the effect was first noticed. Fig.2 shows the finite element mesh for a flow over a long hump; it is highly compressed vertically to allow for representation of the rapid variations of variables. The ground is considered rough and the upstream conditions are representative of atmospheric winds. Fig.3 shows pressure variations along the lower layers of elements after 500 time steps, and it can be seen that successive layers are out of phase, indicating a checkerboard mode, which continues to grow with time.

The obvious way to overcome the problem was to apply the CB suppressor, described previously. Fig.4 shows the results of this after 1000 time steps, where the residual CB effect is small and not growing with time. The remaining in-phase wiggles in pressure are caused by the absence of upwinding in this 2D code. When the change in ground elevation was increased to 1.8m, the unsuppressed computation diverged rapidly, while the CB suppressor produced stable convergence.

Attempts are also being made to develop a special wall element to span the gap between the bottom nodes and the rough wall. This would be similar in kind to that used for smooth walls in FIDAP, and would allow Dirichlet conditions to be imposed on all velocity components, thus avoiding any reference to pressure in the ground boundary conditions.

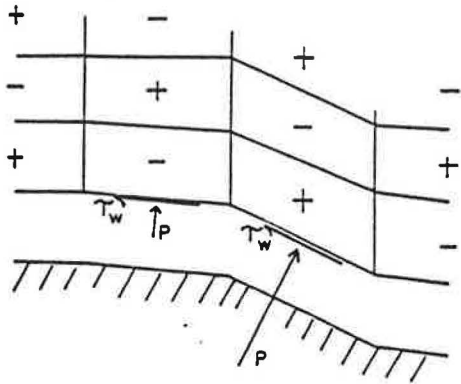


Fig. 1: Boundary forces and CB mode (dy negative)

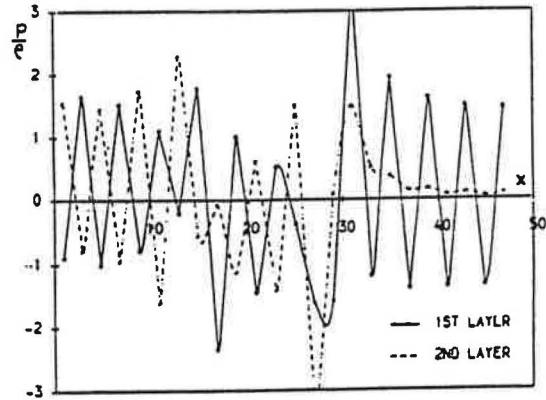


Fig. 3: Pressures in bottom layers of elements, after 500 steps, no CB suppressor

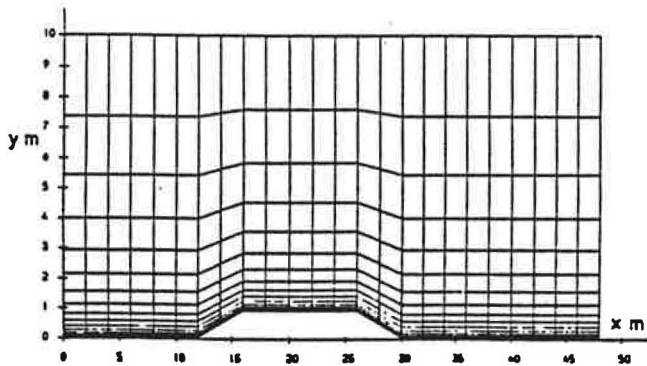


Fig. 2: Element mesh for wind over long hump

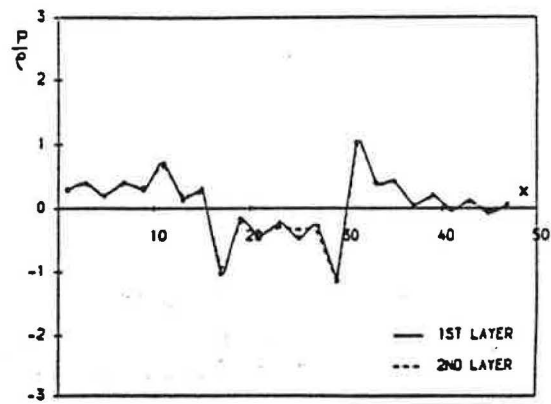


Fig. 4: Pressures in bottom layers of elements after 1000 steps, CB suppressor with  $\beta=100$

## REFERENCES

1. Haroutunian V H, "A time-dependant finite element model for atmospheric dispersion of gases heavier than air", Ph.D. Thesis, UMIST, Manchester 1987.
2. Betts P L and Haroutunian V H, "Finite element calculation of transient dense gas dispersion", in Stably Stratified Flow and Dense Gas Dispersion, ed. J S Puttock, IMA New Series Vol. 15, Oxford U.P., 349-384, 1988.